Hybrid Centralized-Decentralized (HCD) Charging Control of Electric Vehicles

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***Abstract*—Integrating massive electric vehicles (EVs) into power grid requires the charging of EVs to be coordinated to reduce the cost and guarantee the system stability. The coordination becomes more challenging when the EV owners have different charging preferences. To tackle this problem, a hybrid centralized-decentralized (HCD) charging control scheme is designed in this paper which mainly includes three parts as follows. On the centralized charging side, an offline optimal scheduling approach is first presented aiming at minimizing the energy cost while satisfying the charging requirements of EVs. To deal with the system dynamics and uncertainties, a model predictive control (MPC) based adaptive scheduling strategy is then developed to determine the near optimal EV charging profiles in real time. On the decentralized charging side, the interactions between EVs and the charging system controller is modeled as a leader-follower non-cooperative Stackelberg game in which the system controller acts as the leader and the EVs act as followers. The existence of the equilibrium state and its optimality are proved and analyzed. It is shown that by adopting the proposed decentralized charging algorithm, the communication burden between EVs and the system controller is low and the charging scheme is robust to poor communication channels. Last, we investigate how the size of these two charging groups impacts the system utility and propose an algorithm maximizing the total revenues of the whole system. Simulation results evaluate the performances of the scheme and investigate the parameters' impacts on the system utilities. The proposed approach and obtained results may provide guidelines for improving the efficiency of the charging park operation and provide useful insights helping the system operator develop rational investment strategies.**

***Index Terms*—electric vehicles (EVs), hybrid centralized-decentralized charging, model predictive control (MPC), Stackelberg game.**

I. Introduction

Today's transportation sector accounts for a significant portion of petroleum consumption and greenhouse gas emissions worldwide. Statistics show that 63.7% of the petroleum consumed in the world in 2012 was due to the transport sector, which caused emission of 7135 million tons of carbon dioxide into the environment [1]. The world's fossil fuel scarcity, as well as the growing environmental crisis associated with fossil fuel's wide usage, are driving the electrification of transportation and extensive use of electric vehicle (EVs). EVs are emerging as an efficient and sustainable alternative for private and public road transportation [2], [3]. To encourage the purchase of EVs, government of different countries including Australia, Canada, China, Europe Union and U.S.A. subsidize or finance the customers and implement many actions such as tax exemption, transit and parking facility constructions, etc [4].

While the widespread implementation of EVs may provide a solution to the world fossil fuel shortage and air pollution concerns, the growing EV load also brings up multiple technical issues, such as voltage deviations, transformers and line saturations, increase of electrical losses, etc. These issues may jeopardize the security and reliability of the power grid. As a consequence, intelligent charging and scheduling for EVs becomes an important research problem.

A number of technical and regulatory issues, however, have to be resolved before the intelligent charging becomes a commonplace. The arrival of EVs and their required energy amount may appear to be random, which increases the demand side uncertainties. The role of EV owners is also important in the interactions between the charging system and the EVs. From the EV owners' point of view, the degree of satisfaction should be a prior concern. When departures, the EV owner hopes that the EV is charged as much as possible. In addition, EV owners may have various charging habits. Some are prone to individually determine their own charging profiles while others may hope the charging system undertake the charging tasks for them. For instance, a future courier company may assign all the charging tasks of its driverless car fleet to a system controller of the charging park; meanwhile, some private car owners may prefer to control their own charging patterns all by their own. In many cases, these two kinds of users coexist. Therefore, a flexible and efficient EV charging mechanism has to be properly designed to dynamically coordinate the charging of EVs and satisfy the requirements of EV owners.

In this paper, we consider the charging scheduling of a large number of EVs at a charging station. Stimulated by the fact that in practical scenarios, both centralized or decentralized charging architectures have their limitations and EV owners may have various charging preferences, a hybrid centralized-decentralized (HCD) EV charging control method is developed which offers flexible charging choices for customers. In this charging scheme, EV owners can either assign the charging tasks to system controller or individually choose the charging profiles based on their own preferences. In addition, the

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stochastic characteristics of EVs such as the arrival/departure times and charging demands are all taken into account for designing the charging control scheme. Note that the main objective of our study is to properly adjust certain criteria, namely electricity price and energy cap, to intelligently drive the HCD charging system into an economical and stable state.

For the centralized charging scheduling, a local controller is responsible to schedule a group of EVs. Based on the demand side management framework proposed by us in [5], we regard EVs as flexible loads in this paper and formulate the coordinated EV charging problem into a mixed integer quadratic programming (MIQP) problem which aims at minimizing the charging cost of the whole EV fleet over a time horizon. To tackle with the system condition dynamics where EVs' arrival/departure times and their charging demands are uncertain, an online charging scheme is developed based on model predictive control (MPC). The scheme allows more information of EV arrivals (departures) and charging demands to be effectively incorporated into the charging mechanism when such information is available. The calculation burden of the system controller is low.

