Algorithms for Finding Best Locations of Cluster Heads for Minimizing Energy Consumptions in Wireless Sensor Networks

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Abstract

Clustering is a widely adopted energy-saving technique in wireless sensor networks (WSNs). In this paper, we study algorithms for finding the best locations of cluster heads in WSNs to minimize the overall energy consumption. Specifically, based on the assumption that the global information of all the sensors' locations or location distribution is available, algorithms are proposed for finding 1) the best location of the cluster head in a single given cluster; 2) the best formation of a given number of clusters where each cluster head has to communicate with base station directly; and 3) the best formation of a given number of clusters where there can be ad-hoc transmission between cluster heads, respectively. For each case, algorithms are designed for free-space and multipath energy consumption models respectively. Theoretical analysis and extensive simulation results show that the proposed algorithms are fast and can steadily achieve satisfactory results. The calculation results of the proposed algorithms provide a useful benchmark for evaluating various local information-based distributed clustering schemes or schemes based on partial or inaccurate global information.

Keywords: Wireless sensor networks; clustering algorithms; energy efficiency, free-space model, multipath model.

1. Introduction

With the developments of low power and multi-functional sensors, wireless sensor networks (WSNs) are becoming increasingly important in research and applications. WSNs composed of sensors with ability of sensing, data processing and communication have paved the way for wide applications in monitoring, tracking and control, etc.

As sensor nodes have a limited battery power supply and usually cannot be readily replaced or recharged, a main concern in deploying WSNs is to achieve high energy efficiency. Energy consumption is a critical constraint that imposes restrictions on lifetime, communication range and data rate in WSNs.

Clustering is one of the promising techniques [1, 2] for lowering energy consumptions and hence prolonging network lifetime. In a clustered WSN, sensor nodes are partitioned into a certain number of clusters, each of which consists of a cluster head (CH) and non-cluster head members (non-CHs). CH collects information from all the cluster members and then forwards to other CHs or base station (BS), while non-CHs are responsible for sensing environment and transmitting information to the corresponding CH.

Extensive researches have been conducted for achieving high energy efficiency in clustered WSNs (e.g., [3-29]). Existing methods are largely composed of two categories: centralized methods (e.g., [4-11, 29]) and distributed methods (e.g., [3, 5, 12-18]). Centralized methods typically request knowledge of the location of each sensor or the location distribution of all the sensors such that decisions can be made to achieve a certain kind of global optimization. Distributed methods, on the other hand, make most or all decisions based on local information, typically with limited information exchanges between neighborhood sensors. The distributed methods help achieve better scalability of networks, while the centralized methods are useful where location of each sensor or location distribution of all the sensors is known to a central controller. Centralized methods also serve as a good reference for network pre-plan and a useful benchmark for evaluating the performance of distributed methods or methods based on partial/inaccurate global information. In this paper, we focus on studying global information-based centralized methods.

It has been proved that finding the optimal clustering of a WSN for minimizing the overall energy consumption is an NP-complete problem [19]. Numerous centralized algorithms have been proposed and a few representative ones among them are briefly presented here. In LEACH-C (Low Energy Adaptive Clustering Hierarchy-Centralized) [5], BS computes the average node energy and chooses the CHs based on the residual energy and location information of each sensor. The number of clusters is determined beforehand. DSC (Dynamic/Static Clustering) [4] makes improvements to LEACH-C, mainly by lowering the communication overhead in the setup phase. BCDCP (Base Station Controlled Dynamic Clustering Protocol) [6] utilizes a high-energy BS to set up clusters and the routing paths between CHs. The main features of BCDCP include the formation of balanced clusters, BS-supervised randomized rotation of CHs, and CH-to-CH routing for transferring data to BS, etc. AHP (Analytical Hierarchy Process) [30] considers three different factors, namely energy, mobility and the distance to the involved cluster centriod respectively, in calculating the local weight and global weight of sensor nodes. The CHs are chosen by measuring the combined values of these two weights. And CH re-selection is triggered based on node mobility and remaining energy levels. In [31], Memetic Algorithm (MA) was developed to organize the sensors into clusters. The calculations adopt a fitness function which takes a few factors into account including the remaining energy of each sensor, distance to BS, distance between CHs and the time period between clustering operations, etc. In DMCLUSTER (Data Mining Cluster) [9], data mining techniques are adopted to infer the location and topology information from the information of remaining energy. In [32], CHEFL (Cluster Head Election using Fuzzy Logic) was proposed to select CHs according to three fuzzy variables: energy, concentration and centrality. In DCC (Distance-based Crowdedness Clustering) algorithm [33], it is proposed to determine the cluster heads in WSNs based on the crowdedness of sensor nodes with the objective of minimizing the overall energy consumption.

Among the existing results, DCC has the same objective function and adopts largely the same assumptions as what we will adopt in this paper. It is therefore the most closely related existing result.

Other related results include distributed clustering algorithms (e.g. [3, 5, 12-18, 34]), algorithms for maximizing network connectivity (e.g. [35-37]), lifetime (e.g. [38, 39]) and cluster stability (e.g. [40, 41]) respectively, and MAC layer design (e.g. [42, 43]), etc. A good survey of clustering techniques and algorithms is referred to [44].

In this paper, we propose efficient algorithms for finding the best locations of a given number of CHs such that the overall energy consumptions of a given network are minimized. Specifically, algorithms are proposed for finding 1) the best location of CH in a single cluster; 2) the best formation of a given number of clusters in a WSN where each CH has to communicate with BS directly; and 3) the best formation of a given number of clusters where there can be ad-hoc transmission between CHs, respectively. For each case, we consider two different models, namely the *free space* and *multipath* power consumption models (explained in detail later), and propose an algorithm for each of them respectively. Theoretical analyses and extensive simulation results show that the proposed algorithms are efficient and steadily achieve satisfactory performance.

The reminder of this paper is organized as follows. Design and analysis on single-cluster algorithm is proposed in Section 2. In Section 3, the algorithm is extended to multi-cluster networks, with and without ad-hoc transmission between CHs respectively. Extensive simulation results and discussions are presented in Section 4. Finally, Section 5 concludes the paper.

2. Single cluster head allocation (SCHA) algorithm

We first propose an algorithm for finding the optimal location of the CH in a single given cluster. As that in many existing work (e.g., [5]), we consider Friss free space model (also termed as free space model. In this paper, the two names are used interchangeably.) where energy consumption for data transmission is proportional to the square of transmission distance, and multipath model where energy consumption is proportional to the fourth power of transmission distance, respectively. The former model applies to the cases where transmission distance is relatively short, while the latter one better resembles the energy consumption in longer-distance data transmission [45]. Note that in this paper, when we refer to the "multipath model case", we make the assumption that transmissions between CHs and/or between CH and BS follow the multipath model while transmissions between CH and its non-CHs still follow the free space model. This makes sense since a cluster is typically of a limited area. The proposed algorithms however can be easily extended to the case where the multi-path model is also used between CH and its non-CHs. Such extensions are omitted in this paper due to their limited significance in real-life applications.

Unless otherwise specified, we adopt the same assumptions and notations as those in [5]. The definitions of the notations used in equations and their typical values adopted in simulation are summarized in Table 1. For a given cluster with N non-CHs, the position of each node is represented as $(x_i\ , y_i)$, $i = 1,\ 2...N$. The objective of the algorithm is to find the optimal location for CH (an N+1-th node) leading to the minimum overall energy consumption of the cluster.

