Tracking Differentiator via Time Criterion

Hehong Zhang[1], Yunde Xie[2], Gaoxi Xiao[3], Chao Zhai[3], Huangwei Kang[2] and Junqing Tang[4]

Abstract— A discrete-time tracking differentiator (TD) based on a time criterion is presented. Its control law is derived by comparing the time that the initial state is driven to the switching curve or the origin with the sampling period. The advantages of this TD include faster signal-tracking, better noise filtering and derivative extraction. Simulation results show that this TD has smaller errors in signal-tracking and derivative extraction compared with the existing differentiators. Experiments are carried out on the gap sensor of the suspension control system in a magnetic-levitation (maglev) train to demonstrate that the proposed TD can obtain an effective velocity signal from the gap sensor when the accelerometer fails.

Index Terms— Tracking differentiator, time criterion, discrete time, gap sensor, maglev train.

I. INTRODUCTION

The differentiation of a given signal in real time is a well-known yet challenging problem in control engineering and theory [1], [2]. The proportional-integral-derivative (PID) control law developed in the last century still plays an essential role in modern control-engineering practice [3], [4]. However, derivative signals are prone to the noise pollution and derivative controls usually cannot be physically implemented, causing the PID to be degraded to PI control [5], [6]. For this reason, much effort has been devoted to the problem of designing differentiator, such as high-gain observer-based differentiator [7], linear time-derivative tracker [8], super-twisting second-order sliding-mode algorithm [9], robust exact differentiator [10], and finite time-convergent differentiator [11], [12], among others [13]-[15].

A noise-tolerant time-optimal system-based tracking differentiator (TD) was first proposed by J.Q. Han [16], [17]. The advantage of this TD is that it sets a weak condition on the stability of the systems to be constructed for TD and requires a weak condition on the input. In addition, it has advantageous smoothness compared with the obvious chattering problem encountered by sliding-mode-based differentiators [18]. Han used this TD as an important part of the emerging active disturbance rejection control (ADRC)

[1]Hehong Zhang is with Interdisciplinary Graduate School, Nanyang Technological University, Singapore 639798. He was also with College of Mechatronics Engineering and Automation, National University of Defense Technology, Changsha, Hunan 410073, China HZHANG030@e.ntu.edu.sg

[2]Yunde Xie and Huangwei Kang are with Beijing Enterprises Holding Maglev Technology Development Company Limited, Beijing 100124, China xieyunde@outlook.com; huangweikang@maglev.cn

[3]Gaoxi. Xiao and Chao. Zhai are with School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore 639798 EGXXiao@ntu.edu.sg; zhaichao@ntu.edu.sg

[4]Junqing Tang is with ETH Zurich, Future Resilient Systems, Singapore-ETH Centre, 1 CREATE WAY, CREATE Tower, 138602 Singapore junqing.tang@frs.ethz.ch

[18]. He also presented a nonlinear PID control based on a TD [19]. Therein, the TD acted not only as the derivative extraction, but also as a transient profile that the output of a plant could reasonably follow to avoid the setpoint jump in PID. However, to achieve high accuracy of Han's TD in signal-tracking and derivative extraction requests imposing strict constraints on the sampling period. Furthermore, the control law (denoted as $Fhan$) of the TD is determined by comparing the location of the initial state with the isochronic region that is obtained through complicated nonlinear boundary transformation [17].

In this paper, we propose a discrete TD that has characteristics of faster signal-tacking, greater accuracy in derivative extraction and better noise filtering. In addition, the control law is obtained easily by comparing the time that the initial state sequence is driven to the switching curve or the origin with the sampling period.

The remainder of the paper is organized as follows: in Section II, the control law of the TD via a time criterion is presented in detail. In Section III, the structure of the TD and its filtering characteristic are discussed. In Section IV, numerical simulation results are presented to compare the performance of signal-tracking and derivative extraction for the proposed TD versus the existing ones. Experiments are carried out to acquire effective velocity signals from the gap sensor for the suspension system in the maglev train when the accelerometer fails. In Section V, the conclusions and remarks are given.

