A Robust Optimization Approach for Energy Generation Scheduling in Microgrids

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Abstract

In this paper, a cost minimization problem is formulated to intelligently schedule energy generations for microgrids equipped with unstable renewable sources and combined heat and power generators. In such systems, the fluctuant net demands (i.e., the electricity demands not balanced by renewable energies) and heat demands impose unprecedented challenges. To cope with the uncertainty nature of net demand and heat demand, a new flexible uncertainty model is developed. Specifically, we introduce reference distributions according to predictions and field measurements and then define uncertainty sets to confine net and heat demands. The model allows the net demand and heat demand distributions to fluctuate around their reference distributions. Another difficulty existing in this problem is the indeterminate electricity market prices. We develop chance constraint approximations and robust optimization approaches to firstly transform and then solve the prime problem. Numerical results based on real-world data evaluate the impacts of different parameters. It is shown that our energy generation scheduling strategy performs well and the integration of combined heat and power generators effectively reduces the system expenditure. Our research also helps shed some illuminations on the investment policy making for microgrids.

Keywords: microgrid, energy generation scheduling, demand uncertainties, robust optimization, uncertainty set, reference distribution.

1. Introduction

The electricity grid is being restructured to allow high penetration of distributed generators to become more environment friendly and cost effective [1]. The growth and evolution of the power grids is expected to come with the plug-and-play of the basic structure called microgrid. Microgrids can operate in grid-connected mode, in which they are allowed to import power from the electricity grid, or in islanded mode, where they are isolated from the upstream power grid and use their local generators as the source of power supply when needed. There are world-wide deployments of pilot microgrids, especially in Europe, e.g., those reported in [2] and [3]. Reference [2] investigates the key drivers enabling the market feasibility of microgrids in different contexts. While in [3], the technical, social, economic, and environmental benefits provided by microgrids are studied.

Energy generation scheduling to achieve robust and economic power supply is an essential component in microgrids. Two features of microgrids are the integration of large-scale renewable sources and the use of combined heat and power (CHP) generators. Such features, however, impose significant difficulties on the design of intelligent control strategies for microgrids. Traditional generation scheduling schemes are typically based on perfect prediction of future demands [4], which is hardly the case in the microgrids since small-scale demands are hard to predict and renewable energies are highly volatile. Furthermore, although the integration of CHP generators can bring great economic benefits to microgrids by simultaneous production of useful heat and electricity outputs, thereby increasing the overall efficiency and bringing environmental benefits, it brings new uncertainties to the scheduling problem: the heat demand exhibits a new stochastic pattern and makes it more difficult to predict the overall energy demands. On top of these, the real-time pricing in electricity market yields another uncertainty dimension to the scheduling problem. The microgrid has to make a proper strategic decision on the amount of power to be imported so as to cope with the financial risks brought by price uncertainty. Because of these unique challenges, it remains an open issue to design robust and cost-effective energy generation scheduling strategies for microgrids.

1.1. Related Work

Energy generation scheduling is the process of effectively scheduling different energy sources (local generators, central grid, renewable energy generations, etc.) to meet the energy requests at the minimum cost subject to various physical constraints of the power systems. It is a classic problem in electricity system which is composed of two aspects, namely unit commitment (UC) [5] and economic dispatch (ED) [6], respectively. The UC problem involves determining the start-up and shut-down schedules for generator units to be used to meet forecast demand over a short time in future. It is a complex optimization problem with both integer and continuous variables and has been shown to be NP-complete in general. The basic UC methods reported in literature include priority listing method [7], where the generator units are committed according to a priority order based on unit average full load cost; dynamic programming method [8], where the complicated scheduling problem is broken down into a sequence of decision steps over time in a recursive manner; Lagrangian relaxation method [9], where the Lagrangian dual of the UC is maximized with standard sub-gradient techniques and a reserve-feasible dual solution is computed; and integer programming method [10] [11], where binary variables are adopted to model the startup, shutdown and on/off states for every generator unit and every time period, etc. Once the UC problem has determined the start-up and shut-down schedules, the ED problem seeks to find the optimal allocation of electric power outputs from various available generators without alternating their on/off status. In [5], a genetic algorithm (GA) solution to the UC problem is presented. Authors of [6] propose a particle swarm optimization (PSO) method for solving the ED problem in power systems. Readers can refer to comprehensive surveys on UC [12] and ED [13] for more details, in which different methods used in the UC and ED problem-solving techniques are summarized and analyzed.

Conventional energy generation scheduling is typically conducted 24 hours in advance (day ahead) and based on the fact that the system load can be forecast with reasonably good accuracy one day in advance. In microgrids, however, this is no longer the case due to the fact that accurate predictions of small-scale electricity and heat demands, renewable energy supplies and electricity market prices are very difficult, as we stated earlier. Some recent literature has investigated energy generation scheduling of microgrids [14, 15, 16, 17, 18]. In [14], a multi-objective optimization of economic load dispatch for a microgrid is investigated using evolutionary computation. The paper aims at minimizing the emission of the thermal generators and minimizing the total operating cost. In [15], a generalized formulation for intelligent energy management of microgrid is proposed using artificial intelligence techniques jointly with linear-programming-based multi-objective optimization. Similarly, in [16], an intelligent energy management system is proposed for optimal operation of a CHP-based microgrid over a 24-hour time interval. Authors of [17] and [18] also propose different energy management strategies based on different assumptions. The limitation of these results, however, is that they all assume that the energy demands and supplies are known ahead of time, which is rarely the case in practice.

There also exist some studies considering demand and supply uncertainties when scheduling the energy generation. Such work can be categorized into two groups: the stochastic optimization based approaches [19, 20, 21, 22, 23, 24, 25] and robust optimization based approaches [26, 27, 28, 29, 30]. In [19], a stochastic programming approach is adopted in the development of the proposed bidding strategies for microgrid producers and loads. In [20], the authors develop a solution method for scheduling units of a power-generating system to produce electricity by taking into consideration the stochastic nature of the hourly load and its correlation structure. In [21], a stochastic model for the long-term solution of security-constrained unit commitment is proposed. A more complicated scenario can be found in [22], in which an efficient stochastic framework is developed to investigate the effect of uncertainty on the operation management of microgrids. The proposed stochastic framework considers the uncertainties of load forecast error, wind turbine generation, photovoltaic generation and market price concurrently. Authors of [23] examine the impact of the stochastic nature of wind on planning and dispatch of a system. Similarly, authors of [24] compare stochastic and reserve methods and evaluate the benefits of a combined approach for the efficient management of uncertainty in the unit commitment problem. In [25], a two-stage stochastic objective function aiming at minimizing the expected operational cost is implemented. Note that the stochastic optimization approach explicitly incorporate a probability distribution function of the uncertainty, and they often rely on enumerating discrete scenarios of the uncertainty realizations. Such approaches mainly have two practical limitations. First, it may be difficult and costly to obtain an accurate probability distribution of uncertainty. Second, the solution only provides probabilistic guarantees to the system reliability. To obtain a highly reliable guarantee requires a huge number of samples, which poses substantial computational challenges.

In recent literature, robust optimization has received growing attentions as a modeling framework for optimization under uncertainty. In [26], a two-stage adaptive robust unit commitment model is proposed for the security constrained unit commitment problem in the presence of nodal net injection uncertainty. In [27], a robust optimization approach is proposed to accommodate wind output uncertainty, with the objective of providing a robust unit commitment schedule for the thermal generators in the day-ahead market. In [28], a power scheduling approach is proposed based on robust optimization to address the intrinsically stochastic availability of renewable energy sources. References [29] and [30] also present robust optimization based approaches for optimal microgrid management considering wind power or energy consumption uncertainties. Instead of postulating explicit probability distribution, robust optimization confines the random variable in a pre-defined uncertainty set containing the worst-case scenario. For instance, in [26, 27, 28, 31, 32, 29, 30], uncertainties in price prediction or renewable energy generation are presented as interval values with deterministic lower and upper bounds, and the framework developed in [33] and [34] is incorporated to solve the problem. Without requiring an explicit probability distribution, the uncertainty can be characterized more flexibly. In addition, the conservativeness of the solution can easily be controlled and the problem is always computationally tractable both practically and theoretically even for large scale problems.