For the Friss free space model, the energy consumption of a WSN can be calculated as follows

$$\begin{cases} E_{CH} = lE_{elec}N + lE_{DA}N + lE_{elec} + l\xi_{fs}d^2{}_{toBS} \\ \sum E_{non-CH} = lE_{elec}N + l\xi_{fs}\sum d^2{}_{toCH} \end{cases} . \tag{1}$$

Let

$$\begin{cases} C_1 = l(E_{elec} + E_{DA}) \\ C_2 = l\xi_{mp} \\ C_3 = lE_{elec} \\ C_4 = l\xi_{fs} \end{cases} \tag{2}$$

and substitute Eq. (2) into Eq. (1), we have

$$\begin{cases} E_{CH} = C_1 N + C_3 + C_4 d_{toBS}^2 \\ \sum E_{non-CH} = C_3 N + C_4 \sum d_{toCH}^2 \end{cases} . \tag{3}$$

Without loss of generality, we denote the location of BS as the original point $(0,0)$. Denote the ideal location of CH as (x^*, y^*). According to Eq. (3), the overall energy consumption of the given cluster is derived as below.

$$P_{all} = CN + C_3 + C_4(x^{*2} + y^{*2}) + C_4 \sum_{i=1}^{N} [(x_i - x^*)^2 + (y_i - y^*)^2], \tag{4}$$

where

$$C = C_1 + C_3 = 2lE_{elec} + lE_{DA} . \tag{5}$$

Using $F(x, y)$ to represent the overall energy consumption of the cluster as a function of the CH location (x, y), we have

$$F(x, y) = C_4 \sum_{i=1}^{N} [(x - x_i)^2 + (y - y_i)^2] + C_4(x^2 + y^2) . \tag{6}$$

To achieve the optimal value of $F(x, y)$, we shall have

$$\begin{cases} \dfrac{\partial F}{\partial x} = 2C_4 \sum_{i=1}^{N} (x - x_i) + 2C_4 x = 0 \\ \dfrac{\partial F}{\partial y} = 2C_4 \sum_{i=1}^{N} (y - y_i) + 2C_4 y = 0 \end{cases} . \tag{7}$$

Hence

$$\begin{cases} Nx - \sum x_i = -x \\ Ny - \sum y_i = -y \end{cases}, \tag{8}$$

which easily leads to

$$\frac{Nx - \sum x_i}{Ny - \sum y_i} = \frac{x - \frac{\sum x_i}{N}}{y - \frac{\sum y_i}{N}} = \frac{x}{y}\,. \tag{9}$$

From Eq. (9), we have that the optimal position of CH is lying on the straight line connecting BS and the weight center of the cluster, as illustrated in Fig. 1.

Let

$$(x^*, y^*) = (\rho \frac{\sum x_i}{N}, \rho \frac{\sum y_i}{N})\,. \tag{10}$$

Substitute Eq. (10) into Eq. (8), we get that the ideal position of CH is $(\rho \frac{\sum x_i}{N}, \rho \frac{\sum y_i}{N})$, where

$$\rho = \frac{N}{N+1}\,. \tag{11}$$

Substituting Eqs. (10) and (11) into Eq. (4), we get the energy equation as follows

$$P_{all} = CN + C_3 + C_4((\frac{\sum x_i}{N+1})^2 + (\frac{\sum y_i}{N+1})^2) + C_4 \sum_{i=1}^{N} [(x_i - \frac{\sum x_i}{N+1})^2 + (y_i - \frac{\sum y_i}{N+1})^2] \tag{12}$$

An interesting observation from Eq. (12) is that the third and fourth items on the right-hand side may be viewed as calculating the variance of N+1 locations including (x_i, y_i), $i = 1, 2, ..., N$ and (0, 0). Since these two items typically dominate the energy consumption of a sensor cluster, for a cluster with a large enough number of sensors (i.e., a large enough value of N) following the uniformly random distribution, the effects of the location (0,0) may be neglected and consequently, the overall energy consumption is approximately proportional to the number of sensors.

Another observation is that the overall energy consumption of the free space model is symmetric to the best location of CH ($\frac{\sum x_i}{N+1}, \frac{\sum y_i}{N+1}$). Specifically, from Eq. (6) we have

$$\begin{aligned} F(x^* + \Delta x, y^* + \Delta y) - F(x^* - \Delta x, y^* - \Delta y) \\ &= F(\frac{\sum x_i}{N+1} + \Delta x, \frac{\sum y_i}{N+1} + \Delta y) - F(\frac{\sum x_i}{N+1} - \Delta x, \frac{\sum y_i}{N+1} - \Delta y) \\ &= -2(\sum x_i)\cdot(\frac{\sum x_i}{N+1}) - 2(\sum y_i)\cdot(\frac{\sum y_i}{N+1}) - (-2(\sum x_i)\cdot(\frac{\sum x_i}{N+1}) - 2(\sum y_i)\cdot(\frac{\sum y_i}{N+1})) \\ &= 0. \end{aligned} \tag{13}$$

We now proceed to analyze the best CH location for the multipath model. Similar to the earlier discussions, we shall have

$$P_{all} = CN + C_3 + C_2\left(x^{*2} + y^{*2}\right)^2 + C_4\sum_{i=1}^{N}\left[(x_i - x^*)^2 + (y_i - y^*)^2\right]. \tag{14}$$

Let

$$F_1(x, y) = C_2\left(x^{*2} + y^{*2}\right)^2 + C_4\sum_{i=1}^{N}\left[(x_i - x^*)^2 + (y_i - y^*)^2\right]. \tag{15}$$

By adopting analysis similar to that for the free space model case, we have that since

$$\begin{cases} \frac{\partial F_1}{\partial x} = 2\sum_{i=1}^{N}\left(x - x_i\right) + 2\alpha\left(x^2 + y^2\right)\cdot 2x = 0 \\ \frac{\partial F_1}{\partial y} = 2\sum_{i=1}^{N}\left(y - y_i\right) + 2\alpha\left(x^2 + y^2\right)\cdot 2y = 0 \end{cases}, \tag{16}$$

where $\alpha = \frac{C_2}{C_4} = \left(\frac{\in_{mp}}{\in_{fs}}\right)$, the ideal position of CH can be calculated as $(\rho\frac{\sum x_i}{N}, \rho\frac{\sum y_i}{N})$ where

$$\left[\left(\frac{\sum x_i}{N}\right)^2 + \left(\frac{\sum y_i}{N}\right)^2\right] = \frac{(1-\rho).\sum x_i}{2\alpha\rho^3.\frac{\sum x_i}{N}} = \frac{N(1-\rho)}{2\alpha\rho^3}. \tag{17}$$

Let

$$G(\rho)=\frac{C_4 N(1-\rho)}{2C_2\rho^3}. \tag{18}$$

Since

$$G^{'}(\rho)=\frac{C_4 N(2\rho-3)}{2C_2\rho^4}, \tag{19}$$

which is obviously negative for any $0<\rho\leq 1$, we have that $G(\rho)$ is a monotonic function in the interval (0, 1]. The value of ρ satisfying Eq. (17) therefore can be calculated by many numerical computational methods, e.g., the bisection method.

Finally, we summarize the cluster head allocation algorithm as follows:

Single Cluster Head Allocation Algorithm (SCHA)

1. Calculate $\frac{\sum x_i}{N}$ and $\frac{\sum y_i}{N}$.
2. Find out the optimal solution of ρ through Eq. (11) for the free space model and Eq. (17) for the multipath model, respectively.
3. Calculate the optimal CH location $(\rho\frac{\sum x_i}{N},\rho\frac{\sum y_i}{N})$. □

Remark: In the above algorithm, we assume that all the *N* sensors serve as non-CHs and the objective is to find the optimal location of the CH as an *N*+1-th sensor. The algorithm can be easily revised to find the best location of CH where one of the *N* sensors has to serve as CH. For example, we may choose the sensor closest to the ideal position to be the CH. Detailed designs and performance evaluation of such extensions, however, are out of the scope of this paper. Note that, to handle the extension case as stated above, the number of non-CHs needs to be changed from *N* to *N-1* in a few equations. □

The algorithm above is of a low complexity *O(1)* which is independent of cluster size. As implicated from the algorithm, it applies to the cases with either the knowledge of every node's position or only the knowledge of the location distribution of all nodes since the algorithm only needs calculating the weight center of all the sensors.

3. Algorithms for optimizing multi-cluster head locations

3.1 Algorithms for finding multi-cluster head allocations without ad hoc transmission between CHs

The low complexity of the SCHA algorithm allows it to be extensively utilized to minimize the overall energy consumptions of multi-cluster WSNs. In this subsection, we consider the case where there is no ad hoc transmission between CHs.