II. CONTROL LAW FOR TRACKING DIFFERENTIATOR

Consider a standard discrete-time double-integral system as follows

$$x(k+1) = Ax(k) + Bu(k), |u(k)| \leq 1 \qquad (1)$$

where $A = \begin{pmatrix} 1 & h \\ 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 \\ h \end{pmatrix}$. The goal is to derive a high-precision feedback-control law (denoted as Ftd) based on a time criterion in discrete time domain, which is described as follows:

Control Objective: Given the system (1) and its initial state $x(0)$, determine the control signal sequence, $u(0)$, $u(1)$, ..., $u(k)$ by comparing the time that the initial state point is driven back to the switching curve or the origin with the sampling period h such that the state $x(k)$ is driven back to the origin in a finite number of steps, subject to the constraint of $|u(k)| \leq 1$.

The time that any initial state point $M(x_1(0), x_2(0))$ is driven back to the switching curve ($\Gamma(x_1, x_2) = x_1 + \frac{x_2|x_2|}{2}$ [20],

[21]) is denoted as t_A, and the time that the state point located on the switching curve is driven back to the origin is denoted as t_B. We can then determine that $t_A = sx_2(0) + \sqrt{sx_1(0) + \frac{1}{2}x_2^2(0)}$ and $t_B = |x_2(0)|$, where $s = -\text{sign}(x_1(0) + \frac{1}{2}x_2(0)|x_2(0)|)$. The work of determining the control signal sequence that drives any initial state to the origin can be divided into the following two tasks:

Task 1: Determine the control signal sequence when the initial state point $M(x_1(0), x_2(0))$ is not located on the switching curve by comparing the time t_A and the sampling period h.

Task 2: Determine the control signal sequence when the state point $M(x_1(0), x_2(0))$ is located on the switching curve by comparing the time t_B and the sampling period h.

For **Task 1**, when $h \leq t_A$, the control signal is $u = -s$; otherwise, the control signal value should be decreased to guarantee that the state point $M(x_1(0), x_2(0))$ can be driven to the switching curve Γ within the sampling time of h. When the state point is located above the Γ, there exists $s = 1$ and the control signal takes on $u = -u_a$. u_a satisfies the following equations:

$$
\begin{cases}
x_1 = x_1(0) - \frac{1}{2u_a}(x_2^2 - x_2^2(0)) \\
x_2 = x_2(0) - u_a h
\end{cases}
\tag{2}
$$

There exists $x_1 = \frac{1}{2}x_2^2$ when the initial state point is driven back to the switch curve by the corresponding control signal sequence. If u_a is taken as an unknown, then

$$
\frac{1}{2}h^2 u_a^2 + (\frac{1}{2}h^2 - hx_2(0))u_a + \frac{1}{2}x_2^2(0) - x_2(0)h - x_1(0) = 0.
\tag{3}
$$

The discriminant of (3) is

$$
\begin{aligned}
\triangle &= (\frac{1}{2}h^2 - hx_2(0))^2 - 2h^2(\frac{1}{2}x_2^2(0) - x_2(0)h - x_1(0)) \\
&= \frac{1}{4}h^4 + h^3 x_2(0) + 2h^2 x_1(0).
\end{aligned}
$$

Two cases are possible.

1) When $x_2(0) \geq 0$, the discriminant satisfies the condition that

$$
\begin{aligned}
\triangle &> \frac{1}{4}h^4 + 2h^2(x_1(0) + \frac{1}{2}x_2^2(0)) \\
&= \frac{1}{4}h^4 + 2h^2(x_1(0) + \frac{1}{2}x_2(0)|x_2(0)|) > \frac{1}{4}h^4 > 0;
\end{aligned}
$$