In our study, the robust optimization concept is also applied to tackle the uncertainties in energy generation scheduling problem of microgrids. Different from the previous robust optimization works [26, 27, 28, 31, 32, 29, 30] which confine the uncertainty within a lower and upper bounds, in our work, we propose a new uncertainty model to characterize the renewable energy and user demand uncertainties, which can provide more statistical details in describing the underlying uncertainty. Moreover, the proposed uncertainty model is also flexible enough that we can incorporate more information into the uncertainty model when such information is available.

1.2. Main Contributions

In this paper, we consider a robust optimization based energy generation scheduling problem in a CHP-microgrid scenario considering the net demand (the electricity demand not balanced by renewable energy) uncertainty, heat demand uncertainty and electricity price uncertainty. The main contributions of this paper can be briefly summarized as follows:

- We propose a new flexible uncertainty model to capture the fluctuant nature of the net demand and heat demand. Specifically we extract reference distributions as useful references and allow the actual distributions of net demand and heat demand to vary around their references. To the best of our knowledge, this is the first time that distribution uncertainty model is adopted to depict the indeterminacy nature of net demand and heat demand.

- We develop chance constraint approximation and robust optimization approaches based on our uncertainty model to transform the constraints with random variables into typical linear constraints. Then an iterative algorithm is designed to solve the problem.

- Price uncertainty is addressed by adopting robust optimization techniques, which allows the degree of conservatism to be controlled easily. We finally transform the prime problem into a mixed integer linear programming (MILP) problem, which can be solved efficiently by commercial solvers.

- Numerical results based on real-world data evaluate the impacts of different parameters and help provide some insights on designing investment policies for microgrid. It is also shown that the proposed energy generation scheduling strategy achieves considerable cost savings and the integration of CHP generators can effectively reduce the system expenditure.

The remainder of this paper is organized as follows. Section 2 introduces the particulars of the system operation. In Section 3, we introduce the mathematical depiction of the energy generation scheduling problem and the uncertainty models of net and heat demands. Section 4 presents the chance constraint approximation and robust optimization approach for handling the demand balancing and price uncertainty. The simulation results and discussions are shown in Section 5. The parameters and calibration data are drawn from real-world statistics. Finally, we conclude our paper in Section 6.

2. System Model

We consider a microgrid comprising a number of homogeneous CHP generators, a renewable energy generation system and a local heating system. The microgrid is operated on the grid-connected mode, such that it can purchase electricity from the external utility grid when needed. The illustration of the microgrid system is shown in Fig. 1. The main symbols utilized in the paper and their meanings are listed in Table 1. The particulars of the system operation are explained in the following subsections.

Table 1: Notations used in this paper

Symbol	Defination
$\mathcal{A}$	set of CHP generators
a	index of CHP generator, $a \in \mathcal{A}$
c_a^s	start up cost of turning on the generator a
c_a^b	sunk cost of maintaining the generator a
c_a^m	marginal cost for the generator a
$\mathcal{H}$	the set of time slots
$\mathbf{x_a}$	energy generation scheduling vector of CHP a
$\mathbf{y_a}$	state vector of CHP a (binary)
E_a^{min}	the minimum stable output capacity of CHP a
E_a^{max}	the maximum electricity output capacity of CHP a
η_a	heat-electricity ratio for the generator a
p_g	price of heating system for providing one unit of heat
U^h	amount of heat generated from heating system at time h
p_s^h	electricity market price at time h
$\hat{p}_s^h$	lower bound of the predicted electricity market price at time h
d^h	uncertainty range of electricity market price at time h
V^h	electricity obtained from outside power grid at time h
L^h	net demand at time h (random variable)
S^h	heat demand of the microgrid at time h (random variable)
$f_0(L^h)$	electricity demand distribution at time h
$g_h(L^h)$	reference distribution of $f_0(L^h)$
D_h	distance limit of $f_0(L^h)$'s uncertainty set
$\boldsymbol{U}_r(\cdot)$	uncertainty set based on KL divergence
ϵ	fault tolerance limit of the power grid

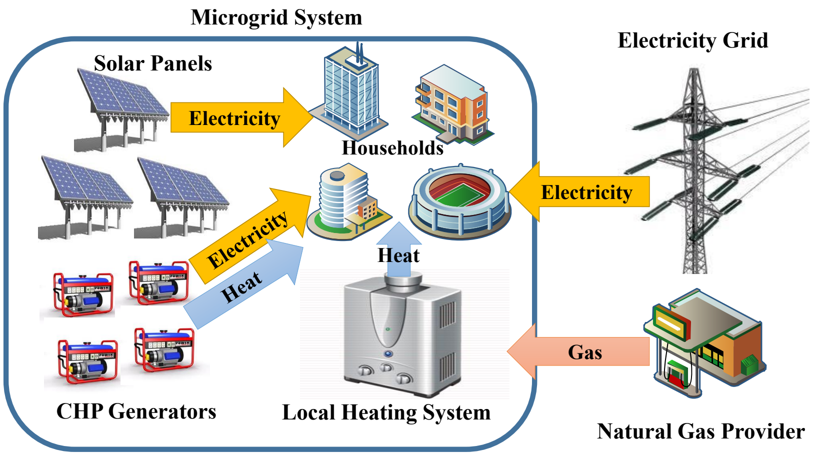

Figure 1: An illustration of a typical microgrid system.

2.1. CHP Generators

We divide time into discrete time slots with equal length. Let $\mathcal{A}$ denote the set of CHP generators. Further denote the start up cost of turning on a generator a as c_a^s, the sunk cost of maintaining the generator a in active mode for one time unit as c_a^b, and the marginal cost for the generator a to produce one unit of electricity as c_a^m. Adopting a general generator model, we define energy generation scheduling vector $\mathbf{x_a}$ and state vector $\mathbf{y_a}$ as follows:

$$\mathbf{x_a} = [x_a^1, x_a^2, ..., x_a^H] \text{ and } \mathbf{y_a} = [y_a^1, y_a^2, ..., y_a^H], \tag{1}$$

where $H \geq 1$ is the scheduling horizon which indicates the number of time slots ahead that are taken into account for decision making in the energy generation scheduling. For each coming time slot $h \in \mathcal{H} = [1, 2, ..., H]$, let a binary variable $y_a^h = 0/1$ denote the state of generator a (on/off) and a variable x_a^h denote the dispatched load to generator a. For each generator a with the maximum electricity output capacity E_a^{max} and the minimum stable generation E_a^{min}, we have

$$y_a^h \cdot E_a^{min} \le x_a^h \le y_a^h \cdot E_a^{max}. \quad (2)$$

The CHP generators can efficiently generate electricity and useful heat energy simultaneously. Let η_a denote the heat-electricity ratio for generator a, which means that the CHP generator a can supply η_a units of heat for free when generating one unit of electricity. Alternatively, heat can be supplied by local heating system at a price of p_g per unit. We use the variable U^h to denote the amount of heat generated from local gas heaters at time slot h. Note that in this paper, we omit the ramping-up and ramping-down constraints of CHP generators since we consider fast response CHP generators such as gas turbines or microturbines, which have fast ramping rates and are able to start from cold to full capacity in 1-10 mins [35].

2.2. *Electricity from External Utility Grid*

The microgrid can import electricity from outside electricity grid for the unbalanced power demand in an on-demand manner. We assume that the electricity market price at time h is p_s^h, which is a bounded random variable that takes value in $[\hat{p}_s^h, \hat{p}_s^h + d^h]$. $\hat{p}_s^h$ denotes the lower bound of the predicted price. $d^h > 0$ denotes that there exists price uncertainty (financial risks) at time h while $d^h = 0$ indicates the price at time h is known in advance. The amount of electricity obtained from electricity grid at time h is denoted as V^h.