If we ignore the energy consumptions for transmitting data from CHs to BS and only try to minimize the energy consumptions by NCHs to transmit data to CHs, the problem reduces to the planar *k*-means problem [46], which is known to be NP-hard. Specifically, the classic planar *k*-means problem is defined as follows. Given *N* points in a plane, find *k* centers which minimize the sum of square of the Euclidean distance of each point to its nearest center. Below we design a heuristic algorithm for the multi-cluster head allocation problem.

Assume a total of *K* clusters need to be set up. The heuristic algorithm is depicted as follows:

Multi-Cluster Head Allocation Algorithm for WSNs without ad-hoc transmission (MCHA-I)

1. Randomly choose *K* locations in the interested area as CHs positions.

2. Each of the other sensors joins the CH with the closest distance to form up clusters.

3. Apply the SCHA algorithm proposed in Section 2 to select the optimal CH location for each cluster.

Implement Step 2 and Step 3 iteratively until no further improvements can be made. □

We repeat the MCHA-I algorithm above for a large enough times, each time with a different initial set of CHs, and finally select among them the one with the lowest energy consumption as the solution.

It can be easily proven that the MCHA-I algorithm converges in finite iterations. Specifically, as we can see, each time when Step 3 is executed, the overall energy consumption either remains unchanged (where each CH is already at the best location for its cluster) or lowered (where a better location is found). The energy may be further lowered or kept unchanged in Step 2 when every sensor joins the cluster with the closest CH. In short, the overall energy consumption monotonically decreases in each iteration until a local optimum is reached.

The time complexity of the MCHA-I algorithm is low. Specifically, the calculations mainly come from step 2, which has a complexity of $O(KN)$. Step 3, as discussed earlier, has a complexity of $O(1)$.

Remark: Starting from a set of randomly selected CHs, the MCHA-I algorithm can only ensure achieve a local optimum solution. Repetitively running the algorithm from different initial sets of CHs helps improve the chance that the global optimal or sub-optimal solution is found. □

3.2 Algorithm for multi-cluster head allocation in WSNs with ad hoc transmission between CHs

We consider the case in multi-cluster WSNs where CHs can transmit to BS through ad-hoc communication with other CHs. The objective is still to minimize the overall energy consumption. There are two options for each CH to communicate to BS in such networks: to communicate directly to BS; or to ad-hoc through another CH. The main idea of the proposed algorithm is to find the optimal solution for each of these two options and then choose among them the one with a lower overall energy consumption. Obviously the SCHA algorithm can be directly applied to find out the optimal solution for the first option. For the second option, we calculate the best solution of CH location where each of the nearby CHs is serving as the next-hop station in ad-hoc transmission and then select the best one among them. It is noteworthy that the best solution means the solution leading to the lowest overall energy consumption, which does not necessarily lead to the lowest energy consumption of the cluster being considered. The algorithm for finding the optimal position of CH for ad-hoc transmission through other CHs can be regarded as a simple extension of the SCHA algorithm, where BS is not located at the origin point but at the position of the next CH in the path of ad hoc transmission to BS (hereafter termed as *next-hop* CH for convenience of discussion). With ad hoc transmission through a certain next-hop CH, the energy consumption of a cluster with N nodes in the free-space model can be derived as follows:

$$\begin{cases} E_{CH} = C_1 N + C_3 + C_4 d_{toNxtCH}^2 \\ \sum E_{non-CH} = C_3 N + C_4 \sum d_{toCH}^2 \end{cases}, \tag{20}$$

The overall power consumption can be expressed as follows:

$$P_{all} = CN + C_3 + C_4[(x^* - x_{nxt})^2 + (y^* - y_{nxt})^2] + C_4 \sum_{i=1}^{N} [(x^* - x_i)^2 + (y^* - y_i)^2], \tag{21}$$

where (x_{nxt}, y_{nxt}) represents the location of the next-hop CH being considered. Definitions of the other notations can be found in Table 1. Let

$$F_2(x, y) = C_4[(x - x_{nxt})^2 + (y - y_{nxt})^2] + C_4 \sum_{i=1}^{N} [(x - x_i)^2 + (y - y_i)^2]. \tag{22}$$

Following the same derivation as that for the SCHA algorithm, we have

$$\begin{cases} x^* = \dfrac{x_{nxt} + \sum x_i}{1 + N} \\ y^* = \dfrac{y_{nxt} + \sum y_i}{1 + N} \end{cases}. \tag{23}$$

Be using similar approach, we can easily develop analysis for the case with multi-path model transmission between CHs. In this paper, we omit discussions on such case since the extension is straightforward.

With the extended SCHA algorithm (termed as ECHA algorithm hereafter) as stated above, we can proceed to develop the CH allocation algorithm for multi-cluster WSNs with ad hoc transmission between CHs. In this paper, we assume that energy consumption for ad-hoc transmission from a CH to BS equals the sum of the power consumption for transmitting from this CH to its next-hop CH (CH_{nxt}) plus the energy consumption for forwarding the aggregated information of its next-hop CH's cluster to BS. An example is depicted in Fig. 2: if CH_2 is serving as the next-hop CH for CH_3, then the energy consumption for ad hoc transmission from CH_3 to BS equals the energy consumption from CH_3 to CH_2 plus the energy consumption for transmitting the aggregated information of CH_2's own cluster to BS, with or without further ad hoc transmission through other CHs.

The multi-cluster head allocation algorithm for WSNs with ad hoc transmission between CHs is depicted as follows.

Multi-Cluster Head Allocation Algorithm for WSNs with ad-hoc transmission between CHs (MCHA-II)

<u>First Iteration:</u>

1. <u>Initialization</u>: Randomly select K sensors as CHs. The other sensors join the cluster with the closest CH.
2. <u>Sorting</u>: Sort all the CHs in an increasing order of their distances to BS. When there is a tie, break it randomly. Denote these clusters as $C_1, C_2, ..., C_K$, and their respective CHs as $CH_1, CH_2, ..., CH_K$.
3. <u>CH location</u>:
   a. Use the SCHA algorithm to find the new CH_1,
   b. For C_i $(1 < i \le K)$

i. Use the SCHA algorithm to find the new CH_i assuming that CH is communicating directly to BS. Calculate the energy consumption by CH_i for forwarding the aggregated information to BS (denoted as P_i^f) as well as cluster C_i's overall energy consumption (denoted as P_i^{all}).

ii. For each $1 \le j < i$, use the ECHA algorithm to find the new CH_i assuming that CH_i uses CH_j as the next-hop CH in its ad-hoc transmission to BS. Calculate the corresponding $P_i^{all} + P_j^f$ (Note that, as mentioned earlier, we assume that P_j^f equals the additional energy consumption CH_i imposes on the other CHs for the case where it uses CH_j as the next-hop CH in its ad hoc transmission to BS).

iii. Compare power consumption calculated in Steps 3.b.i and 3.b.ii and select among them the one with the lowest value of $P_i^{all} + P_j^f$. If the best solution has been achieved in Step 3.b.ii where CH_i communicates with BS through a certain CH_{j*} as the next-hop CH, update P_i^f as being equaling to P_{j*}^f plus the energy consumption for communicating between CH_i and CH_{j*}.

4. <u>Re-clustering</u>: Each sensor joins the cluster with the closest CH.

<u>Following Iterations:</u>

In the following iterations, we basically repeat Steps 2-4 of the first iteration, with only one revision in the SCHA and ECHA algorithms: if in the last iteration, CH_{j*} has been chosen as the next-hop CH for CH_i's ad-hoc transmission to BS, then in the Step 3 of the next iteration, the new CH_{j*} should be calculated as if there were a non-CH of cluster C_{j*} at the position of CH_i. If CH_i is not only forwarding its own cluster's aggregated information to CH_j but *k* other clusters' information as well, the new CH_{j*} should be calculated as if there were *k*+1 non-CHs of cluster C_{j*} at the position of CH_i. For example, in Fig. 2, if according to the calculation results, CH_2 should send its own cluster's information as well as information from CH_3 and CH_4 to CH_1, in the next iteration, the ideal position of CH_1 should be calculated as if there were three non-CHs of cluster C_1 at the position of CH_2. □

The iterative calculations terminate when no further improvements could be made.

