A High-Precision Discrete Tracking Differentiator and Its Application in Processing PMU Data

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***Abstract*—A new and simple tracking differentiator (TD) with high precision based on discrete time optimal control (DTOC) law is proposed. The DTOC law is constructed in the form of the state feedback for a discrete-time double-integral system by using the state back-stepping approach. Zero-order hold of the control signal is introduced to improve the precision of discretization model. The control signal sequence in this approach is determined by the linearized criterion according to the position of the initial state point on the phase plane. The state estimation filtering characteristics of the TD are analyzed. The field phasor measurement units (PMUs) data are processed using the proposed TD. Not requiring complex power system modeling and historical data, the proposed TD is suitable for real-time synchrophasor estimation application especially when the states are corrupted by noise.**

***Index Terms*—tracking differentiator (TD), discrete time optimal control (DTOC), phasor measurement units (PMUs), filtering.**

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I. Introduction

It is known that the powerful yet primitive proportional-integral-derivative (PID) control law developed in the last century still plays an essential role in modern control engineering practice [1], [2]. However, since derivative signals are prone to corruption by noise and derivative control is usually not physically implementable, the PID control is usually degraded to PI control. To deal with this, there has been a lot of research done on differentiation trackers such as the high-gain observer-based differentiator [3], the super-twisting second-order sliding mode algorithm [4], linear time-derivative tracker [5], robust exact differentiation [6] and so forth [7], [8].

Firstly proposed by Han [9], the advantage of this noise-tolerant time optimal control (TOC)-based tracking differentiator (TD) is that it sets a weak condition on the stability of the systems to be constructed for TD and requires a weak condition on the input [10], [11]. In addition, it also has the advantage of smoothness compared with the sliding-mode-based differentiators with the chattering problem. However, the discrete time optimal control (DTOC) law ($Fhan$) of the TD proposed by Han is determined by comparing the position of the initial state with the isochronic region obtained through non-linear boundary transformation. This makes the structure of a TD to be complex with non-linear calculations, including square-root calculations. In this paper, the mathematical derivation of a new closed-form DTOC law is presented. Furthermore, zero-order hold of the control signal is introduced to improve the precision of the discretization model. Unlike the control law $Fhan$, the DTOC law is based on a linearized criterion that depends upon the position of the initial state point on the phase plane. In doing so, the new control law has a simpler structure that is much easier to be applied in practical engineering scenarios.

A power grid is an ever-growing complex infrastructure experiencing large changes such as integration of renewable generation, load growth, integration of electric vehicles and energy storage, to name a few [12]. A real-time accurate state estimation (SE) has deemed to be crucial to better operate the grid under such changes, increase the effectiveness of system utilization, ensure the security of supply and prevent blackouts [13]. Driven by these concerns, an increasing number of phasor measurement units (PMUs) are being deployed in power systems [14]. These devices provide accurate and synchronized voltage/current phasors and are thus capable of directly measuring the power-system state [15], [16]. This paper turns to the model-free control theory and proposes a discrete tracking differentiator to filter and estimate the real-time states of a power system.

The paper is organized as follows: the background on TOC for a continuous double integral system and its problem on direction digitization are introduced in Section II. The construction of the DTOC algorithm is introduced in Section III. In Section IV, comparison simulation results between Han's TD and the proposed one are presented to show the performance of signal-tracking filtering and differentiation acquisition, followed by experiment results on PMU field data from real power systems. Finally, Section V concludes the paper.

II. Background

In this section, some early results and basic concepts on the time optimal control (TOC) are reviewed. In particular, the TOC of the continuous double-integral system has received considerable attention in literature, which is defined as follow:

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = u, |u| \le r \end{cases} \tag{1}$$

where $x(t) = [x_1(t), x_2(t)]^T \in R^2$. The resulting control law that drives any initial state point to the origin in the minimum time is [17], [18]

$$u(x_1, x_2, r) = -r\mathrm{sign}(\mathit{\Gamma}(\mathrm{x}_1, \mathrm{x}_2)) \tag{2}$$

where $\mathit{\Gamma}(x_1, x_2) = x_1 + \frac{x_2|x_2|}{2r}$ is the switching curve. Denote $T(x_1, x_2)$ as the time that any state point $M(x_1, x_2)$ reaches the origin

$$T(x_1, x_2) = \frac{x_2}{r}s + \frac{2}{\sqrt{r}}\sqrt{\frac{x_2^2}{2r} + sx_1} \tag{3}$$