2.3. Fluctuant Electricity and Heat Demand

Renewable energy generation can be regarded as a non-positive demand [4]. Denote the net demand at time h as L^h, which is a random variable of which the probability distribution may not be known. Similarly, the heat demand of the microgrid S^h is also random. Accurate prediction of small-scale demands and renewable energy generation is difficult to obtain due to limited management resources and their unpredictable nature. We need a proper uncertainty model to capture the indeterminacy properties of net and heat demands. A central requirement to the microgrid is to set the generation source power such that the electricity and heat supplies could meet the demands. This statement can be described as

$$V^h + \sum_{a \in \mathcal{A}} x_a^h \geq L^h \tag{3}$$

$$U^h + \sum_{a \in \mathcal{A}} \eta_a \cdot x_a^h \geq S^h. \tag{4}$$

3. Problem Formulation

In this section, a cost minimization problem formulation which incorporates CHP generation constraints, uncertain net demand, uncertain heat demand and time varying electricity prices is first given. The uncertainty model for describing the randomness of net demand and heat demand is then demonstrated.

3.1. Cost Minimization Formulation

The microgrid aims to minimize the operation cost of the whole system over the entire time horizon. The cost minimization formulation is defined as follows:

$$
\begin{aligned}
\min_{\mathbf{X},\mathbf{Y},\mathbf{V},\mathbf{U}} \quad & \sum_{h=1}^{H} \Bigg\{ p_g \cdot U^h + p_s^h \cdot V^h + \\
& \sum_{a \in \mathcal{A}} \left[c_a^m \cdot x_a^h + c_a^b \cdot y_a^h + c_a^s \cdot (y_a^h - y_a^{h-1})^+ \right] \Bigg\} \\
\text{s.t.} \quad & (2)\ (3)\ (4),\ y_a^h \in \{0,1\} \\
& x_a^h, V^h, U^h \in \mathbb{R}_0^+, h \in \mathcal{H}, a \in \mathcal{A},
\end{aligned} \tag{5}
$$

where $\mathbf{X} = [\mathbf{x_1}, \mathbf{x_2}, ..., \mathbf{x_a}, ...]^{\mathrm{T}}$ and $\mathbf{Y} = [\mathbf{y_1}, \mathbf{y_2}, ..., \mathbf{y_a}, ...]^{\mathrm{T}}$ are matrices of decision vectors $\mathbf{x_a}$ and $\mathbf{y_a}$ for $a \in \mathcal{A}$, respectively; $\mathbf{V} = [V^1, V^2, ..., V^h, ...]$ and $\mathbf{U} = [U^1, U^2, ..., U^h, ...]$ are vectors of decision variables V^h and U^h for $h \in \mathcal{H}$, respectively; $(\cdot)^+$ is a function where $(x)^+ = \max(0, x)$. The cost function comprises the cost of electricity from outside power grid, the cost of generating heat from local heat generators, and the operation and start-up cost of CHP generators for the entire time horizon H.

A difficulty in solving this problem lies in the correlation term $(y_a^h - y_a^{h-1})^+$. By introducing an auxiliary variable z_a^h into the problem formulation, an equivalent expression can be obtained as:

$$\begin{aligned}
\min_{\mathbf{X},\mathbf{Y},\mathbf{Z},\mathbf{V},\mathbf{U}} \quad & \sum_{h=1}^{H} \Bigg\{ p_g \cdot U^h + p_s^h \cdot V^h + \\
& \sum_{a \in \mathcal{A}} \left[c_a^m \cdot x_a^h + c_a^b \cdot y_a^h + c_a^s \cdot z_a^h \right] \Bigg\} \\
\text{s.t.} \quad & z_a^h \geq 0,\ z_a^h \geq y_a^h - y_a^{h-1} \\
& (2)\ (3)\ (4),\ y_a^h, z_a^h \in \{0,1\} \\
& x_a^h, V^h, U^h \in \mathbb{R}_0^+, h \in \mathcal{H}, a \in \mathcal{A},
\end{aligned} \tag{6}$$

where $\mathbf{Z}_{|\mathcal{A}| \times H}$ is the matrix of auxiliary variable z_a^h for $a \in \mathcal{A}$, $h \in \mathcal{H}$. The objective for introducing an auxiliary variable z_a^h into problem formulation (5) is to have an equivalent, solvable problem without the correlation term $(y_a^h - y_a^{h-1})^+$. Another difficulty in solving problem (5) is the indeterminacy of net demand L^h and heat demand S^h existing in constraints (3) and (4). Note that to optimize over the space defined by (3) and (4) amounts to solving an optimization problem with potentially large or even infinite number of constraints. Obviously, this realization of uncertainties is intractable. Next, we develop a practical and flexible model to capture the uncertainties of L^h and S^h.

3.2. Probability Distribution Measure of Uncertainties

It is generally difficult to characterize the net demand and heat demand. In our optimization model, operations on the random variables L^h and S^h are cumbersome and computationally intractable. Moreover, in practice, we may not know the precise distributions of L^h and S^h. Solutions based on assumed distributions hence may not be justified. We usually measure the variability of a random variable using its variance or second moments which, however, may not provide sufficient details in describing the random variables. In this paper, we extract a reference distribution, rather than moment statistics, from historical data and predictable information, to capture the distribution properties. Since net demand and heat demand distributions may fluctuate over time and hard to be described in closed-form expressions, we adopt empirical distributions as useful references and allow the actual distributions to fluctuate around them. For example, we may assume that the net demand distribution $f_0(L^h)$ is shifting around a known distribution $g_h(L^h)$, which can be obtained based on predictions and long-term field measurements. In the following part of this paper, we only show the way to deal with random variable L^h. The method to tackle with random variable S^h is exactly the same.

The discrepancy between $f_0(L^h)$ and its reference $g_h(L^h)$ can be described by a probabilistic distance measure: the Kullback-Leibler (KL) divergence [36], which is a non-symmetric measure of the difference between two probability distributions. Name these two distributions as $f(L^h)$ and $g(L^h)$, respectively. Generally, one of the distributions, say, $f(L^h)$, represents the real distribution through precise modeling, while the reference $g(L^h)$ is a closed-form approximation based on the theoretic assumptions and simplifications. The definition of the KL divergence between two continuous distributions is given as follows:

$$
\begin{aligned}
&D_{KL}(f(L^h), g(L^h)) = \\
&\int_{L^h \in S} [\ln f(L^h) - \ln g(L^h)] f(L^h) \mathrm{d}L^h,
\end{aligned} \tag{7}
$$

where S is the integral domain. When distributions $f(L^h)$ and $g(L^h)$ are close to each other, the distance measure is close to zero. Adopting the KL divergence, we define the distribution uncertainty set as follows:

$$\begin{aligned} &\boldsymbol{U}_r(g(L^h), D_0) = \\ &\{f(L^h) \mid \mathbb{E}_f[\ln f(L^h) - \ln g(L^h)] \leq D_0\}, \end{aligned} \tag{8}$$

where $D_0 > 0$ represents a distance limit which may be obtained from empirical data or real-time measurement. It indicates net demand's variation level. If the net demand is highly volatile, we have less confidence on the reference distribution and thus may set a larger distance limit.

Considering the electricity demand distribution $f_0(L^h)$ with reference distribution $g_h(L^h)$ and distance limit D_h, we have the following constraints for electricity demand distribution $f_0(L^h)$:

$$\mathbb{E}_{f_0}[\ln f_0(L^h) - \ln g_h(L^h)] \leq D_h \tag{9}$$

$$\mathbb{E}_{f_0}[1] = 1. \tag{10}$$

Equation (10) represents the fact that the integral of a probability density function over the entire space is equal to 1. With (9) and (10), we are now ready to transform the constraint (3) (similarly for (4)) to allow efficient solution of the problem (6).

Note that in the proposed approach, renewable energy is treated as a non-positive demand. We integrate user demand and renewable energy generation together and denote it as the net demand. The combined uncertainties from both user and supply sides are described by an uncertainty set as defined in (9) and (10).