Again, it is easy to prove that the overall energy consumption monotonically decreases in each iteration. Therefore, the algorithm converges to a local optimum in a finite number of iterations. Repetitively running the algorithm from different initial sets of CHs helps improve the chance that the global optimal or sub-optimal solution is achieved.

The time complexity of each iteration in the MCHA-II algorithm remains the same as that of the MCHA-I algorithm at $O(KN)$, which comes from step 4. The step 3.b.ii has a complexity of $O(K^2)$ which is lower than $O(KN)$.

4. Simulation results and discussions

The performances of the proposed algorithms have been evaluated by extensive numerical simulations. Due to length limit, only a few representative results are presented in this paper while the conclusions hold in all the other cases we have tested. All figures and tables showing average values of a number of realizations are presented with 95% confidence interval unless otherwise specified. The confidence intervals of the proposed algorithms are generally quite small; in some figures they may be smaller than the symbols, making these figures look like not showing confidence interval. In this paper, we simulate the cases where sensors are distributed in a square region with a uniformly random distribution. The proposed algorithms certainly also work for any other non-uniform distribution in less regular regions.

We firstly compare the results obtained by the SCHA algorithm with the solutions of the brute force method. Specifically, we calculate the optimal location of CH by using the SCHA algorithm and then assign the sensor closest to the optimal location as the CH. While using the brute force method, we assign each of N sensors as CH and find among them the best one. The results are presented in Fig. 3. We assume that the sensor distribution areas are $0 \le x, y \le 100$ for the free space model and $100 \le x, y \le 200$ for the multi-path model respectively; the length unit is meter. Without loss of generality, BS is always located at $(0, 0)$. There are 100 realizations for each of the two models, each realization with a different randomly generated network; and we show the average results of all the realizations. As shown in Figs. 3(a) and 3(b), the results obtained from the SCHA algorithm match those from the brute force method, which verifies the correctness of the SCHA algorithm. The calculation time for each realization has always been in sub-second scale on a PC with Intel Core 2 Duo CPU 2.66GHz and 2.0GB of RAM. From Fig. 3 we also see that the overall energy consumption is approximately proportional to the number of nodes, which verifies our discussion in Section 2.

In Fig. 4, we illustrate the relationship between the overall energy consumption and the CH location in relevance to the optimal location. We can see that for the case where sensors follow a uniformly random distribution, the overall energy consumption is symmetric to the optimal location of $(\frac{\sum x_i}{N+1}, \frac{\sum y_i}{N+1})$, as we have proved in Section 2.

We then present simulation results obtained by using the MCHA-I algorithm, assuming there is no ad hoc transmission between CHs. The sensor distribution areas remain the same as those for the single cluster networks, and 4 clusters need to be formed up by 100 sensors randomly distributed in the areas. For each case, 100 independent simulation realizations have been carried out, each time with a different randomly selected set of 4 sensors as initial CHs. Fig. 5(a) and Fig. 5(b) depict the best results of these 100 realizations for the free space and multi-path model cases respectively. Specifically, we show for the realization which achieves the best result the fast convergence of overall energy consumptions in only a few iterations. To further demonstrate the efficiency of the algorithm, Figs. 6(a) and 6(b) illustrate the distribution of the numbers of iterations needed to achieve the final results in all the 100 realizations. As we can see, the majority of the 100 realizations converge in no more than 7 iterations (94% for the free space model and 91% for the multi-path model respectively), and we never need to have more than 9 iterations before the calculations converge.

As discussed earlier in Section 3, for multi-cluster networks, the proposed algorithm can only ensure achieving local optimum in each realization. It is therefore important to have an idea on how quickly the results can be improved with the number of realizations. Figure 7 shows for 100 different randomly generated networks, the improvements of the best results with the number of realizations, each realization with a different set of randomly selected CHs in the initialization step of the MCHA-I algorithm. We see that having 60-100 realizations is generally sufficient for achieving satisfactory results while having further more realizations may not significantly help. Also we note that for the multi-cluster networks, the calculation results on different networks have a larger 95% confidence interval than that for a single-cluster network though it is still quite small: the 95% confidence interval is smaller than 1% of the overall energy consumption, meaning that the optimal clustering does not change drastically in different networks as long as these networks are with the same number of sensors distributed in the same region following the same uniform random distribution.

As aforementioned in Section 1, among all the existing methods, DCC algorithm adopts the most similar assumptions and design objective to those of the proposed algorithm, more specifically, the MCHA-I algorithm. Therefore we compare the performances of these two methods. We test 9 different cases where the number of sensors varies from 100 to 900 and they are dispersed in the area of $0 \leq x, y \leq 200$. For each case, we firstly find out the optimal number of clusters by using the DCC algorithm (i.e., the number of clusters is optimized for the DCC algorithm), and then adopt the same number of clusters in the MCHA-I algorithm. To simplify the discussions, we assume that the multipath model case is adopted in calculating the energy consumptions which means transmissions between CHs and/or between CH and BS follow the multipath model while transmissions between CH and its non-CHs still follow the free space model. While using the MCHA-I algorithm, we implement 100 independent realizations for each network and choose the best solution among them. Figure 8 shows the average results from 100 different randomly generated networks. Specifically, Fig. 8(a) shows the results where sensors have a uniform random distribution in the region; while Fig. 8(b) is for the case where the *x*- and *y*-coordinates of each sensor independently follow an exponential distribution with an expected value of 30 (For convenience, hereafter we term such case as the *exponential distribution model*.). We see that MCHA-I outperforms DCC by 5.09%-15.62% in the overall energy consumptions for the uniform distribution model and by 6.80%-8.89% for the exponential distribution model. Note that for both algorithms the confidence intervals are rather small for both of the two distribution models, making them hardly visible in the figures.

We now further evaluate the efficiency of MCHA-II algorithm and equally important, how much the ad hoc transmission between CHs can help lower overall energy consumptions. We assume that the interested areas remain the same as before and 4 clusters are to be formed up. We find that MCHA-II algorithm converges almost as quickly as the MCHA-I algorithm. Due to length limit, we omit detailed results on the convergence speed. As to the effects of allowing ad hoc transmission between CHs on lowering overall energy consumptions, the results are summarized in Tables 2 and 3 for the free space and multi-path models respectively.

As we can see in Tables 2 and 3, for both of the two models, allowing ad hoc transmission between CHs helps lower overall energy consumptions. The effects however are much more significant in the multi-path model case than those in the free space model case. This can be easily understood: in the multi-path model case, the energy consumption for CHs to communicate to BS is proportional to the fourth power of the distance between them; hence it accounts for a substantial portion of the overall energy consumption. Ad hoc transmission between CHs, by drastically reducing this portion, helps lower the overall energy consumption by 4.90%-9.31%. Such effects however become much less significant in the free space model case where energy consumption for CHs to communicate BS accounts for only a small fraction of the overall consumption: reducing this small fraction only helps lower the overall energy consumption by a marginal value of 0.03%-0.88%.

Lastly we discuss on the relationship between the number of clusters and the overall energy consumption for both the free space and multipath model cases, with and without ad hoc transmissions between CHs respectively. We still adopt the same parameter values as those in earlier calculations and assume that sensors have a uniform random distribution in the areas. Figures 9(a) and 9(b) illustrate the results for the free space and multipath model cases respectively. Figure 9(a) shows that, for the free space model case, the overall energy consumptions firstly decrease with the number of clusters, achieving the minimum values where there are 5 (w/o ad hoc transmission) or 6 (with ad hoc transmission) clusters, and then go up. As to the multipath model case, the observations are very different: the energy consumptions steadily go up with the number of clusters. This can be easily understood: in the multipath model case, the energy consumption for the transmission from CH to BS dominates; thus having more clusters increases the overall energy consumptions. Note that, however, this is based on the assumption that the energy consumption for data transmission from non-CHs to CHs always fits into the free space model. When there are too few clusters, some non-CHs may be far away from any CH, making the free space model invalid for describing the energy consumptions for data transmission from these non-CHs to their CHs. Optimization algorithms can be designed for distance-based, combined free space/multipath models, which are not very difficult yet request rather lengthy discussions on algorithm design and performance evaluations. Such discussions therefore have to be left to a separate report.