For the decentralized charging scheduling, we model the interactions between the system controller and EVs into a leader-follower noncooperative Stackelberg game. The game aims at maximizing the profit of the charging system and utilities of the EV owners. We prove the existence of the generalized Stackelberg equilibrium (GSE) where both the leader and followers reach their equilibrium states. It is shown that the GSE also represents the socially optimal[1] solution. Moreover, a decentralized EV charging algorithm is developed. We show that the communication burden between EVs and the system controller is low and the proposed decentralized charging control scheme is robust to poor communication quality.

Moreover, we investigate the interactions between these two charging groups. It is shown that an optimal energy cap exists for the decentralized charging group which maximizes the entire system's revenue. An optimal energy allocation algorithm is proposed to find such energy cap. Furthermore, how the vehicle number in each charging group impacts the system utility is specifically studied. The proposed approach and obtained results may provide guidelines to improve the operation efficiency of the charging park and provide useful insights helping the system operator develop rational investment strategies.

The remainder of this paper is organized as follows: In Section II, we review the recent literature concerning the EV charging control strategies. Section III introduces the system model of the HCD EV charging mechanism. In Section IV, we present the problem formulation of the HCD charging control of EVs. In Section V, the algorithms to solve the HCD EV charging problem are introduced. The simulation results and discussions are presented in Section VI. Finally, we conclude our paper in Section VII.

[1]A strategy is called socially optimal iff it minimizes the sum of all the cost.

II. Related Work

The existing EVs' charging control schemes can be roughly classified into two categories: centralized charging strategies and decentralized charging strategies. The main idea of centralized control is to utilize centralized infrastructure to collect information from all EVs and centrally optimize EVs' charging considering the grid technical constraints. In such a strategy, the master controller makes decisions about the rate and time of charging EVs to get the optimal solution [6]–[20]. Esmaili *et al.* [6]–[9] develop various centralized charging strategies for different optimization objectives, including saving system cost, minimizing CO_2 emission, reducing power loss, adjusting power frequency and satisfying EV owners, etc. Various optimization methods and heuristic algorithms have been adopted to solve such problems. In [10], a hierarchical control scheme is proposed for managing EVs' charging station loads in a distribution network while minimizing energy cost and abiding by substation supply constraints. The scheduling is based on the forecast load information. Reference [11] proposes a DP (dynamic programming)-based optimization method of charging an EV fleet modelled as a single, so-called aggregated battery. However, in all aforementioned papers, the dynamics of the EVs' arrival/departure times and charging patterns are not considered; Qi *et al.* [12]–[15] adopt receding horizon control based techniques to tackle the uncertainties in the dynamic charging systems. References [16]–[18] develop online algorithms for coordinating EVs' charging to save the system cost and lessen EVs' harmful impacts on the distribution network. Note that these papers specifically consider the dynamics of EVs' charging system. Jin *et al.* [19] study EV charging scheduling problems from a customer's perspective by jointly considering the aggregator's revenue and customers' demands and costs. Paper [20] studies risk-aware day-ahead scheduling and real-time dispatch for plug-in EVs, aiming to jointly optimize the EV charging cost and the risk of load mismatch between the forecast and the actual EV loads. Different from previous papers, both static and dynamic charging scenarios are considered in [19] and [20]. For centralized charging strategy, the size of the optimization problem increases with the number of EVs. Accurate information collection from a large number of EVs may also impose a challenge. Designing an effective centralized EV charging strategy therefore remains as a difficult problem.