The proposed model also allows some convenient extensions to include and handle more components in the microgrid systems. For example, to incorporate the reserve constraint into the proposed model, we only need to add the reserve constraints, which are linear functions, into the formulation (5) and then add a quadratic reserve cost into the objective function [37]. The new problem could still be transformed into a mixed integer programming (MIP) problem and the algorithm to be introduced in the next section can still be applied with virtually no change.

Remark: Proper estimations of reference distribution and distance limit may be obtained by various methods, for instance, the Kernel Density Estimation (KDE), which is a non-parametric way to estimate the probability density function of a random variable [38, 39]. KDE handles the fundamental data smoothing problem where inferences about the population are made based on finite data sampling. Adopting such a method typically involves analyzing a large amount of historical data. Detailed discussions on such approaches, however, are beyond the scope of this paper.

4. Optimization Algorithms

In this section, we present the optimization algorithms for solving problem (6). We first develop a robust approach for handling constraints (3) and (4), and then decompose (6) into a subproblem and a main problem to allow easier solution.

Finally, a robust approach for tackling the financial risk inducted by time varying electricity market clearing prices is demonstrated.

4.1. Robust Approach for Constraints (3) and (4)

As shown in (3), the net demand balance can be expressed as $V^h + \sum_{a\in\mathcal{A}} x_a^h \geq L^h$. In practice, a decision criterion is to properly set decision $V^h + \sum_{a\in\mathcal{A}} x_a^h$ to allow good confidence that (3) is satisfied. To achieve that, we may introduce a small value ϵ to control the degree of conservatism and change the above expression into a chance constraint:

$$\mathbf{P}(L^h \geq V^h + \sum_{a\in\mathcal{A}} x_a^h) \leq \epsilon \tag{11}$$

where ϵ is the fault tolerance limit of the power grid, representing the acceptable probability that the desirable power supply is not attained. Then we can have this expression that

$$\max_{f_0(L^h)\in\boldsymbol{U}_r(g_h,D_h)} \mathbf{P}(L^h \geq V^h + \sum_{a\in\mathcal{A}} x_a^h) \leq \epsilon, \tag{12}$$

which is equivalent to:

$$\max_{f_0(L^h)\in\boldsymbol{U}_r(g_h,D_h)} \int_{V^h+\sum_{a\in\mathcal{A}} x_a^h}^{+\infty} f_0(L^h)\mathrm{d}L^h \leq \epsilon. \tag{13}$$

Defining $\mathcal{L}^h = V^h + \sum_{a\in\mathcal{A}} x_a^h$ as the robust electricity supply (ES) decision, which equals the amount of electricity generated and imported at time slot h, we introduce an auxiliary function as follows:

$$h(L^h, \mathcal{L}^h) = \begin{cases} 0, & L^h \leq \mathcal{L}^h; \\ 1, & L^h > \mathcal{L}^h. \end{cases} \tag{14}$$

The left part of inequality (13) then can be formulated into an optimization problem:

$$\max_{f_0(L^h)} \quad \int_0^{+\infty} h(L^h, \mathcal{L}^h) \cdot f_0(L^h) \mathrm{d}L^h \tag{15}$$

$$\text{s.t.} \quad \mathbb{E}_{f_0}[\ln f_0(L^h) - \ln g_h(L^h)] \leq D_h \tag{16}$$

$$\mathbb{E}_{f_0}[1] = 1 \tag{17}$$

Define $K_f^h(\mathcal{L}^h) = \max_{f_0(L^h) \in \boldsymbol{U}_r(g_h, D_h)} \int_0^{+\infty} h(L^h, \mathcal{L}^h) \cdot f_0(L^h) \mathrm{d}L^h$ as the worst-case fault probability. We can then get a worst-case mapping $\mathcal{M}_{wc}^h$ which maps the robust ES decision $\mathcal{L}^h$ to $K_f^h(\mathcal{L}^h)$:

$$\mathcal{M}_{wc}^h : \quad \mathcal{L}^h \longrightarrow K_f^h(\mathcal{L}^h). \tag{18}$$

Note that the degree of conservatism depends on the values of fault tolerance limit ϵ and the distance limit of uncertainty set D_h. When a less conservative control sequence is desired, we shall set a higher fault tolerance limit and a more lenient distance limit. A tradeoff exists between the degree of conservation and the reliability of the decision making.

4.2. Sub-Problem: Determine the Robust ES Decision Threshold

Since there exists a random variable L^h in the constraint, we cannot solve energy generation scheduling problem (6) directly. As aforementioned, we decompose the problem into a subproblem and a main problem. The goal of the sub-problem is to determine the robust ES decision threshold $\mathcal{L}^{h*}$ so that the constraint (3) can be transformed into a solvable form.

Theorem 1: Problem (15)-(17) is a convex optimization problem.

The proof of this theorem is shown in Appendix-A. Through Theorem 1 and Slater's condition, we can see that strong duality holds for problem (15)-(17). Adopting the Lagrangian method, we can obtain the worst-case fault probability $K_f^h(\mathcal{L}^h)$ as follows:

$$K_f^h(\mathcal{L}^h) = \min_{\tau,\eta} \max_{f_0(L^h)} \mathbb{E}_{f_0}\left[h(L^h, \mathcal{L}^h) - \eta - \tau \ln\frac{f_0(L^h)}{g_h(L^h)}\right] + \tau D_h + \eta,$$

where $\tau \geq 0$ and η are Lagrangian multipliers associated with constraints (16) and (17), respectively. Let

$$\mathcal{P}(L^h, f_0, \tau, \eta) = \mathbb{E}_{f_0}\left[h(L^h, \mathcal{L}^h) - \eta - \tau \ln\frac{f_0(L^h)}{g_h(L^h)}\right],$$

the derivative of $\mathcal{P}(\mathcal{L}^h, f_0, \tau, \eta)$ with respect to f_0 can be derived as

$$\begin{aligned}\frac{\partial \mathcal{P}}{\partial f_0} &= \lim_{t\to 0}\frac{1}{t}\left[\mathcal{P}\big(f_0(L^h) + t\cdot g_0(L^h)\big) - \mathcal{P}\big(f_0(L^h)\big)\right] \\ &= \int_0^{+\infty}\left(h(L^h, \mathcal{L}^h) - \tau\ln\frac{f_0(L^h)}{g_h(L^h)} - \eta - \tau\right) g_0(L^h)\mathrm{d}L^h.\end{aligned}$$

Adopting the Karush-Kuhn-Tucker (KKT) optimality condition, we have

$$h(L^h, \mathcal{L}^h) - \tau\ln\frac{f_0(L^h)}{g_h(L^h)} - \eta - \tau = 0 \tag{19}$$

$$\int_0^{+\infty} f_0(L^h)\mathrm{d}L^h = 1 \tag{20}$$

$$\mathbb{E}\left[\ln\frac{f_0(L^h)}{g_h(L^h)}\right] - D_h \leq 0 \tag{21}$$

$$\tau\cdot\left(D_h - \mathbb{E}\left[\ln\frac{f_0(L^h)}{g_h(L^h)}\right]\right) = 0 \tag{22}$$

From (19), the optimal distribution function can be expressed as follows:

$$f_0^*(L^h) = g_h(L^h)\exp\left(\frac{h(L^h, \mathcal{L}^h) - \eta}{\tau} - 1\right). \tag{23}$$

The dual variables (τ, η) in (23) should be chosen properly such that conditions (20)-(22) are satisfied. Specifically, we have the following results.

Theorem 2: The choice of (τ, η) is a solution of the following nonlinear equations.

$$H_1(\tau, \eta) = R(\mathcal{L}^h)e^{-\eta/\tau} + S(\mathcal{L}^h)e^{(1-\eta)/\tau} - 1 = 0 \tag{24}$$

$$H_2(\tau, \eta) = S(\mathcal{L}^h)e^{(1-\eta)/\tau} - \eta - \tau(1 + D_h) = 0, \tag{25}$$

where $S(\mathcal{L}^h) = (1-G_h(\mathcal{L}^h))\exp(-1)$, $R(\mathcal{L}^h) = G_h(\mathcal{L}^h)\exp(-1)$, and $G_h(\mathcal{L}^h) = \int_{L^h \leq \mathcal{L}^h} g_h(L^h)\mathrm{d}L^h$ denotes the cumulative distribution function of reference distribution $g_h(L^h)$.