Figure 9 also shows that for both the free space and multipath model cases, having ad hoc transmission between CHs helps lower overall energy consumption. Specifically, when the number of clusters varies from 2 to 10, having ad hoc transmission between CHs lowers the overall energy consumption by 0.13% to 1.52% in the free space model case, and 4.07% to 19.75% in the multipath model case.

5. Conclusions

In this paper, we studied algorithms for finding the best locations of cluster heads in WSNs based on global information of sensor locations, with the main objective of minimizing the overall energy consumption of the whole network. We considered different cases with a single or multiple clusters, with and without ad hoc transmission between CHs respectively. For each case, algorithms were designed for both free space and multipath models, respectively. Theoretical analysis and extensive simulation results verified the correctness and high efficiency of all the proposed algorithms. The calculation results of the proposed algorithms provided a useful benchmark for evaluating various local information-based distributed clustering schemes or schemes based on partial or inaccurate global information.

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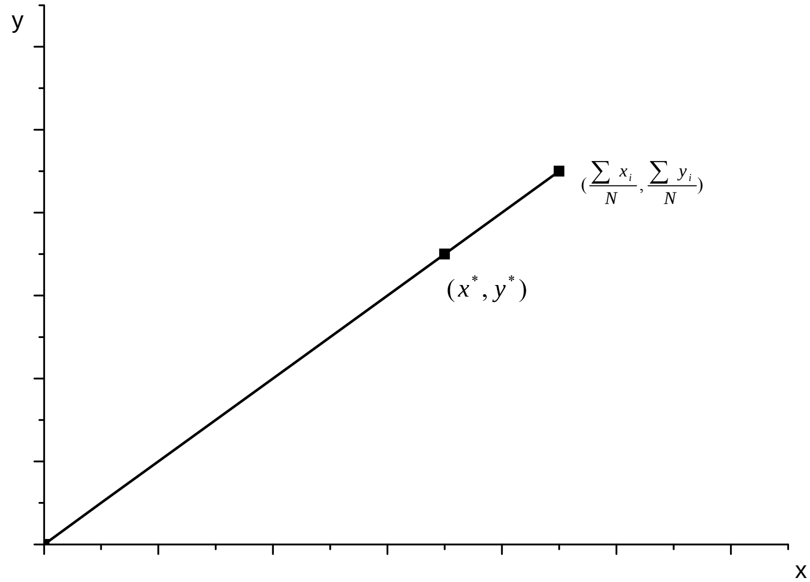


Fig. 1: Illustration of the ideal position of CH.

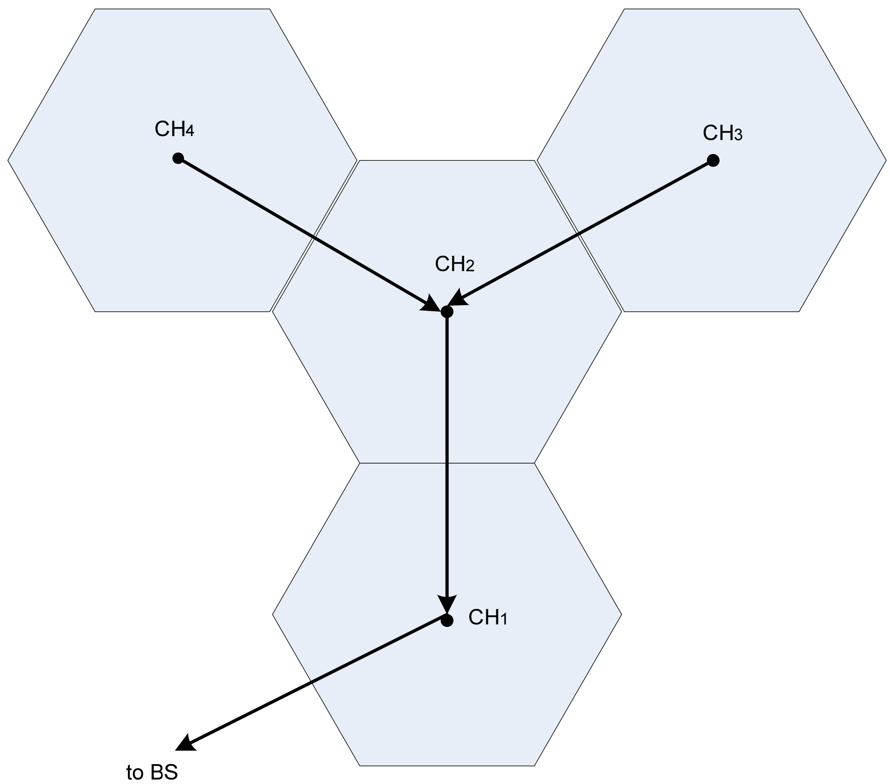


Fig. 2: An example of ad hoc transmission between CHs.