2) When $x_2(0) < 0$, $x_1(0) - \frac{1}{2}x_2^2(0) > 0$ can be derived because $x_1(0) + \frac{1}{2}x_2(0)|x_2(0)| > 0$. Therefore,

$$
\begin{aligned}
\triangle &= \frac{1}{4}h^4 + h^3 x_2(0) + 2h^2 x_1(0) \\
&> \frac{1}{4}h^4 + h^3 x_2(0) + 2h^2 \frac{1}{2}x_2^2(0) = h^2(x_2(0) + \frac{1}{2}h)^2 \geq 0.
\end{aligned}
$$

According to two cases analysed above, the discriminant can always satisfy $\triangle > 0$. Furthermore, there exists two unequal real-number roots that satisfy $x_2(h) < 0$. Because $x_2(h) =$

$x_2(0) - u_a h = \frac{h}{2} \pm \frac{\sqrt{\triangle}}{h} < 0$, the positive root shall be excluded and the expression of u_a can be obtained as follows:

$$
u_a = -\frac{1}{2} + \frac{1}{h}x_2(0) + \frac{1}{2}\sqrt{1 + \frac{4}{h}x_2(0) + \frac{8}{h^2}x_1(0)}.
\tag{4}
$$

Similarly, when the state point M is located below Γ where $s = -1$, the control input is $u = u_a$. u_a can be derived as follows:

$$
u_a = -\frac{1}{2} - \frac{1}{h}x_2(0) + \frac{1}{2}\sqrt{1 - \frac{4}{h}x_2(0) - \frac{8}{h^2}x_1(0)}.
\tag{5}
$$

The resulting expression of u_a is

$$
u_a = -\frac{1}{2} - \frac{s}{h}x_2(0) + \frac{1}{2}\sqrt{1 + (\frac{4}{h}x_2(0) - \frac{8}{h^2}x_1(0))s}
\tag{6}
$$

where the parameter s has the same definition as described in the previous section.

For **Task 2**, when the state point M is located on the switching curve Γ, it satisfies the condition

$$
\begin{cases}
x_1(t) = \frac{1}{2}x_2^2(t) \\
x_2(t) = x_2(0) + ut = 0, t = h.
\end{cases}
\tag{7}
$$

When $h \leq t_B$ ($t_B = |x_2(0)|$), the control law is $u = -\text{sign}(x_2(0))$. However, when $t > t_B$, the control signal value should be decreased to guarantee that the state point M can be driven back to the origin in one step. Therefore, the control signal cannot be the constant value but must satisfy the following condition:

$$
\begin{cases}
x_1(t) = x_1(0) + x_2(0)t + \int_0^t \int_0^\tau u(\sigma)d\sigma d\tau = 0 \\
x_2(t) = x_2(0) + \int_0^t u(\tau)d\tau = 0, t = h.
\end{cases}
\tag{8}
$$

For simplicity, supposing $u = a + bt$, and substituting it into (8), we have

$$
\begin{cases}
x_2(0) + ah + \frac{1}{2}bh^2 = 0 \\
x_1(0) + x_2(0)h + \frac{1}{2}ah^2 + \frac{1}{6}bh^3 = 0.
\end{cases}
\tag{9}
$$

This leads to

$$
\begin{cases}
a = -\frac{2}{h^2}(2x_2(0)h + 3x_1(0)) \\
b = \frac{6}{h^3}(x_2(0)h + 2x_1(0)).
\end{cases}
\tag{10}
$$

From the above, we can obtain the control law based on a time criterion for a standard discrete-time double-integral system (1). This is denoted here as $u(k) = Ftd(x_1(k), x_2(k), r, h)$, where the parameters r and h have the same definitions as described in the previous section. The resulting control law is

$$
\begin{cases}
u(k) = Ftd(x_1(k), x_2(k), r, h) \\
x_1(k+1) = x_1(k) + hx_2(k) + \frac{1}{2}u(k)h^2 - \frac{1}{3}bh^3 \\
x_2(k+1) = x_2(k) + u(k)h - \frac{1}{2}bh^3.
\end{cases}
\tag{11}
$$

For a general discrete time double-integral system, one can obtain its control law *Ftd* by substituting the x_1 and x_2 with z_1 and z_2 respectively, where $z_1 = \frac{x_1}{r}$ and $z_2 = \frac{x_2}{r}$.