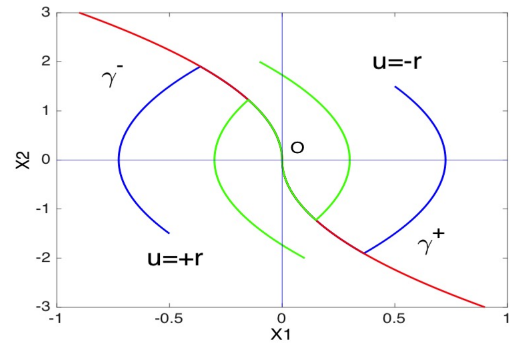

Fig. 1. Illustrations of the switching curve and the optimal trajectories

where $s = \mathrm{sign}(\mathrm{x}_1 + \frac{\mathrm{x}_2|\mathrm{x}_2|}{2\mathrm{r}})$. For continuous-time plant (1), we choose $T(x_1, x_2)$ as Lyapunov function. Thus we have

$$\begin{cases} \frac{\partial T}{\partial x_1} = \frac{s}{\sqrt{r}\sqrt{(x_2^2/2r)+sx_1}} \\ \frac{\partial T}{\partial x_2} = \frac{s}{r} + \frac{x_2/r}{\sqrt{r}\sqrt{(x_2^2/2r)+sx_1}} \\ \frac{dT}{dt} = \frac{\partial T}{\partial x_1}\dot{x_1} + \frac{\partial T}{\partial x_2}\dot{x_2} = -1 \end{cases}$$

There exists $T(x_1(t), x_2(t)) = -t + T(x_1(0), x_2(0))$, which indicates that any state can reach the origin along the optimal trajectory in finite time. The switching curve $\mathit{\Gamma} = \gamma^+ \cup \gamma^-$ and the optimal trajectories are shown in Fig. 1.

This time optimal control method has many advantages over linear controller: 1) the state arrives at the steady state in minimal and finite time; 2) superior disturbance rejection robustness against dynamic uncertainties. It can also be easily extended to the tracking problem by replacing x_1 and x_2 in (2) with $x_1 - v$ and $x_2 - \dot{v}$, respectively. Here v and $\dot{v}$ are the desired state trajectories. However, with the rapid development of computer control technology, most control algorithms are implemented in discrete time domain today. Direct digitization of the continuous TOC solution proves to be problematic in practice because of the high-frequency chattering of the control signal.

For driving the initial state back to the origin in the continuous system in (1), the control signal switches between its two extreme values around the switching curve $\Gamma(x_1, x_2)$ in (2). That is, when the initial state is located over the switching curve, the control signal takes on the extreme values, i.e., $u = -r$; otherwise, the control signal takes on $u = +r$. The control signal switches the sign after reaching the switching curve. For a continuous system, the control signal can switch instantaneously. For a discrete time system, however, the process of sign switching of the control signal will take place within the sampling period h. During the process, the corresponding state sequences would locate in a certain region (denoted as Ω) near the switching curve. The control signals for the state sequences in the region Ω are determined by the linearized criterion. The control signal varies from a certain positive (negative) value to a negative (positive) value when the control signal u passes from one side of the region Ω to the other. All initial state sequences located outside the region Ω when the control signal takes on extreme values, i.e., $u = +r$ or $u = -r$, would locate at certain curves, referred as boundary curves Γ_A and Γ_B. The region Ω is surrounded by these boundary curves. All states that correspond to $u = 0$ constitute another curve, which is referred to as the control characteristic curve Γ_C. We will introduce the above regions and curves in the next section.

III. Discrete Tracking Differentiator

In this section, a new and simple tracking differentiator (TD) with high precision based on discrete time optimal control (DTOC) law is proposed. In order to improve the precision of discretization model, zero-order hold of the control signal is introduced.