The proof for Theorem 2 is straightforward by substituting (23) to (20)-(22). However, it is still rather difficult to obtain an explicit solution from (24) and (25). Hence we propose the Newton iteration method as detailed in Algorithm 1.

Once we determine the solutions for (24) and (25) in Theorem 2, we can obtain the worst-case fault probability from (19) and (22) as follows:

$$K_f^h(\mathcal{L}^h) = \mathbb{E}_{f_0^*}[h(L^h, \mathcal{L}^h)] = (1 + D_h)\tau + \eta. \tag{26}$$

Our next step is to find the robust ES decision threshold $\mathcal{L}^{h^*}$ such that $K_f^h(\mathcal{L}^{h^*}) = \epsilon$, which involves the calculation of inverse function of $K_f^h(\mathcal{L}^h)$ that is not directly possible from (26). The following property of the function $K_f^h(\mathcal{L}^h)$, however, may help us design such a search method.

Theorem 3: The worst-case fault probability $K_f^h(\mathcal{L}^h)$ is non-decreasing with respect to the robust ES decision $\mathcal{L}^h$.

It is straightforward to derive Theorem 3 since $\mathrm{d}K_f^h(\mathcal{L}^h)/\mathrm{d}\mathcal{L}^h = \mathrm{d}\mathbb{E}_{f_0^*}[h(L^h, \mathcal{L}^h)]/\mathrm{d}\mathcal{L}^h = f_0^*(\mathcal{L}^h) \geq 0$. Though direct solution is not available, the monotonicity of $K_f^h(\mathcal{L})$ enlightens us a bisection method to search for the solution for $K_f^h(\mathcal{L}^h) = \epsilon$. The main idea is to perform the search within an interval of $[0, \rho]$, where ρ is an empirical constant such that $K_f^h(\rho) > \epsilon$.

Details of the algorithm for searching the robust ES decision threshold are presented in Algorithm 1. Note that, from the 3rd to the 11th lines of the algorithm, we use Newton iteration to solve the equation in Theorem 2 and obtain the worst-case probability with fixed robust ES decision. Then we compare the worst-case probability at $\mathcal{L}^h{}_{-}$ and $\mathcal{L}^{h^-}$ with the fault tolerance limit ϵ, respectively. The comparison results help shrink the search region as shown in lines 12-14.

Once the robust ES decision threshold $\mathcal{L}^{h^*}$ for the constraint (3) is obtained (and similarly, robust heat supply (HS) decision threshold $\mathcal{S}^{h*}$ for constraint (4) is obtained), we can approximate (3) and (4) with the following two constraints:

$$V^h + \sum_{a \in \mathcal{A}} x_a^h \geq \mathcal{L}^{h*} \tag{27}$$

$$U^h + \sum_{a \in \mathcal{A}} \eta_a \cdot x_a^h \geq \mathcal{S}^{h*}. \tag{28}$$

4.3. Main Problem: Robust Approach for the Uncertain Electricity Prices

There exist financial risks associated with real time electricity price uncertainty where p_s^h are unknown quantities. We adopt certain intervals at the $\alpha-$confidence level for prices $p_s^h \in [\hat{p}_s^h, \hat{p}_s^h + d^h]$, $h \in \mathcal{H}$ and formulate the well defined robust model [33] [34]. Specifically, we tackle the following optimization problem rather than the original formulation (6):

$$
\begin{aligned}
\min \quad & \sum_{h=1}^{H} \Big\{ p_g \cdot U^h + \hat{p}_s^h \cdot V^h + \sum_{a \in \mathcal{A}} \Big[c_a^m \cdot x_a^h \\
& + c_a^b \cdot y_a^h + c_a^s \cdot z_a^h \Big] \Big\} + \phi \cdot \Gamma + \sum_{h \in J_0} e^h \\
\text{s.t.} \quad & \phi + e^h \geq d^h \cdot k^h, \quad \forall h \in J_0 \\
& -k^h \leq V^h \leq k^h \\
& e^h \geq 0, \quad k^h \geq 0, \quad \phi \geq 0, \quad z_a^h \geq 0, \quad \forall h \in J_0 \\
& z_a^h \geq y_a^h - y_a^{h-1} \\
& (2)\ (27)\ (28),\ y_a^h, z_a^h \in \{0,1\} \\
& x_a^h, V^h, U^h, k^h, e^h, \Gamma \in \mathbb{R}_0^+, h \in \mathcal{H}, a \in \mathcal{A}.
\end{aligned}
\tag{29}
$$

Robust problem (29) is obtained using duality properties and exact linear equivalences. It represents the worse case while considering that electricity prices can be uncertain in at most Γ slots. $J_0 = \{h|\ d^h > 0\}$ is the set of electricity price p_s^h, $h \in \mathcal{H}$ that are subject to parameter uncertainty. Variable e^h is the dual variable of the initial problem (6) used to consider the known bounds of electricity prices, while ϕ and k^h are auxiliary variables used to obtain equivalent linear expression. Readers can refer to Appendix-B for detailed description of how to obtain this robust problem from problem (6). Γ is a parameter that controls the level of robustness in the objective function. This parameter is assumed to be integer and takes value in the set $\{0, 1, 2, ..., |J_0|\}$, i.e., between zero and the number of unknown electricity prices. In this case, when $\Gamma = 0$, the influence of price uncertainty in the objective function is ignored; when $\Gamma = |J_0|$, all possible price deviations are taken into account, which is the most conservative case. In general, a higher value of Γ increases the level of robustness at the expense of a higher cost. Note that constraints (3) and (4) with random variables in the initial formulation (6) are approximated and replaced by (27) (28) with no random variable. This problem is a mixed integer linear programming (MILP) problem, which can be effectively tackled by cutting plane method, branch and bounded method, etc.

5. Simulation Results and Discussions

In this section, we present simulation results based on real world traces to assess the performance of the proposed energy generation scheduling scheme and evaluate the effects of different parameters.

5.1. Parameters and Settings

5.1.1. Net Demand and Heat Demand Trace

We obtain the electricity and gas demand statistics from [40]. We focus on a college at Forecasting Climate Zone (FCZ) 09. The electricity within this zone is supplied by the Southern California Edison company. This trace contains hourly electricity demand and heat demand of the college in year 2002. We assume there are solar panels in the microgrid system. The area of solar panel in this microgrid system is set to be 3.75×10^4 m^2. The energy conversion efficiency is 0.8. The solar radiation intensity data is adopted from [41]. We employ electricity demand, heat demand and solar power data of a typical month in winter (January) and estimate the distributions of net demand (electricity demand minus solar energy) and heat demand in each hour based on the samples using Kernel Density Estimation [42]. We find that in all the time slots (hours), the distribution functions of net demand and heat demand are close to be normal distribution. Thus, the reference distribution of net demand and heat demand is set to be normal distribution.

5.1.2. CHP Generator Characteristics

The parameters of CHP generators are set based on the statistics in [43]. The maximum output of a CHP generator is $E_a^{max} = 3.5$ MWh and the minimum stable output is $E_a^{min} = 1.5$ MWh. The marginal cost for producing one unit of electricity is $c_a^m = 0.051$ \$/KWh, which is obtained using the fuel price and the energy conversion efficiency. The sunk cost for CHP generator keeping in active mode is $c_a^b = 110$ \$/h, which includes the capital cost, operation cost and maintenance cost. We set the start up cost to be $c_a^s = 560\ \$$ and the heat-electricity ratio to be $\eta = 2.065$ [43]. Finally, it is assumed there are 8 CHP generators in this microgird system unless otherwise stated.