In contrast, the vehicle owners directly control their EVs' charging patterns employing the decentralized charging strategies [21]–[37]. Yang *et al.* [21] focus on the scheduling of EV charging process among different charging stations and each station can be supplied by both renewable energy generators and a distribution network. Gan *et al.* [22] propose a decentralized algorithm to schedule EV charging to fill the electric load valley. This charging control strategy iteratively solves an optimal control problem in which the charging rate of each vehicle can vary continuously within its upper and lower bounds. In each iteration, each EV updates its own charging profile according to the control signal broadcast by the utility, and the utility company alters the control signal to guide their updates. In [23]–[29], various decentralized charging frameworks to coordinate charging demand of EVs are implemented based on game theory concepts. In [30], a decentralized online valley filling algorithm for EV charging is proposed. An optimal power flow (OPF) framework is adopted to model the network constraint that rises from charging EVs at different locations. Similar to [30], decentralized EV charging schemes with valley filling objective can also be found in [31] and [32]. Considering the selfish nature of people, authors of [33] define some weighting factors in the objective function of EV charging management problem aiming at modeling users' convenience in the presented optimization procedure. Xi *et al.* [34] study a decentralized price-based EV charging control. A pricing scheme that conveys price and quantity information to the load aggregator is developed. In [35], a novel online coordination method for the charging of plug-in EVs in smart distribution networks is proposed. An innovative parking lot prediction unit is developed adopting $M/G/\infty$ queuing model[2]. In [36], the authors formulate the EV charging problem as a convex optimization problem and then propose a decentralized water-filling-based algorithm to solve it. A receding horizon approach (similar to [12]–[14]) is utilized to handle the random arrival of EVs and the inaccuracy of the forecast non-EV load. Although the decentralized charging strategy offers more ownership authority to EV owners, it may not ensure the optimality in the charging of EVs and brings security concerns of the power grid [6], [23], [37].

There are also some literature taking the vehicle-to-grid (V2G) technology into consideration when scheduling the charging of EVs. For instance, in [38], a step-by-step methodology based on a mixed integer linear programming formulation is presented to solve the optimal charging coordination of EV considering V2G technology. Tang *et al.* [39] propose a new modelling method of EVs and an optimal V2G charging control strategy under large EV population. Compared with previous studies, the HCD EV charging scheme proposed in this paper offers flexible charging choices for customers, where EV owners can either delegate the charging tasks to system controller or individually choose the charging profiles based on their own preferences. The stochastic characteristics of EVs such as the arrival/departure times and charging demands are taken into account. By adopting the proposed algorithm, the communication burden between EVs and the system controller is low; and the proposed charging control is robust to poor communication channels. Also note that the proposed HCD charging scheme is applicable to handle other system objectives and is not restricted to any specific random patterns of EVs.

III. System Model

We consider an intelligent charging system (e.g., a charging park) which offers two charging options for the customers: 1) centralized charging: utilizing centralized infrastructure to collect information from EVs and centrally optimize EV charging considering the grid technical constraints; 2) decentralized charging: the vehicle owners directly control their EVs' charging patterns according to their own preferences. The general structure of the system is shown in Fig. 1. The system controller, the local controller(s) and EVs are the main components in this HCD charging control system. The abbreviations used in the paper and their meanings are listed in Table I. The particulars of the system operation and the main roles of these components are explained in the following subsections.

[2]An $M/G/\infty$ queue is a queue model where arrivals are Markovian (modeled by a Poisson process), service time has a general distribution and there are infinite number of servers

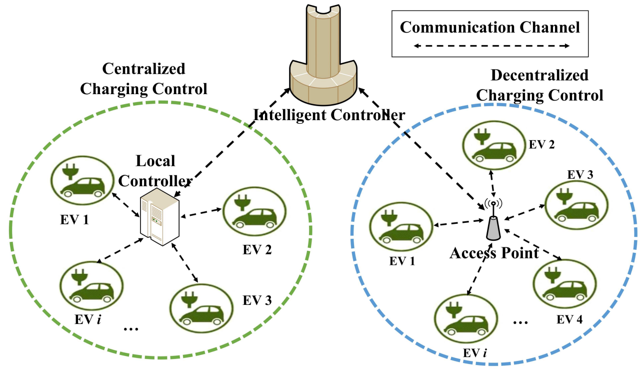


Fig. 1: Illustration of the system architecture

TABLE I: Abbreviations used in this paper

Abbreviation	Meaning
EV	electric vehicle
HCD	hybrid centralized-decentralized
MPC	model predictive control
GSE	generalized Stackelberg equilibrium
SOC	state of charge
GNE	generalized Nash equilibrium
GNEP	generalized Nash equilibrium problem
MIQP	mixed integer quadratic programming
GSG	generalized Stackelberg game
NE	Nash equilibrium
VE	variational equilibrium
VI	variational inequality

A. Centralized Charging Control Model

A local controller is responsible for scheduling the charging patterns of a group of EVs on behalf of their owners. If the number of EVs is large, the EV fleet can be classified into several groups (e.g., according to their geographical locations) and one local controller is responsible for the charging tasks of one EV group. The local controller and EVs (the local controller and the system controller as well) are connected through two-way communication infrastructures (e.g., a local area network (LAN)). The operation time of the charging system is divided into discrete time intervals with equal length, i.e., time slot. The length of a time slot is denoted by η, which can vary from 5 mins to half an hour based on the charging traffic conditions [40]. EVs can be regarded as flexible loads. Let $\mathcal{A}$ denote the set of EVs which participate in the centralized charging scheme. Adopting a general scheduling model [5], we define EV charging scheduling vector $\mathbf{x_a}$ and state vector $\mathbf{y_a}$ as follows:

$$\mathbf{x_a} = [x_a^1, x_a^2, ..., x_a^H] \text{ and } \mathbf{y_a} = [y_a^1, y_a^2, ..., y_a^H], \tag{1}$$