A. Problem Formulation

Considering zero-order hold of the control signal, we have that the discretization of (1) is

$$x(k+1) = Ax(k) + Bu(k), |u(k)| \leq r \tag{4}$$

where $A = \begin{bmatrix} 1 & h \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} h^2/2 \\ h \end{bmatrix}$ and $x(k) = [x_1(k), x_2(k)]^T$. The objective here is to derive a time optimal control law directly in discrete time domain. The problem is defined as follows:

DTOC Law: Given the system (4) and its initial state $x(0)$, determine the control signal sequence, $u(0)$, $u(1)$, ..., $u(k)$, such that the state $x(k)$ is driven back to the origin in a minimum and finite number of steps, subject to the constraint of $|u(k)| \leq r$. That is, finding $u(k^*)$, $|u(k)| \leq r$, such that $k^* = \min\{k | x(k+1) = 0\}$.

The basic idea in deriving the DTOC law is to find the control signal sequence for any initial state point $x(0) \in \Omega$ or $x(0) \notin \Omega$. The whole task is divided into two parts: **i**: Determine the boundary curves of the region Ω based on state backstepping approach, i.e., the representation of the initial condition $x(0) = [x_1(0), x_2(0)]^T$ in term of h and r, from which the state can be driven back to the origin in $(k+1)$ steps. **ii**: For any given initial condition $x(0) \in \Omega$ or $x(0) \notin \Omega$, find the corresponding control signal sequence as a function of $x(0)$.

B. Determination of Boundary Curves and the control characteristic curve

For any initial state sequence, there is at least one admissible control sequence, $u(0)$, $u(1)$, ..., $u(k)$ that can make the solution of (4) satisfy $x(k+1) = 0$. The solution of (4) with the initial condition $x(0)$ is

$$x(k+1) = A^{k+1}x(0) + \sum_{i=0}^{k} A^{k-i}Bu(i) \tag{5}$$

where $x(0) = [x_1(0), x_2(0)]^T$ and $i = 0, 1, 2, ..., k$. It manifests that $x(k+1) = 0$. Therefore, the initial condition satisfies

$$x(0) = -\sum_{i=0}^{k} A^{-i-1}Bu(i) \tag{6}$$

Based on the state back-stepping approach above, we determine the two boundary curves Γ_A and Γ_B as well as the control characteristic curve Γ_C as follows.

For any initial state located above the switching curve and entered the region Ω, suppose that the control signal sequence in the first step takes on $u(0) = -\alpha_1 r$, where α_1 is a variable, and from the second step on, the control sequence takes on $u(i) = +r$ $i = 1, 2, ..., k$. According to (6), we can obtain:

$$\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} h^2 r(-\frac{\alpha_1}{2} + \frac{k}{2} + \frac{k}{2}(k+1)) \\ hr(\alpha_1 - k) \end{bmatrix} \tag{7}$$

Simplifying $x(0)$ into x and eliminating the variable k, we can get

$$x_1 - \frac{x_2^2}{2r} + hx_2(\alpha_1 + 1) - \frac{\alpha_1(\alpha_1+1)}{2} rh^2 = 0 \tag{8}$$

Similarly, for any initial state located above the switching curve and entered the region Ω, suppose that the control signal sequence in the first step takes on $u(0) = +\alpha_1 r$, where α_1 is a variable, and from the second step on, the control sequence takes on $u(i) = -r$ $i = 1, 2, ..., k$, we can obtain

$$\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} h^2 r(\frac{\alpha_1}{2} - \frac{k}{2} - \frac{k}{2}(k+1)) \\ hr(-\alpha_1 + k) \end{bmatrix} \tag{9}$$

Simplifying $x(0)$ into x and eliminating the variable k, we have

$$x_1 + \frac{x_2^2}{2r} + hx_2(\alpha_1 + 1) + \frac{\alpha_1(\alpha_1+1)}{2} rh^2 = 0 \tag{10}$$

Furthermore, according to (8) and (10), we have

$$x_1 + \frac{x_2|x_2|}{2r} + hx_2(\alpha_1 + 1) + \frac{\alpha_1(\alpha_1+1)}{2} rh^2 \mathrm{sign}(x_2) = 0 \tag{11}$$

The boundary curves and the control characteristic curve depend on the value of the control signal sequence in the first step, that is, the value of α_1. When $\alpha_1 = -1$, the boundary curve $\Gamma_A : x_1 + \frac{x_2|x_2|}{2r} = 0$ can be obtained. When $\alpha_1 = 0$, we have the control characteristic curve $\Gamma_C : x_1 + \frac{x_2|x_2|}{2r} + hx_2 = 0$. When $\alpha_1 = 1$, the boundary curve $\Gamma_B : x_1 + \frac{x_2|x_2|}{2r} + 2hx_2 + rh^2 \mathrm{sign}(x_2) = 0$ can be obtained.