Based on the control law above, the following TD can be constructed:

$$\begin{cases} u(k) = Ftd(x_1(k) - v(k), c_1 x_2(k), r_0, c_0 h) \\ \\ x_1(k+1) = x_1(k) + h x_2(k) + \frac{1}{2} u(k) h^2 - \frac{1}{3} b h^3 \quad (12) \\ \\ x_2(k+1) = x_2(k) + u(k) h - \frac{1}{2} b h^3. \end{cases}$$

where r_0 is the quickness factor, c_1 is the damping factor, c_0 is the filtering factor, and v is the given signal.

III. STRUCTURE ANALYSIS OF DISCRETE TIME TRACKING DIFFERENTIATOR

For a given signal sequence $v(k)(k = 0, 1, 2, ...)$, the discrete-time TD in (12) can be transformed into an approximately linear one by taking on the proper parameter r, as follows:

$$x(k+1) = Gx(k) + Hv(k), k = 0, 1, 2... \quad (13)$$

where $x(k) = [x_1(k), x_2(k)]^T$, $H = [\frac{1}{c_0^2}, \frac{2}{c_0^2 h}]^T$ and $G = [G_1, G_2]^T$, where $G_1 = [1 - \frac{1}{c_0^2}, (1 - \frac{c_1}{c_0^2})h]$ and $G_2 = [-\frac{2}{c_0^2 h}, 1 - \frac{2c_1}{c_0}]$.

We assume that the given signal is $v(t) = \sum_{i=1}^{N} A_i e^{j(w_i t + \phi_{i0})} + \xi(t)$, where $A_i, w_i \in \mathbb{R}^+$, $\phi_{i0} \in \mathbb{R}$, and $\xi(t)$ is a width-steady process. Then we have

$$x(k) = G^k p_0 + \sum_{i=1}^{N} (e^{j w_i h} I_2 - G)^{-1} H A_i e^{j(w_i kh + \phi_{i0})} + \eta(k), \quad (14)$$

where $\eta(k+1) = G\eta(k) + H\xi(k), k = 0, 1, 2, ...$, and p_0 is determined by the initial conditions $x(0)$ and $\eta(0)$. The sufficient and necessary condition for convergence of (14) is that the spectral radius of matrix G satisfies $\rho(G) < 1$. It can be derived that $x_1(k) = CG^k p_0 + \sum_{i=1}^{N} C(e^{j w_i h} I_2 - G)^{-1} H A_i e^{j(w_i kh + \phi_{i0})} + C\eta(k)$ by choosing $C = [1, 0]$. When the transfer function of the discrete-time system is denoted as $\Phi(z) = C(zI_2 - G)^{-1}H$, $x_1(k)$ can be expressed as follows

$$x_1(k) = CG^k p_0 + \sum_{i=1}^{N} \Phi(e^{j w_i h}) A_i e^{j(w_i kh + \phi_{i0})} + C\eta(k). \quad (15)$$

The characteristics of magnitude-frequency and phase-frequency for the TD in (12) are analysed in the followings. Given a sine signal $v(t) = Ae^{j(wt + \phi_0)}$, the tracking output signal y_{out} is also a sine signal when the particular signal sequence is long enough and the spectral radius of matrix G satisfies $\rho(G) < 1$. We suppose that the output signal is $y_{out} = \beta v(t - \tau_0)$, where β is the dynamic amplifier factor and τ_0 is the time delay. By choosing the filtering factor c_0 to satisfy the condition $c_0 wh \ll 1$, we have

$$\begin{cases} \beta = \frac{1}{1 + 0.5 c_0^2 (c_1^2 - 1) w^2 h^2} \\ \tau_0 = (c_0 c_1 - 1)h \end{cases} \quad (16)$$