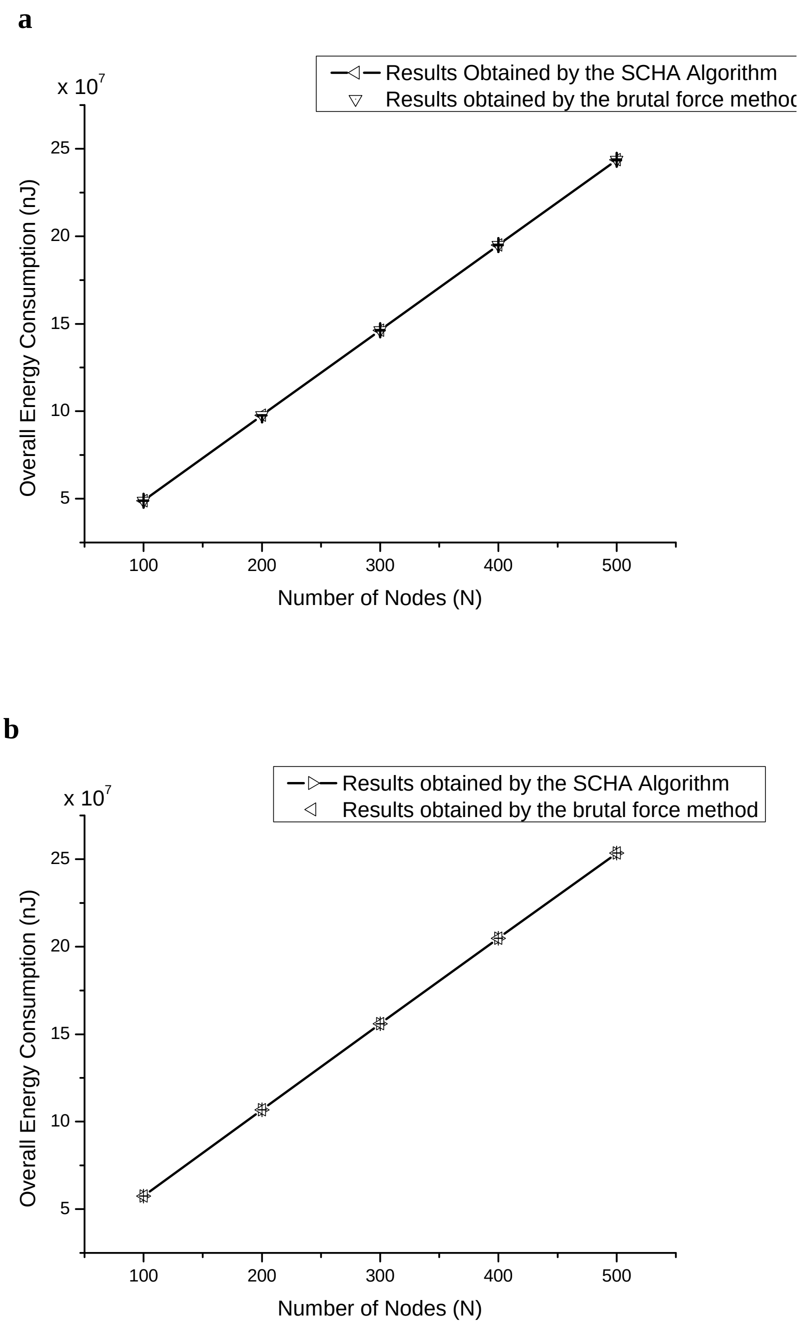
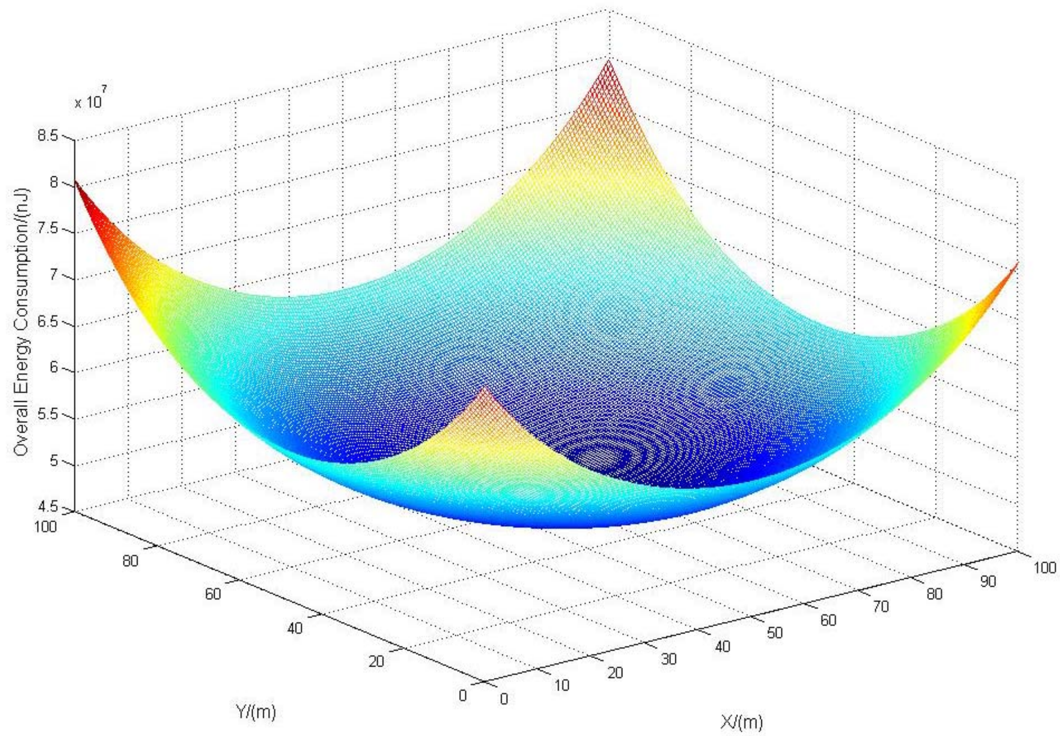
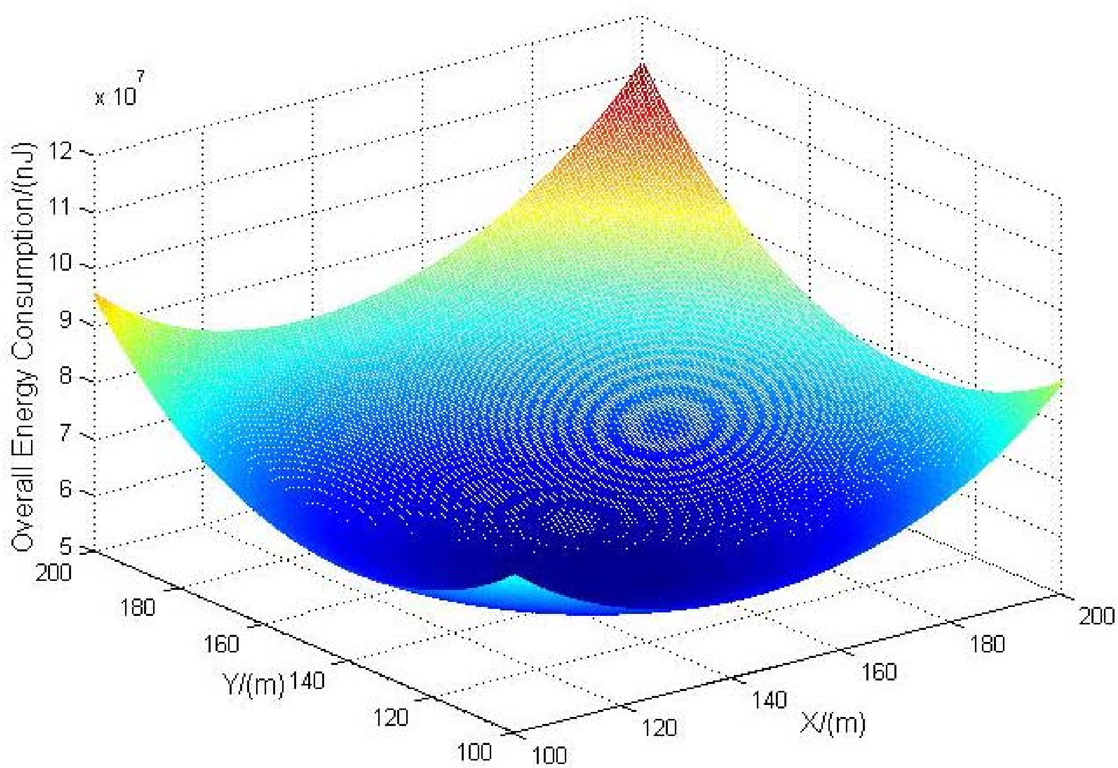


Fig. 3: Comparisons of the energy consumptions in a single cluster calculated by the SCHA algorithm and the brute force method respectively: (a) free space model in the region of $0 \le x, y \le 100$; (b) multi-path model in the region of $100 \le x, y \le 200$.

a


x 10^7
8.5
8
7.5
7
6.5
6
5.5
5
4.5
Overall Energy Consumption/(nJ)
100
80
60
40
20
0
Y/(m)
0
10
20
30
40
50
60
70
80
90
100
X/(m)

b


x 10^7
12
11
10
9
8
7
6
5
Overall Energy Consumption/(nJ)
200
180
160
140
120
100
Y/(m)
100
120
140
160
180
200
X/(m)

c.