The above two boundary curves of the region Ω and the control characteristic curve are determined with the state backstepping method, and they are shown on the phase plane in Fig. 2.

C. Construction of the Discrete TOC Law

In this subsection, the DTOC law is obtained constructively based on the boundary curves, the control characteristic curve and regions proposed above. As shown in Fig. 2 , we assume that for any initial state $M(x_1, x_2)$ in the fourth quadrant $(x_1 > 0, x_2 < 0)$, there is an auxiliary line $x_2 = x_2(M)$ that intersects with the boundary curves and the control

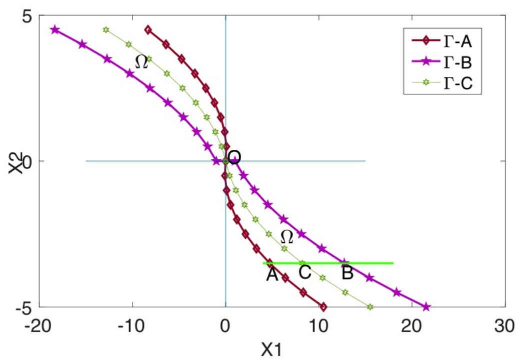

Fig. 2. Illustrations of Γ_A, Γ_B, Γ_C and the region Ω.

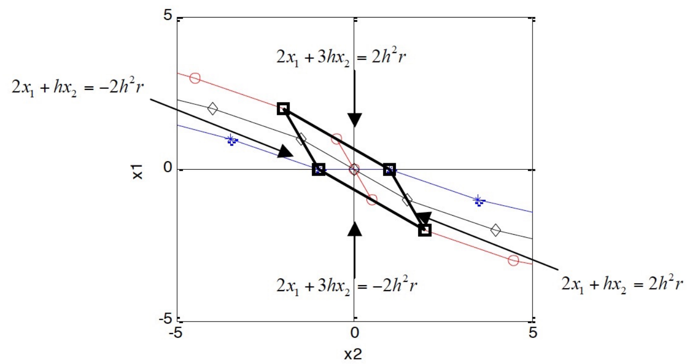

Fig. 3. Illustration of Ω_2.

characteristic curve at points A, C and B (in the direction of x_1). Their x-axis value x_A, x_B and x_C are

$$\begin{cases} x_A = \frac{x_2^2}{2r} \\ x_B = \frac{x_2^2}{2r} + 2h|x_2| + h^2 r \\ x_C = \frac{x_2^2}{2r} + h|x_2| \end{cases} \tag{12}$$

For any initial state $M(x_1, x_2)$ satisfying $x_1 < x_A$ or $x_1 > x_B$, the control signal is taken as $u = +r$ or $u = -r$. For any initial state $M(x_1, x_2)$ satisfying $x_1 \in [x_A, x_C]$, the control signal can be determined as follows:

$$u = -r\alpha \mathrm{sign}(\mathrm{x}_2) \tag{13}$$

where $\alpha = \frac{x_C - x_1}{x_C - x_A}$. For any initial state $M(x_1, x_2)$ satisfying $x_1 \in [x_C, x_B]$, the control signal is calculated as:

$$u = r\beta \mathrm{sign}(\mathrm{x}_2) \tag{14}$$

where $\beta = \frac{x_1 - x_C}{x_B - x_C}$. When the initial state $M(x_1, x_2)$ is in the second quadrant, the control signal sequence can be constructed similarly.

However, when the initial state $M(x_1, x_2)$ (located outside the region Ω) is in the first or third quadrant, there are two different cases for choosing the control signal. When $M(x_1, x_2)$ cannot be driven back to the origin within two steps, that is, the initial state does not satisfy the condition $x_1^2 + x_2^2 = 0$, let $u = -r\mathrm{sign}(x_1 + hx_2)$. When $M(x_1, x_2)$ can be driven back to the origin within two steps, the initial state $x(0)$ and the corresponding control signal sequence satisfies (6), i.e.,

$$\begin{cases} x_1(1) = x_1(0) + hx_2(0) + \frac{h^2}{2}u(0) \\ x_2(1) = x_2(0) + hu(0) \\ x_1(2) = x_1(1) + hx_2(1) + \frac{h^2}{2}u(1) \\ x_2(2) = x_2(1) + hu(1). \end{cases}$$