Furthermore, when the damping factor $c_1 = 1$, we have $\beta = 1$ and $\tau_0 = (c_0 - 1)h$. Based on the conditions above, the characteristics of magnitude-frequency and phase-frequency are shown in Fig. 1. We can see that the proposed tracking differentiator has good filtering ability when the filtering factor c_0 is properly selected.
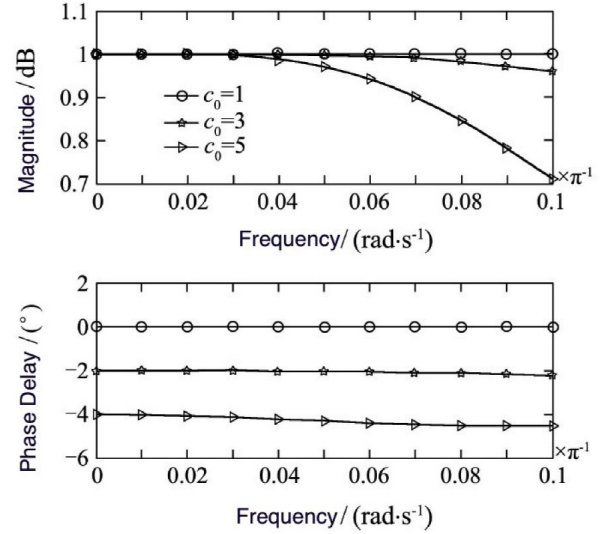

Fig. 1. Characteristic curves of magnitude-frequency and phase-frequency for TD.

The following path is taken to compare the filtering characteristics between the discrete-time TD *Ftd* and *Fhan* [16], [17]. Here, only the width-steady random process is considered. For a discrete-time linear-tracking differentiator, width-steady input leads to width-steady output. When the random input sequence $\xi(t)$ is a white noise sequence, $R_\xi = Q\delta(\tau)$, where $\delta(\tau)$ is the *Kronecker* delta function [22], [23] and Q is the constant matrix. The variance matrix satisfies the equation $R_\eta(k+1) = GR_\eta(k)G^T + HQH^T$. When the constant k is large enough, $R_\eta(k)$ converges to the constant matrix, i.e.,

$$R_\eta = GR_\eta G^T + HQH^T. \quad (17)$$

The above equation is a Lyapunov function of a discrete-time system, demonstrating the relationship between the output sequence's variance R and the filtering factor c_0. For the *Fhan* algorithm, the matrices G and H are

$$G = \begin{pmatrix} 1 & h \\ \frac{1}{c_0 h} & 1 - \frac{2}{c_0} \end{pmatrix}, H = \begin{pmatrix} 0 \\ \frac{1}{c_0^2 h} \end{pmatrix}$$

respectively. By assuming that the density of the white noise power spectrum is $Q = 1$, the result shown in Fig. 2 is achieved.

As shown in Fig. 2, the tracking differentiator can filter random noises when the proper filtering factor c_0 is selected. The filtering capacity is better for the *Ftd* algorithm than for the *Fhan* algorithm.
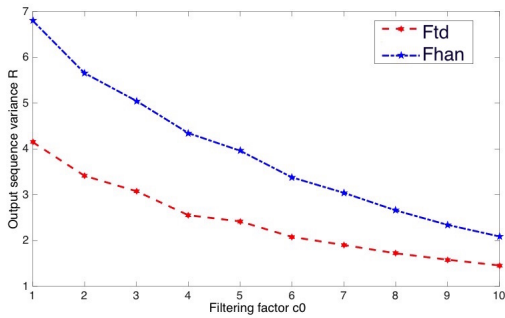
Fig. 2. Output sequence variance R vs filtering factor c_0 for two algorithms.
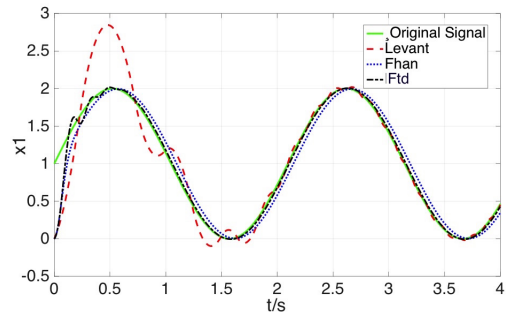

Fig. 3. Comparison result in signal-tracking with three different differentiators.