5.1.3. Electricity and Gas Prices

The electricity price trace is obtained from [44] and the gas price data is obtained from [45]. In our paper, we adopt the electricity market prices of central New York Control Area (NYCA) on a typical day in January. We set $\hat{p}_s^h$ and d^h be equal to the lower bound and variation range of electricity market price at hour h, respectively. In addition, the natural gas price is set to be $p_g = 6.075$ \$/mmBTU.

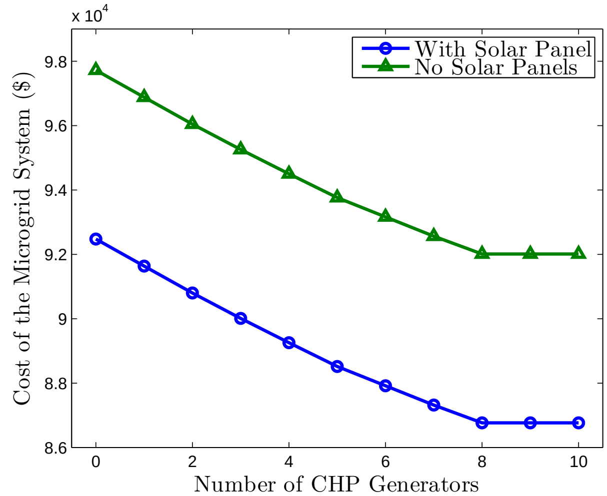

Figure 2: Cost reduction for different number of CHPs

5.2. Results and Discussions

5.2.1. Robust ES Threshold and Robust HS Threshold

We first solve the sub-problem and obtain the robust ES threshold $\mathcal{L}^{h*}$ and robust HS threshold $\mathcal{S}^{h*}$ for solving the main problem. The reference distributions of net demand and heat demand are normal and are estimated from sample data. The distance limit of net demand and heat demand uncertainty sets is 10^{-1}. The fault tolerance limit of net demand supply is 10^{-2} while the fault tolerance limit of heat demand supply is 10^{-1}. Given reference distributions, distance limits, and fault tolerance limits, we obtain $\mathcal{L}^{h*}$ and $\mathcal{S}^{h*}$ based on **Algorithm** 1. The results are shown in Table 2.

Table 2: Parameters of distribution uncertainty sets and corresponding ES and HS thresholds (unit: MWh for electricity and mmBTU for heat. $\overline{m}_E^h$ and σ_E^h are mean and standard deviation of net demand reference distribution, respectively. $\overline{m}_H^h$ and σ_H^h are mean and standard deviation of heat demand reference distribution, respectively)

Time Slot	$\overline{m}_E^h$	σ_E^h	$\mathcal{L}^{h*}$	$\overline{m}_H^h$	σ_H^h	$\mathcal{S}^{h*}$
1	18.44	0.1059	18.98	63.88	8.3372	81.65
2	18.08	0.0965	18.57	51.96	5.0481	62.72
3	18.06	0.1005	18.58	43.63	1.7780	47.42
4	18.43	0.1246	19.07	46.62	1.8902	50.64
5	20.60	0.1456	21.34	50.39	1.7311	54.08
6	24.67	0.3807	26.61	80.35	7.5946	96.53
7	32.18	1.6355	40.52	124.93	1.4380	127.99
8	44.08	1.9485	53.50	283.69	8.0012	300.74
9	64.06	3.7971	78.77	285.91	6.4596	299.67
10	58.64	2.2394	60.54	254.82	7.5097	270.82
11	59.28	2.3199	58.88	219.39	10.7104	242.21
12	58.73	2.2730	56.27	195.55	10.1975	217.28
13	58.68	2.2121	54.18	183.64	11.0907	207.27
14	58.77	2.3731	56.66	177.02	11.6296	201.79
15	58.70	2.4761	61.38	171.43	12.0786	197.17
16	57.91	2.5475	64.65	167.69	12.1597	193.59
17	57.32	2.2805	67.77	166.47	12.6110	193.34
18	55.41	2.0156	65.69	169.83	14.0442	199.75
19	53.16	2.2647	64.72	176.10	14.0746	206.09
20	47.58	2.5553	60.62	184.35	14.3077	214.83
21	41.59	3.3157	58.51	190.49	15.3283	223.14
22	35.99	3.4268	53.47	198.32	15.0698	230.43
23	27.40	2.9277	42.34	111.43	10.2832	133.33
24	20.05	0.2638	21.40	78.80	7.7375	95.29

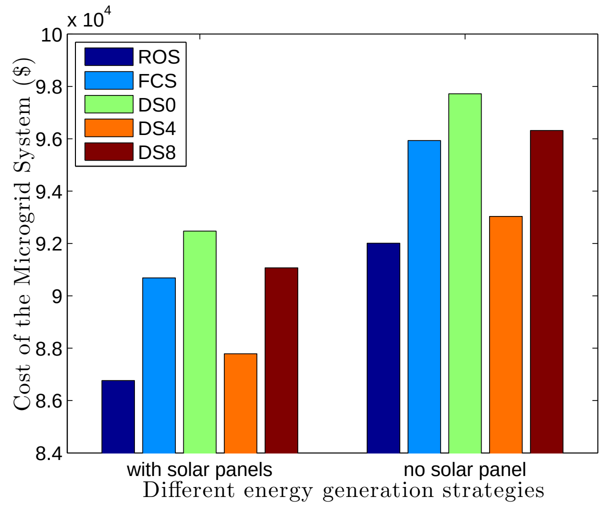

Figure 3: Costs of different generation scheduling strategies

5.2.2. Potential Benefits of CHP Generators and Solar Panels

Once we obtain robust ES threshold $\mathcal{L}^{h*}$ and robust HS threshold $\mathcal{S}^{h*}$, we are ready to adopt the robust optimization approach to study the energy generation scheduling problem (29) with respect to real time electricity market prices. Problem (29) is solved using the data provided in the previous subsection 5.2.1. The problem is solved using MOSEK optimization toolbox 7.0 on an Intel workstation with 6 processors clocking at 3.2 GHZ and 16 GB of RAM.

We first try to investigate the potential savings with CHP generators and solar panels. In particular, we conduct two sets of experiments. Both sets of experiments have nearly the same default settings, except that solar panels in the microgrid are enabled in the first set, but not in the second one. We vary the number of CHP generators installed in the microgrid from 0 to 10 and compute the total cost of the system in a day. The results are shown in Fig. 2. It is observed that having 8 CHP generators with full capacity 28 MW is sufficient to obtain nearly all the cost saving benefits. Thus, we may suggest that installed CHP generator capacity should be about half of the peak demand (The peak demand of a day in January is around 60 MW.). The intuitive reason is that most of the time, demands are much lower than the peaks. This result can shed some light on making investment decisions in microgrids. Note that the leftmost points in the two curves denote the case where microgrid only uses external electricity and local heat generators (without CHP generators). System cost in this case can be interpreted as a cost benchmark. The results show that CHP can bring a saving of 6.2% (around $\$5700$ per day) to the system. Finally, by comparing the two curves in Fig. 2, we find that the one day cost reduction achieved by solar panels is about 6.05% (around $\$5200$ per day).

5.2.3. Comparisons of Different Generation Scheduling Strategies

We compare 3 energy generation scheduling strategies: (1) the proposed robust optimal strategy (ROS); (2) fixed choice strategy (FCS): making one fixed choice of the generation level for entire duration for each generator. The system cost induced by this strategy has been used as a benchmark in literature [46]; (3) deterministic strategy (DS): A fixed number of CHP generators are switched on for the entire time horizon. The microgrid has to properly schedule the output level of active CHP generators, imported energy and local heat generators to meet electricity and heat demand. Specifically, we consider 3 schemes with

0, 4 and 8 CHP generator(s) in active mode and termed as DS0, DS4 and DS8, respectively. We emphasize that the microgrid always tries to find the optimal control sequences under any of these three generation scheduling strategies and the scheduling choices of the last two methods for comparison (i.e., FCS and DS) are made in hindsight. In addition, all the three scheduling strategies adopt the same parameter settings. The cost comparison results are depicted in Fig. 3.