Furthermore, when $M(x_1, x_2)$ can be driven back to the origin within two steps, the corresponding control signals can be derived as follows:

$$\begin{cases} u(0) = -\frac{2x_1(0) + 3hx_2(0)}{2h^2} \\ u(1) = -\frac{2x_1(1) + 3hx_2(1)}{2h^2}. \end{cases} \tag{15}$$

The region in which any $x(0)$ can be driven back to the origin within two steps, denotes as Ω_2 (see Fig. 3), is surrounded by two pairs of parallel lines $2x_1 + hx_2 = \pm 2h^2 r$ and $2x_1 + 3hx_2 = \pm h^2 r$.

Now, any initial state $M(x_1, x_2)$ on the $x_1 - x_2$ plane can be driven back to the origin in a minimum and finite number of steps according to the control signal sequence above. The complete DTOC law is described as follows:

Step 1: Setting $y_1 = 2x_1 + 3hx_2, y_2 = 2x_1 + hx_2$, if $|y_1| > 2h^2 r$ or $|y_2| > 2h^2 r$, then $M(x_1, x_2)$ cannot be driven back to the origin within two steps, i.e., $M(x_1, x_2) \notin \Omega_2$, and go to next step; otherwise, go to Step 5;

Step 2: If the initial state $M(x_1, x_2)$ satisfies $x_1 x_2 \geq 0$ and $M(x_1, x_2) \notin \Omega_2 \cup \Omega$, then the control signal takes on $u = -r\mathrm{sign}(x_1 + x_2)$;

Step 3: Determine the boundary of the region Ω, i.e., $x_A = \frac{x_2^2}{2r}$, $x_B = \frac{x_2^2}{2r} + 2h|x_2| + h^2 r$ and $x_C = \frac{x_2^2}{2r} + h|x_2|$;

Step 4: If $|x_1| > x_B$, then the control signal takes on $u = -r\mathrm{sign}(x_1)$; if $|x_1| < x_A$, then the control signal takes on $u = r\mathrm{sign}(x_1)$; if $x_1 \in [x_A, x_C]$, the control signal takes on $u = -r\alpha\mathrm{sign}(x_2)$; if $x_1 \in [x_C, x_B]$, $u = +r\beta\mathrm{sign}(x_2)$, where $\alpha = \frac{x_C - x_1}{x_C - x_A}$ and $\beta = \frac{x_1 - x_C}{x_B - x_C}$;

Step 5: If the initial state $M(x_1, x_2) \in \Omega_2$, then the control signal takes on $u(i) = \frac{2x_1(i) + 3hx_2(i)}{2h^2}$, where $i = 0, 1$;

Step 6: The algorithm ends.

From the deduction above, the mathematical derivation of a closed-form discrete time optimal control law (**DTOC Law**) as a function of x_1, x_2, r and h, denoted as $u(k) = Ftd3(x_1(k), x_2(k), r, h)$, is obtained. Based on the control law above, we can then construct the following TD:

$$\begin{cases} u(k) = Ftd3(x_1(k) - v(k), x_2(k), r, c_0 h) \\ x_1(k+1) = x_1(k) + hx_2(k) + \frac{1}{2}h^2 u(k) \\ x_2(k+1) = x_2(k) + hu(k) \end{cases} \tag{16}$$

where r is the quickness factor, c_0 is the filtering factor, h is sampling period and v is the given signal.

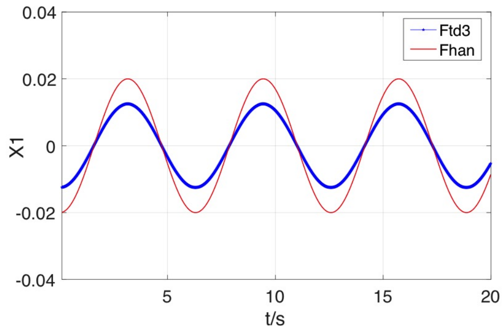

Fig. 4. Errors of signal tracking filtering output.