Fig. 4. Comparison result in differentiation acquisition with three different differentiators.

IV. NUMERICAL SIMULATION AND EXPERIMENT RESULTS

The numerical simulations are conducted to compare the proposed differentiator versus the existing ones in signal-tracking and differentiation acquisition. Experiments are also carried out on the gap sensor of the suspension system of a maglev train to determine whether the proposed TD can acquire an effective velocity signal from the gap sensor when an accelerometer fails.

A. Numerical Simulation

In this subsection, we present some numerical simulation results to compare the following three differentiators.

DI. Robust exact differentiator using sliding mode technique from [10].

$$\begin{cases} \dot{x}_1 = x_2 - \alpha |x_1 - v|^{0.5}\text{sign}(x_1 - v), \\ \dot{x}_2 = -\beta\,\text{sign}(x_1 - v) \end{cases}$$

DII. Tracking differentiator based on discrete time optimal control $Fhan$ from [16], [17].

$$\begin{cases} u(k) = Fhan(x_1(k) - v(k), x_2(k), r_0, c_0 h), \\ x_1(k+1) = x_1(k) + h x_2(k), \\ x_2(k+1) = x_2(k) + h u(k), |u(k)| \leq r \end{cases}$$

DIII. Tracking differentiator based on discrete time optimal control Ftd proposed in this letter.

$$\begin{cases} u(k) = Ftd(x_1(k) - v(k), c_1 x_2(k), r_0, c_0 h), \\ x_1(k+1) = x_1(k) + h x_2(k) + \frac{1}{2}u(k)h^2 - \frac{1}{3}bh^3, \\ x_2(k+1) = x_2(k) + u(k)h - \frac{1}{2}bh^3. \end{cases}$$

The Matlab program of Euler method is adopted in investigation. We choose the same initial value $(x_1(0) = 0, x_2(0) = 2)$ and the input signal sequence $v(t) = 1 + \sin(3t) + \gamma(t)$ in all simulations, where $\gamma(t)$ is the evenly distributed white noise with an intensity of 0.001. For differentiator **DI**, the parameters are $\alpha = 1.5$, $\beta = 36$; for differentiator **DII** and differentiator **DIII**, the sampling step is $h = 0.005$, the quickness factor is $r_0 = 200$, the damping factor is $c_1 = 2$, and the filtering factor is $c_0 = 5$. As mentioned in **proposition 2.1**, the control signal in differentiator **DIII** satisfies $|u(k)| \leq r$. The comparison results in signal-tracking and differentiation

acquisition for three differentiators are plotted in Figs. 3 and 4.

From above simulation results, we see that tracking differentiator **DII** and differentiator **DIII** based on discrete time optimal control are smoother than differentiator **DI** in which the discontinuous function produces chattering problem. From the perspectives of static errors and phase delays, differentiator **DIII** performs better in signal tracking and differentiation acquisition than differentiator **DII**.

B. Experiment Results

The experiments are intended to evaluate the practical engineering application of the proposed tracking differentiator during the operation of a maglev train. The experimental platform is shown in Fig. 5, where the maglev train adopts the real-time development environment RTW toolbox supported by Matlab software. The integrated electromechanical system of this train comprises the vehicle structure, the bogie, the track, and the suspension system [24]-[26]. The three electromagnets of the suspension system are regulated by three sets of controllers. The platform can carry out the single-point or multi-point modularity suspension control experiments because three suspension points are mechanically decoupled.