As we observe in Fig. 3, ROS can achieve a cost saving of 4.5% (about $\$3900$ per day), 6.5% (about $\$5700$ per day), 1.2% (about $\$1000$ per day) and 5.0% (about $\$4300$ per day) compared with FCS, DS0, DS4 and DS8, respectively (equipped with solar panels). Moreover, we note that only using external electricity (DS0) or switching on all the local generators (DS8) are not economical. Another interesting observation is that the cost of DS8 is lower than that of DS0. This shows that when all the CHP generators are switched on, although a significant amount of electricity may be wasted in the off-peak time slots, the strategy nevertheless still achieves better performance than the case where all electricity is imported from outside power grid. This justifies the economic potential of using local CHP generators. Obviously, DS4 achieves more cost savings than DS0 and DS8. This is because that when half of the CHP generators are turned on, a considerable proportion of the electricity demand can be supplied by CHP generators and the energy loss in off-peak hours is relatively low than that in DS8.

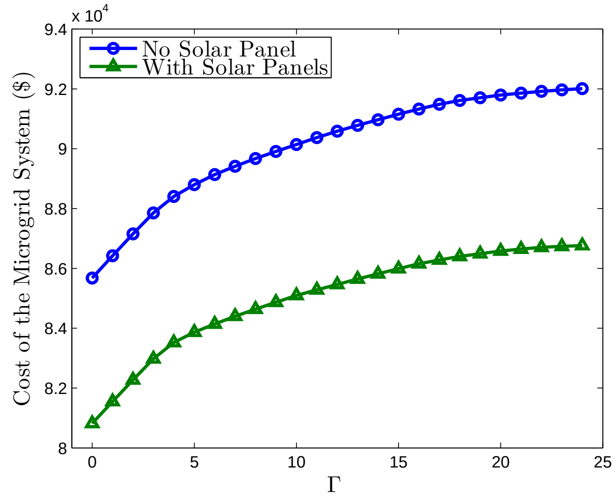

Figure 4: System cost with respect to robustness level Γ.

5.2.4. The Impact of Robustness Level Γ

The sensitivities of the electricity cost with respect to robustness level Γ are depicted in Fig. 4. We set $|J_0| = 24$, i.e., price uncertainty may exist in all time slots of the day. We are interested in finding an optimal solution which optimizes against all scenarios under which a number Γ of the electricity prices can vary in such a way as to maximally influence the objective. We vary the value of Γ from 0 to 24 in formulation (29) and obtain the optimal system cost. Remember that the value of Γ indicates the number of worst-case prices during the 24 time slots. $\Gamma = 0$ corresponds to the lowest robustness level while $\Gamma = 24$ corresponds to the highest robustness level. Apparently, the system cost is an increasing function of Γ. The incremental cost when the robustness level grows is the price for tackling the financial risks. We observe that to fully overcome the financial risks (i.e. the

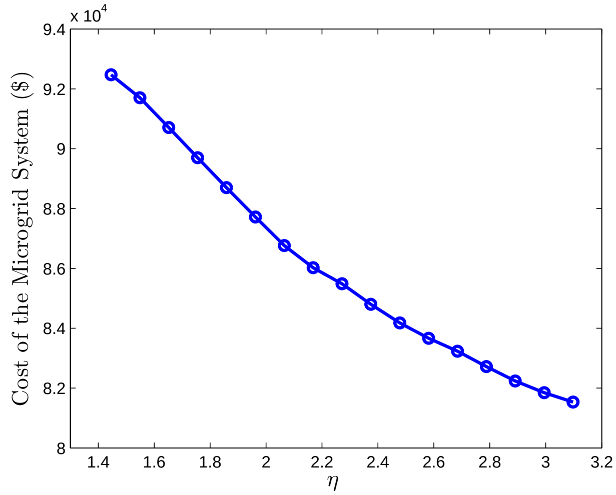

Figure 5: Cost profile with respect to different η

most conservatism condition), the microgrid has to pay additional 7.35% (about $\$5900$ per day) expenditures. However the rise rate of the cost curve slows down when Γ increases. The reason is that when Γ increases, the protection level for the robust solution increases, then the probability that the robust solution is not favorable declines. Hence, it becomes less costly to protect the microgrid against the financial risk. We also compare the costs of two scenarios where solar panels are available and not available, respectively. The difference between these costs is called cost gap. It is interesting to note that cost gap only rises marginally when Γ increases. This shows that the uncertainty of solar energy has little impacts on the financial risks of the system since the indeterminacy of it has been alleviated by the proposed robust approach in the sub-problem.

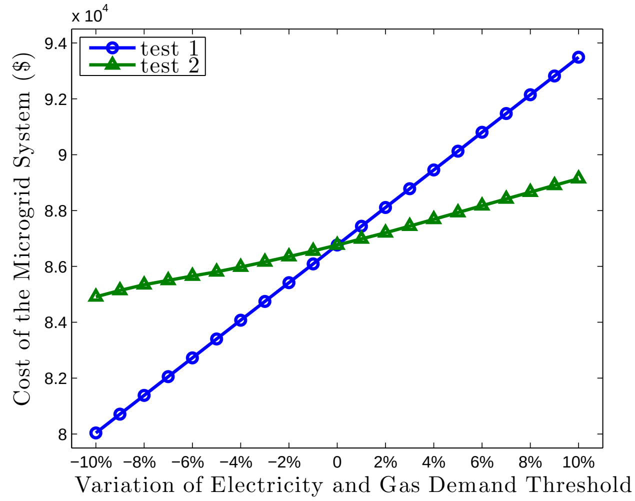

Figure 6: Cost sensitivity with the variation of $\mathcal{L}^*$ and $\mathcal{S}^*$

5.2.5. The Impacts of Heat-Electricity Ratio η

Figure 5 depicts the reduction in cost versus heat-electricity ratio η. It appears that system cost decreases when η grows. The reason is that a larger η means CHP generators can provide more heat for free. In this case, the microgrid can reduce the reliance on local heat generators, which can be seen from Eq. (28). Meanwhile, we observe that the decrease rate slows down when η increases. This observation is intuitive since when η is large, nearly all the heat demands can be supplied by CHP generators for free. Therefore, additional free heat cannot bring significant benefits as the heat may be wasted.

5.2.6. System Cost Sensitivity to the Robust ES and HS Thresholds

In Fig. 6, we illustrate the relationship between the system cost and variation of $\mathcal{L}^{h*}$ and $\mathcal{S}^{h*}$. Specifically, we conduct two tests. In the first test, $\mathcal{S}^{h*}$ remains unchanged and we vary the value of $\mathcal{L}^{h*}$; while in the second one, $\mathcal{L}^{h*}$ remains constant and $\mathcal{S}^{h*}$ varies. It is observed that the system cost has a nearly linear relationship with $\mathcal{L}^{h*}$ and $\mathcal{S}^{h*}$, which is consistent with the theoretical formulation (29). From (29) we see that the objective function have linear relationships with variables V^h, U^h and $\sum_{a\in\mathcal{A}} x_a^h$, $h \in \mathcal{H}$. However, due to the tradeoff between using local CHP generators and outside electricity when we vary $\mathcal{L}^{h*}$ and $\mathcal{S}^{h*}$, the relation between system cost and $\mathcal{L}^{h*}$ ($\mathcal{S}^{h*}$ as well) is only approximately linear. Also note that system cost is more sensitive to the variation of $\mathcal{L}^{h*}$. Since a large proportion of heat demands are satisfied by CHP generators for free, the system expenditure on heating is much lower than that on generating or buying electricity. Hence, the variation of heat demand has lower impacts on the system cost.