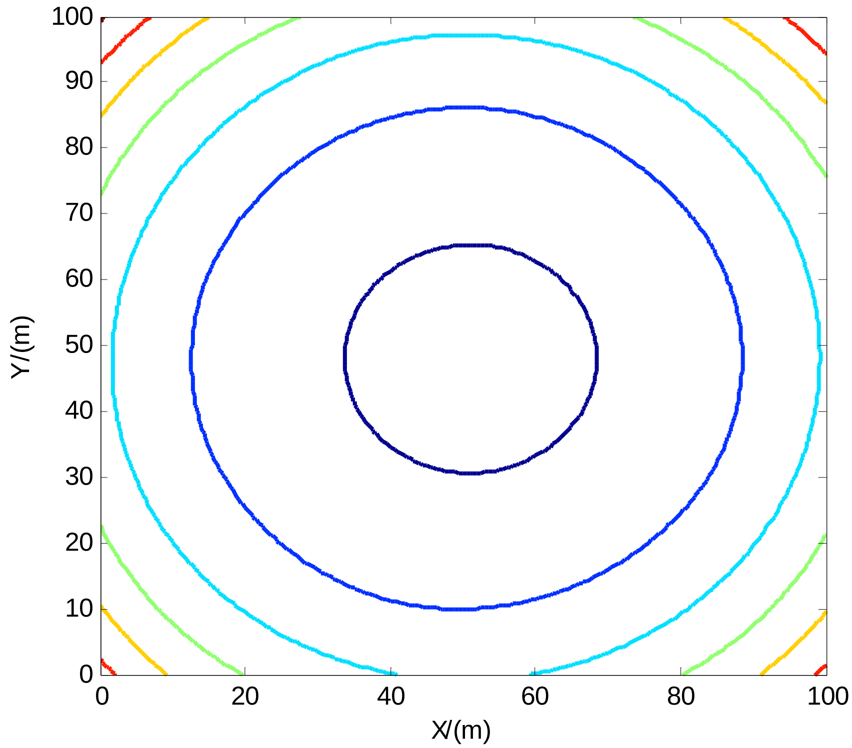
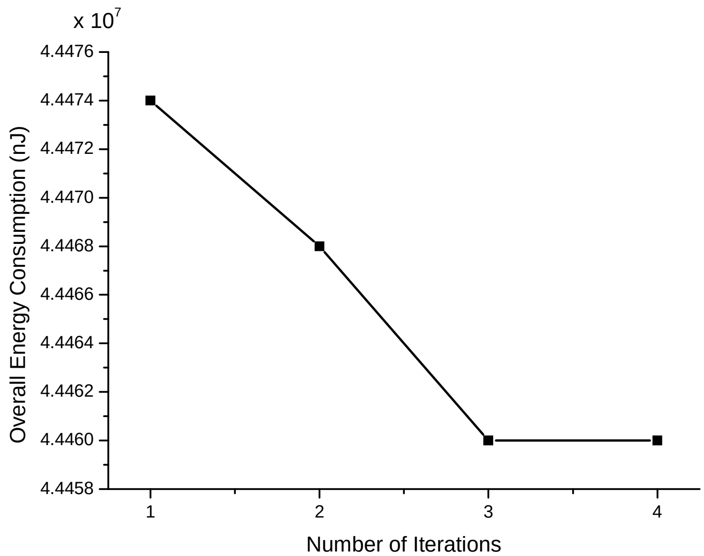


Fig. 4: The overall energy consumption corresponding to different CH locations in a 100-sensor, single-cluster network with uniform random distribution: (a) free space model; (b) multi-path model; (c) the contour line of energy consumptions in the free space model.

a



b

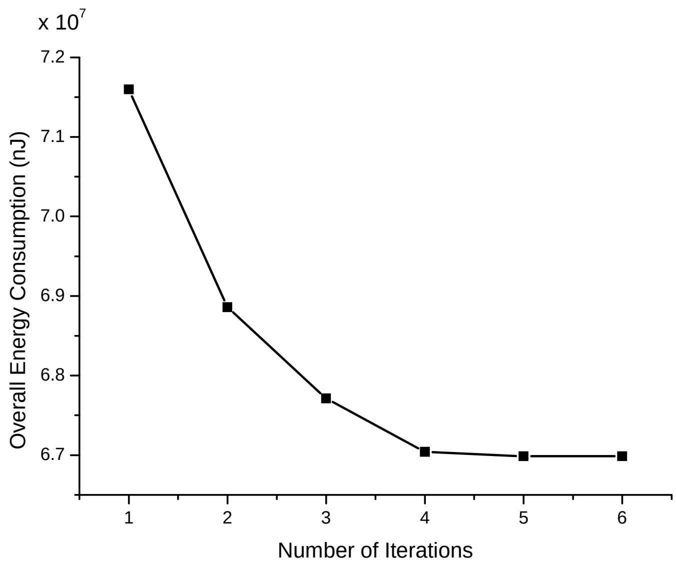
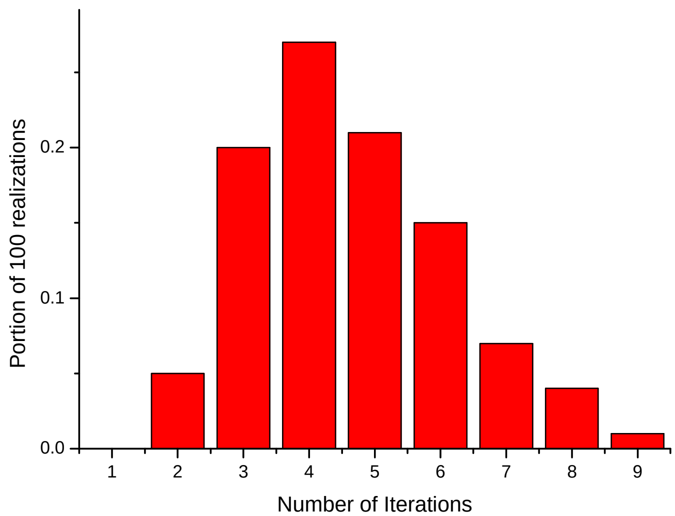


Fig. 5: Performance of the MCHA-I algorithm in a 100-sensor network with 4 clusters. The curves show the best results of 100 independent realizations: (a) free space model in the region of $0 \leq x, y \leq 100$; (b) multi-path model in the region of $100 \leq x, y \leq 200$.

a



b

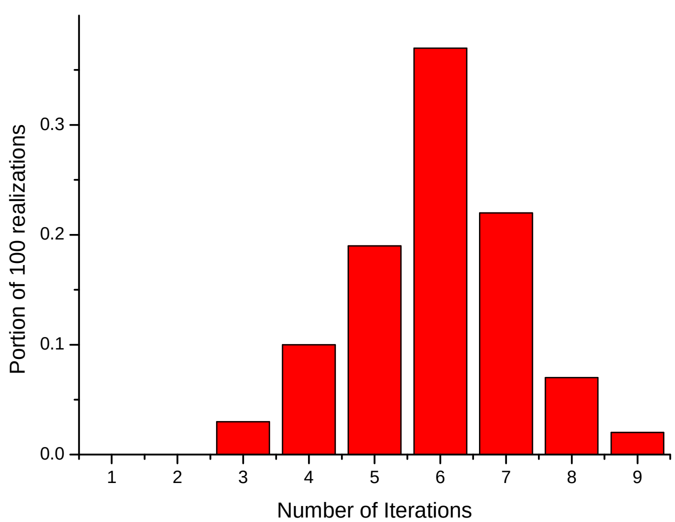


Fig. 6: Convergence speed of the MCHA-I algorithm: distribution of the number of iterations for the calculations to converge in 100 independent realizations in a 100-sensor network. (a) free space model in the region of $0 \le x, y \le 100$; (b) multi-path model in the region of $100 \le x, y \le 200$.

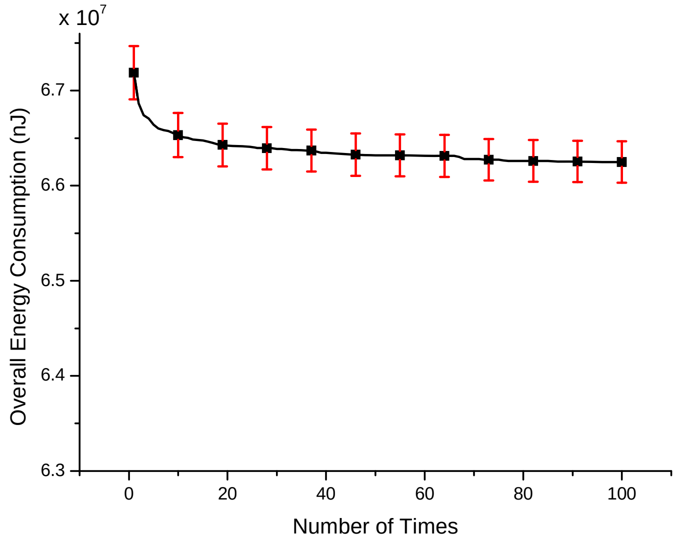
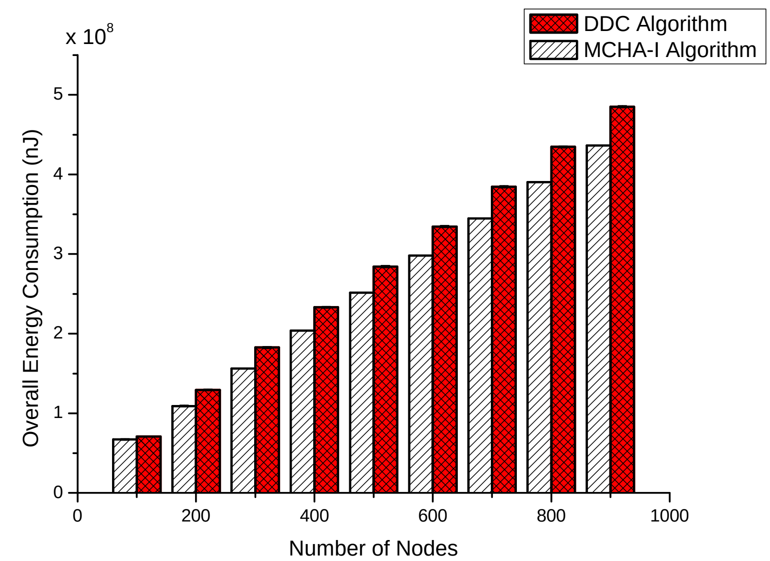