The suspension sensor group collects the gap, acceleration, and current signals from the suspension system. This closed-loop feedback-control scheme utilizes the PID control law [27], for which the velocity signal comes from the integral of the acceleration signal. Without that velocity signal, the suspension system cannot guarantee suspension stability. However, in a practical engineering scenario, the

Fig. 5. Experimental platform of the maglev train.

accelerometer is more likely to fail because of poor operating conditions. Furthermore, redundant technology is not used for the accelerometer because it is too expensive. In our experiments, the proposed TD is proposed to track the gap signal and produce the velocity signal if an accelerometer fails.

The A/D module is set with a sampling frequency of $F_s = 2kHz$ to collect the gap signals and acceleration signals when the train is moving with load fluctuation and train body might not be stabilized at $3mm$. The proposed TD is used to track the gap signals and produce the corresponding velocity signals, where the filtering factor is $c_0 = 8$, the damping factor is $c_1 = 2$ and the quickness factor is $r = 650$. The comparative results are presented in Fig. 6. As
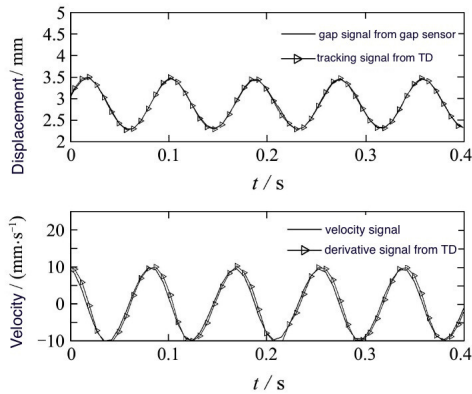

Fig. 6. Comparison between tracking and derivative signal obtained by TD and gap and velocity signal collected by sensors.

shown in Fig. 6, the proposed TD is able to track the gap signals quickly with small tracking errors. It also produces the desired velocity signals with only a small phase delay. Therefore, if the accelerometer fails, we believe that the proposed TD can acquire the velocity signals effectively with the gap signals.

V. CONCLUSIONS

We proposed a new and simple control law that is effective for applications in discrete-time, double-integral systems.

This algorithm enabled us to develop a novel tracking differentiator. Numerical simulation results demonstrated that, compared with the existing differentiators, the proposed noise-tolerant TD has faster signal-tracking, and better noise filtering and derivative extraction. Experiment results showed that the proposed noise-tolerant TD can produce effective velocity signals for the suspension system of a maglev train when the accelerometer fails. Note that the utility of the TD is not limited to signal-tracking and differentiation acquisition, it can also be adopted in constructing controllers. Future work will include analysing the accuracy of this tracking differentiator and further exploring its practical engineering applications.