6. Conclusions

In this paper, we studied the energy generation scheduling problem in a microgrid scenario to minimize the cost and maintain system stability. To tackle the randomness of net demand and heat demand, we introduced reference distributions and then defined distribution uncertainty sets to confine the fluctuations. Such a model allows convenient handling of volatile demands as long as the demand profiles are not too intensely different from the predictions or empirical knowledge. The uncertainty in electricity price was addressed by bounded random variables. We developed chance constraint approximations and robust optimization algorithms to firstly transform and then solve the problem. Numerical results based on real-world data indicate the satisfactory efficiency of the proposed energy scheduling strategy and the cost benefits of CHP generators. Moreover, the impacts of different parameters have been carefully evaluated. Such evaluations, as we believe, shall provide useful insights helping microgrid operators develop rational investment strategies.

In our future work, we will consider a microgrid where there are energy storages (batteries and heat accumulators) in the system. In such a system, the energy storages will impose their own cost; meanwhile they may to a certain extent alleviate the uncertainty problem caused by the fluctuations of the net demand and heat demand, especially when the storages are of a large enough capacity. The optimization problem for this scenario therefore becomes significantly different from the one we considered in this paper and worth further studies.

Appendix A. Proof of Theorem 1

Proof. Rewrite (15)-(17) as follows:

$$\begin{aligned} \max_{f_0(L^h)} \quad & \int_0^{+\infty} h(L^h, \mathcal{L}^h) \cdot f_0(L^h) \mathrm{d}L^h \qquad \text{(A.1)} \\ \text{s.t.} \quad & \int_0^{+\infty} [\ln f_0(L^h) - \ln g_h(L^h)] f_0(L^h) \mathrm{d}L^h \le D_h \\ & \int_0^{+\infty} f_0(L^h) \mathrm{d}L^h = 1. \end{aligned}$$

We can see that the objective function and equality constraint function are affined with respect to $f_0(L^h)$. Next we show that the inequality constraint function is convex.

Lemma: If $f : \mathbf{R^n} \longrightarrow \mathbf{R}$ is convex, then the perspective of f, which is denoted as a function $g : \mathbf{R^{n+1}} \longrightarrow \mathbf{R}$ that

$$g(x,t) = tf(x/t), \tag{A.2}$$

with domain

$$\mathbf{dom}\ g = \{(x,t)|x/t \in \mathbf{dom}\ f,\ t > 0\} \tag{A.3}$$

preserves convexity.

That is to say, if f is a convex function, so is its perspective function g. Similarly, if f is concave, so is g. This can be proved in several ways, e.g., by direct verification of the defining inequality or using epigraphs and the perspective mapping on $\mathbf{R^{n+1}}$. Readers can refer to [47] for more detailed discussions.

We consider the convex function $f(x) = -\ln x$ on $\mathbf{R}_{++}$. Its perspective is

$$g(x,t) = -t \ln(x/t) = t \ln(t/x) = t(\ln t - \ln x) \tag{A.4}$$

and it is convex on $\mathbf{R^2_{++}}$. The function g is called the relative entropy of t and x. Then we have that the KL divergence $\int_{x\in S}[\ln f(x) - \ln g(x)]f(x)\mathrm{d}x$ between distribution $f(x)$ and $g(x)$ is convex in $f(x)$ (and $g(x)$ as well). In this case, we claim that the inequality constraint is convex with respect to distribution $f_0(L^h)$.

$\square$

Appendix B. Reformulation of Problem (6)

Specifically, the robust counterpart of Problem (6) is as follows:

$$\begin{aligned}
\min_{\mathbf{X},\mathbf{Y},\mathbf{Z},\mathbf{V},\mathbf{U}} \quad & \sum_{h=1}^{H} \Bigg\{ p_g \cdot U^h + \hat{p}_s^h \cdot V^h + \\
& \sum_{a\in\mathcal{A}} \left[c_a^m \cdot x_a^h + c_a^b \cdot y_a^h + c_a^s \cdot z_a^h \right] \Bigg\} \\
& + \max_{\{W_0 | W_0 \subseteq J_0, |W_0| \leq \Gamma\}} \left\{ \sum_{h\in W_0} d^h \cdot V^h \right\} \\
\text{s.t.} \quad & z_a^h \geq 0,\ z_a^h \geq y_a^h - y_a^{h-1} \\
& (2)\ (3)\ (4),\ y_a^h, z_a^h \in \{0,1\} \\
& x_a^h, V^h, U^h \in \mathbb{R}_0^+, h \in \mathcal{H}, a \in \mathcal{A},
\end{aligned} \tag{B.1}$$

Theorem 4: Problem (B.1) has an equivalent MIP formulation as (29).

Proof. Given a vector $\mathbf{V}^*$, we can convert the last part of Problem (B.1)'s objective function to a linear one as follows:

$$\begin{aligned}
\beta_0(\mathbf{V}^*) &= \max \left\{ \sum_{h\in W_0} d^h \cdot V^{h*} : W_0 \subseteq J_0, |W_0| \leq \Gamma\} \right\} \\
&= \max \Bigg\{ \sum_{h\in J_0} d^h \cdot V^{h*} \cdot \phi_h : \sum_{h\in J_0} \phi_h \leq \Gamma, \\
&\qquad 0 \leq \phi_h \leq 1, \forall h \in J_0 \Bigg\}.
\end{aligned} \tag{B.2}$$

Next, the dual of Problem (B.2) is:

$$\begin{aligned}
\min \quad & \sum_{h\in J_0} e^h + \Gamma \cdot \phi \\
\text{s.t.} \quad & \phi + e^h \geq d^h \cdot V^{h*} \\
& \phi \geq 0, e^h \geq 0, \forall h \in J_0.
\end{aligned} \tag{B.3}$$

By strong duality, we have:

$$\beta_0(\mathbf{V}^*) = \min\left\{\sum_{h\in J_0} e^h + \Gamma\cdot\phi : \phi + e^h \geq d^h \cdot V^{h*}, \phi \geq 0, e^h \geq 0, \forall h \in J_0\right\}. \tag{B.4}$$

Substituting (B.4) to Problem (B.1), we obtain that Problem (B.1) is equivalent to Problem (29). □

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Algorithm 1 Search for robust ES decision threshold $\mathcal{L}^{h^*}$

Input: Reference distribution $g_h(L^h)$;

Distance limit D_h; Search radius ρ;

Load balance fault tolerance limit ϵ; Tolerance ε.

Output: Robust ES decision threshold such that $K_f^h(\mathcal{L}^{h^*}) = \epsilon$;

Begin

initialize $\mathcal{L}^h{}_- = 0$, $\mathcal{L}^{h^-} = \rho$, and set $\mathbf{H}(\tau, \eta) = [H_1(\tau, \eta), H_2(\tau, \eta)]^T$

while $|\mathcal{L}^h{}_- - \mathcal{L}^{h^-}| > \varepsilon$

  set $\bar{\mathcal{L}}^h = \frac{\mathcal{L}^h{}_- + \mathcal{L}^{h^-}}{2}$, initiate the time iteration $k = 1$

  while $\mathbf{H}(\tau, \eta) > \varepsilon$

    evaluate $\mathbf{H}(\tau, \eta)$ and Jacobian matrix $\mathbf{J}(\tau, \eta)$

    solve $\mathbf{J}(\tau, \eta)\Delta \mathbf{x_k} = -\mathbf{H}(\tau, \eta)$

    update $\tau_{k+1} = [\tau_k + \Delta\tau_k]^+$, $\eta_{k+1} = \eta_k + \Delta\eta_k$

    set $k = k + 1$

  end while

  update $K_f^h(\bar{\mathcal{L}}^h) = (1 + D_h)\tau_{k+1} + \eta_{k+1}$

  if $\left(K_f^h(\bar{\mathcal{L}}^h) - \epsilon\right)\left(K_f^h(\mathcal{L}^{h^-}) - \epsilon\right) < 0$

    then set $\mathcal{L}^h{}_- = \bar{\mathcal{L}}^h$ **else** set $\mathcal{L}^{h^-} = \bar{\mathcal{L}}^h$ **end if**

  if $|K_f^h(\bar{\mathcal{L}}^h) - \epsilon| < \varepsilon$ break **end if**

end while

set $\mathcal{L}^{h^*} = \bar{\mathcal{L}}^h$

End