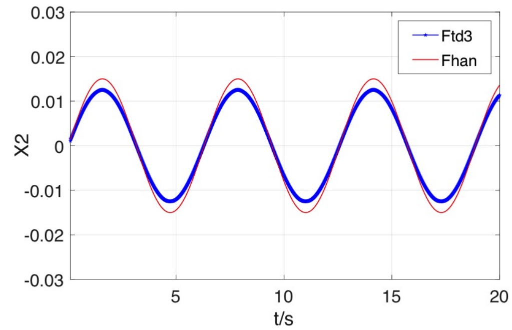

Fig. 5. Errors of signal differentiation acquisition.

IV. Numerical Simulations and PMU data Processing

We conducted numerical simulations to compare the performance of the proposed differentiator with that of the existing ones in signal-tracking filtering and differentiation acquisition. We also conducted experiments involving using this model-free TD as a state estimation filter to filter and estimate the field PMU data.

A. Numerical Simulations

DI. Tracking differentiator based on discrete-time optimal control $Fhan$ from [9].

$$\begin{cases} u(k) = Fhan(x_1(k) - v(k), x_2(k), r, c_0 h), \\ x_1(k+1) = x_1(k) + h x_2(k), \\ x_2(k+1) = x_2(k) + h u(k), |u(k)| \leq r \end{cases}$$

DII. The proposed tracking differentiator $Ftd3$.

$$\begin{cases} u(k) = Ftd3(x_1(k) - v(k), x_2(k), r, c_0 h) \\ \\ x_1(k+1) = x_1(k) + h x_2(k) + \frac{1}{2} h^2 u(k) \\ \\ x_2(k+1) = x_2(k) + h u(k), |u(k)| \leq r \end{cases}$$

The same initial value ($x_1(0) = 0$, $x_2(0) = 2$) and input signal sequence $v(t) = \sin(3t) + \gamma(t)$ are selected for all simulations, where $\gamma(t)$ denotes the evenly distributed white noise with an intensity of 0.05. For differentiator **DI** and differentiator **DII**, the sampling period is $h = 0.005s$, the quickness factor is $r_0 = 200$, and the filtering factor is $c_0 = 5$. The results for comparing the errors of signal-tracking and differentiation acquisition of these two differentiators are plotted in Fig. 4 and Fig. 5.

From these simulation results, we see that, based on discrete-time optimal control, tracking differentiator **DII** performs better in signal-tracking filtering and differentiation acquisition than differentiator **DI**.

B. PMU data Processing

In this subsection, we use the proposed discrete tracking differentiator to filter real-time PMU data. The data including current and voltage phasors are recorded during normal operation at a 20-KV power plant in Jiangsu province, China. The sampling frequency is 4960 Hz. In the processing, the parameters for the proposed tracking differentiator are adapted as follows: the sampling period is $h = 0.001s$, the quickness factor is $r_0 = 800$, and the filtering factor is $c_0 = 8$. The field PMU data and the corresponding processing results are plotted in Fig. 6.

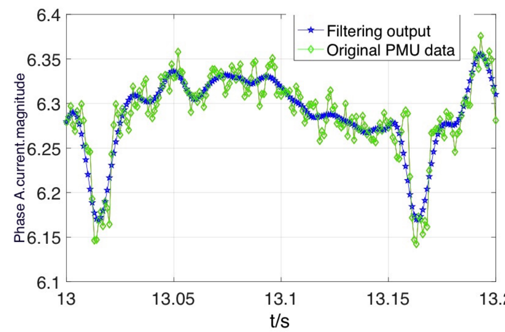

Fig. 6. The output of current magnitude state estimation filtering using the real-time PMU data.

From Fig. 6, we can see that using the proposed tracking differentiator, the filtering output of state estimation is much smoother than the original PMU data.

V. Conclusions

In this paper, we proposed a novel discrete time optimal control (DTOC) law-based tracking differentiator (TD). Zero-order hold of the control signal is introduced to improve the precision of discretization model. This closed-form nonlinear state feedback clearly demonstrates that time optimal control in discrete time is not necessarily bang-bang control and this characteristic makes the new control law advantageous in engineering applications. Numerical simulation results show that, compared with Han's TD, the proposed TD performs better in signal-tracking filtering and differentiation acquisition. Using the proposed TD, the output of state estimation filtering of the field PMU data is smooth. Without taking complex power system modeling and historical data into account or requesting robustness of parameters' setting, the proposed TD may provide a promising approach to filter the real-time PMU data, enabling better state estimation and fault detection in power systems.

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