REFERENCES

[1] Davila, Jorge, Leonid Fridman, and Arie Levant, "Second-order sliding-mode observer for mechanical systems," *IEEE Trans. Autom. Control*, vol. 50, no. 11, pp. 1785-1789, 2005.
[2] Zhai Chao, and Yiguang Hong, "Decentralized sweep coverage algorithm for uncertain region of multi-agent systems," *American Control Conference (ACC)*, 2012.
[3] Astrom, Karl Johan, and Tore Hagglund, "The future of PID control," *Control Eng. Pract.*, vol. 9, no. 11, pp. 1163-1175, 2001.
[4] Silva, Guillermo J., Aniruddha Datta, and Shankar P. Bhattacharyya, "New results on the synthesis of PID controllers," *IEEE Trans. Autom. Control*, vol. 47, no. 2, pp. 241-252, 2002.
[5] Sun, Li, Donghai Li, and Kwang Y. Lee, "Enhanced decentralized PI control for fluidized bed combustor via advanced disturbance observer," *Control Eng. Pract.*, vol. 42, pp. 128-139, 2015.
[6] Silva, Guillermo J., Aniruddha Datta, and Shankar P. Bhattacharyya, "PI stabilization of first-order systems with time delay," *Automatica*, vol. 37, no. 12, pp. 2025-2031, 2001.
[7] Deza, F., et al, "High gain estimation for nonlinear systems," *Syst Control Lett*, vol. 18, no. 4, pp. 295-299, 1992.
[8] Ibrir, Salim, "Linear time-derivative trackers," *Automatica*, vol. 40, no. 3, pp. 397-405, 2004.
[9] Gonzalez, Tenoch, Jaime A. Moreno, and Leonid Fridman, "Variable gain super-twisting sliding mode control," *IEEE Trans. Autom. Control*, vol. 57, no. 8, pp. 2100-2105, 2012.
[10] Levant, Arie, "Robust exact differentiation via sliding mode technique," *Automatica*, vol. 34, no. 3, pp. 379-384, 1998.
[11] Levant, Arie, "Non-homogeneous finite-time-convergent differentiator," *CDC/CCC 2009. Proceedings of the 48th IEEE Conference on. IEEE, 2009*, 2009.
[12] Wang, Xinhua, Zengqiang Chen, and Geng Yang, "Finite-time-convergent differentiator based on singular perturbation technique," *IEEE Trans. Autom. Control*, vol. 52, no. 9, pp. 1731-1737, 2007.
[13] Alwi, Halim, and Christopher Edwards, "An adaptive sliding mode differentiator for actuator oscillatory failure case reconstruction," *Automatica*, vol. 49, no. 2, pp. 642-651, 2013.
[14] Polyakov, Andrey, Denis Efimov, and Wilfrid Perruquetti, "Homogeneous differentiator design using implicit Lyapunov function method," *Control Conference (ECC), 2014 European. IEEE, 2014*, 2014.
[15] Shtessel, Yuri, et al, "Higher-order sliding mode controllers and differentiators," *Sliding Mode Control and Observation*. Springer New York, 2014.

[16] Han, Jingqing, "From PID to active disturbance rejection control," *IEEE Trans. Ind. Electron.*, vol. 56, no. 3, pp. 1731-1737, 2009.

[17] Hehong Zhang, Yunde Xie, Gaoxi Xiao, and Chai Zhai, "Closed-Form Solution of Discrete Time Optimal Control and Its Convergence," *IET Control Theory Appl.*, November 2017, DOI10.1049/iet-cta.2017.0749.

[18] Guo, Bao-Zhu, and Zhi-Liang Zhao, "Active Disturbance Rejection Control for Nonlinear Systems: An Introduction," *John Wiley and Sons*, 2016.

[19] Han, Jingqing, "Nonlinear PID controller," *Acta Automatica Sinica*, vol. 20, no. 4, pp. 487-490, 1994.

[20] Zhou, Kemin, John Comstock Doyle, and Keith Glover, "Robust and optimal control," *New Jersey: Prentice hall*, vol. 40, 1996.

[21] Bellman, Richard, Irving Glicksberg, and Oliver Gross, "On the bang-bang control problem," *Quarterly of Applied Mathematics*, vol. 14, no. 1, pp. 11-18, 1956.

[22] Fontana, Marco, and K. Alan Loper, "Kronecker function rings: a general approach," *Lecture Notes in Pure and Applied Mathematics*, pp. 189-206, 2001.

[23] Zoubir, Abdelhak M., and B. Boashash, "The bootstrap and its application in signal processing," *IEEE Signal Proc Mag.*, vol. 15, no. 1, pp. 56-76, 1998.

[24] Lee, Hyung-Woo, Ki-Chan Kim, and Ju Lee, "Review of maglev train technologies," *IEEE Trans. Magn.*, vol. 42, no. 7, pp. 1917-1925, 2006.

[25] Hehong Zhang, Yunde Xie, and Zhiqiang Long, "Fault detection based on tracking differentiator applied on the suspension system of maglev train," *Mathematical Problems in Engineering*, 2015.

[26] Hehong Zhang, and Zhiqiang Long, "Control of high-precision magnetic levitation positioning stage," *Journal of Engineering of Heilongjiang University*, (2015): 017.

[27] Liu, Hengkun, Xiao Zhang, and Wensen Chang, "PID control to maglev train system," *Industrial and Information Systems, 2009. IIS'09. International Conference on. IEEE, 2009*, 2009.