Fig. 7: Overall energy consumption vs. the number of realizations of the MCHA-I algorithm. Assume 100 sensors are distributed in the region of $100 \le x,\ y \le 200$ and 4 clusters need to be set up.

(a)



(b)

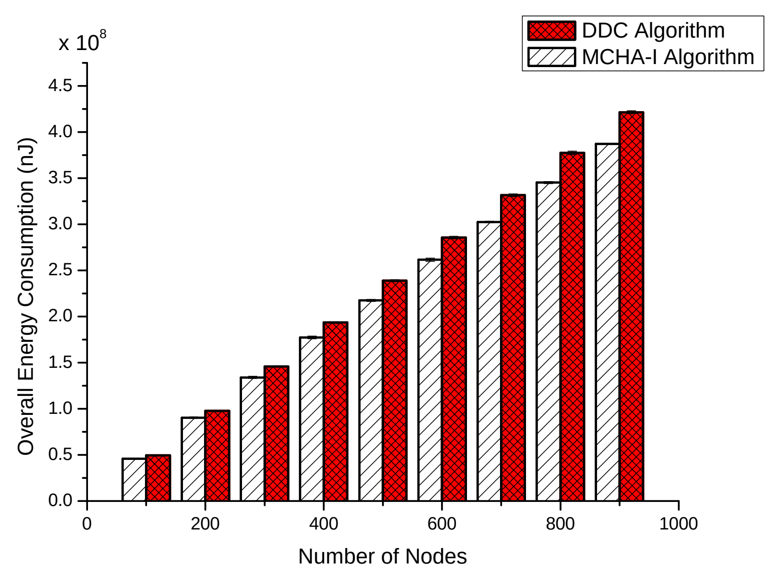
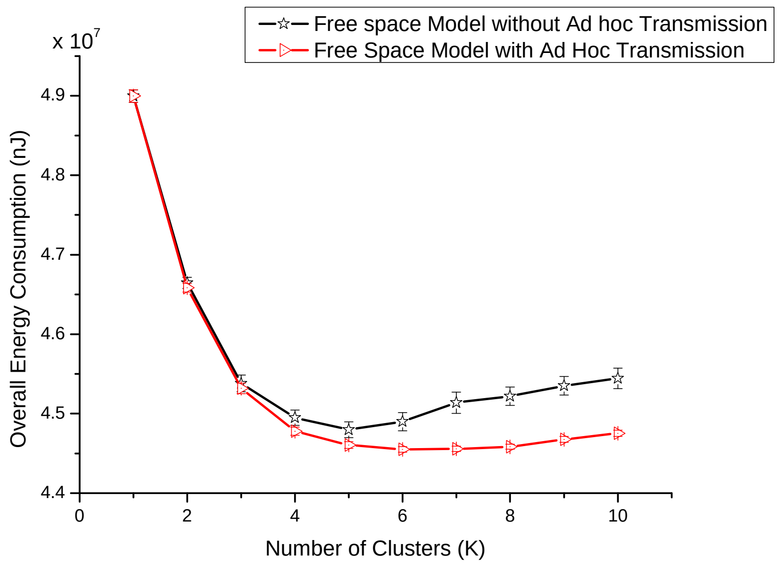


Fig. 8: Performance comparisons between the DCC and the MCHA-I algorithms with different numbers of sensors in the region of $0 \le x, y \le 200$: (a) uniform distribution model; (b) exponential distribution model.

(a)



(b)

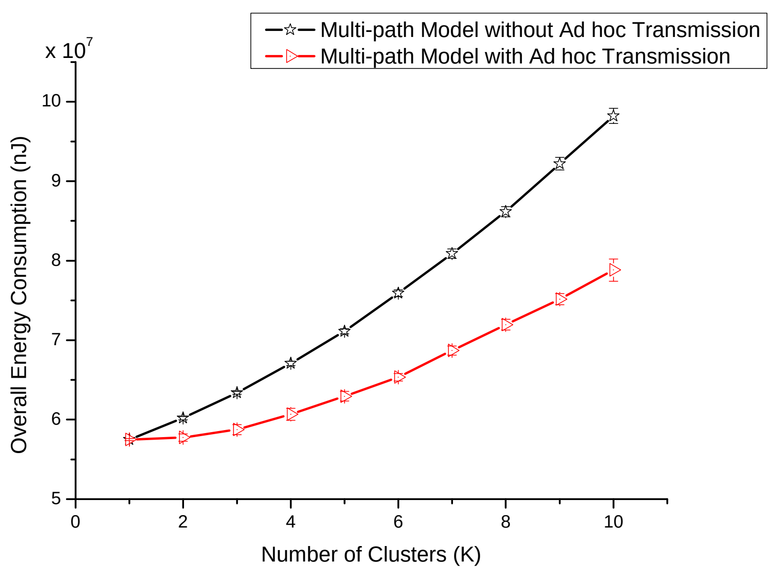


Fig. 9: Overall energy consumption vs. the number of clusters in a 100-sensor network with uniform random distribution in the areas: (a) free space model in the region of $0 \le x, y \le 100$; (b) multi path model in the region of $100 \le x, y \le 200$.

Table 1
Summaries of the definitions of notations and their typical values

Notations	Meanings	Values
l	No. of bits in each data message	4000 bits
N	No. of nodes in each cluster	N.A.
E_{elec}	Energy consumption for processing each bit of data	50 nJ/bit
E_{DA}	Energy consumption for data aggregation	5 nJ/bit/signal
ξ_{mp}	Coefficient of amplifier energy in multi-path model	0.0013 pJ/bit/m^4
ξ_{fs}	Coefficient of amplifier energy in free space model	10 pJ/bit/m^2
d	Distance between communicating nodes	N.A.
E_{CH}	Energy consumption for CH	N.A.
E_{non-CH}	Energy consumption for non-CH member node	N.A.

Table 2

Comparisons of the overall energy consumption for the free space model cases with and without ad hoc transmission between CHs respectively. Assume sensors are distributed in the region of $0 \le x, y \le 100$ and 4 clusters need to be set up. The results in the table are average results from 100 independent realizations.

Power Consumption \ Number of nodes	100	200	300	400	500
W/o ad-hoc transmission (nJ x 10^7)	4.4949	8.9221	13.2400	17.5710	22.015
With ad-hoc transmission (nJ x 10^7)	4.4778	8.8434	13.2360	17.5630	21.9850

Table 3

Comparisons of the overall energy consumption for the multipath model cases with and without ad hoc transmission between CHs respectively. Assume sensors are distributed in the region of $100 \le x, y \le 200$ and 4 clusters need to be set up. The results in the table are average results from 100 independent realizations.

Power Consumption \ Number of Nodes	100	200	300	400	500
W/o ad-hoc transmission (nJ x 10^7)	6.7554	11.5390	16.1820	20.7080	25.3640
With ad-hoc transmission (nJ x 10^7)	6.1262	10.8870	15.2480	19.7620	24.1200