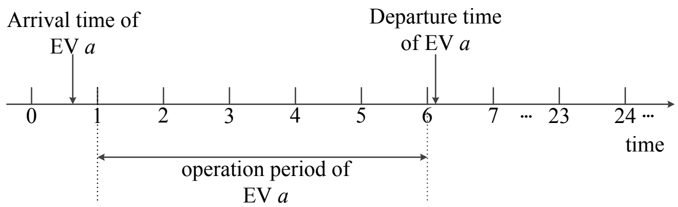


Fig. 2: Available charging period of EV a

where $H \geq 1$ is the scheduling horizon which indicates the number of time slots ahead that are taken into account for decision making in the EVs' charging scheduling. For each coming time slot $h \in \mathcal{H} = [1, 2, ..., H]$, let a binary variable $y_a^h = 0/1$ denote the state of EV a (not charging/charging) and a variable x_a^h denote the scheduled energy to be charged to EV a at time h. For each EV a with the maximum allowable charging rate R_a^{max} and the minimum charging rate R_a^{min}, we have that

$$y_a^h \cdot R_a^{min} \cdot \eta \leq x_a^h \leq y_a^h \cdot R_a^{max} \cdot \eta. \tag{2}$$

Let t_a^s and t_a^f denote the arrival time and departure time of EV a, respectively. Since we divide time into multiple discrete time slots, the available charging time of EV a, denoted by $\mathcal{T}_a$, is defined as the set of continuous time slots fall between the plug-in time t_a^s and plug out time t_a^f, as depicted in Fig. 2. Obviously we have $y_a^h = 0$ if $h \notin \mathcal{T}_a$. Further denote the battery energy capacity (which is defined as the maximum amount of energy can be stored in the battery), initial battery energy and desired departure battery energy of EV a by E_a^{cap}, E_a^s and E_a^d, respectively. Obviously, we have $E_a^d \leq E_a^{cap}$. The desired departure state of charge (SOC) of EV a is defined as $\gamma_a^d = E_a^d / E_a^{cap}$, where $0 < \gamma_a^d \leq 1$. The local controller can automatically detect the arrival time t_a^s, battery capacity E_a^{cap} and initial battery energy E_a^s of EV a when it connects to the charging plug. The departure time t_a^f, desired departure SOC γ_a^d are provided to the local controller by the owner of EV a before the charging begins. Given t_a^s and t_a^f, the available charging period $\mathcal{T}_a$ can be easily obtained. From the above descriptions, we have the following constraints intuitively:

$$E_a^s + \sum_{h \in \mathcal{T}_a} x_a^h \geq E_a^d, \tag{3}$$

$$E_a^d \leq E_a^{cap}. \tag{4}$$

Considering the fact that customers are risk averse [41], [42], they would be reluctant to join the scheme if they face the financial risks associated with electricity price uncertainty (i.e., their EV may be charged during periods when the electricity prices are high). Thus, it is assumed that the local controller offers flat electricity price p_c (which is announced in advance) for the EVs in this centralized charging scheme.

B. Decentralized Charging Control Model

Let $\mathcal{B}(h)$ denote the set of EVs engaging in the decentralized charging scheme at time h. EVs are able to communicate with the system controller via two-way communication channels, as illustrated in Fig. 1. For a particular time slot $h \in \mathcal{H}$, the system controller has a limited energy E_m^h that it can provide to the $B(h)$ connected vehicles for charging, where $B(h) = |\mathcal{B}(h)|$; the system controller charges the EVs a price of p_d^h for one unit of electricity; for each EV $b \in \mathcal{B}(h)$, let x_b^h denote the amount of energy it requests from the charging system so as to meet its energy requirements. The energy demand x_b^h may vary for different EVs based on different parameters such as battery capacity E_b^{cap}, current SOC γ_b^c, desired unplug SOC γ_b^d, the time varying electricity price p_d^h as well as the travel plans (two EVs may have different travel plans and may have different energy demands). We assume that EVs will only request the amount of energy according to their immediate need for charging and EVs compete with each other for the limited scarce available energy. Thus, the following constraints must be satisfied for the total amount of energy EVs charged at time slot h:

$$\sum_{b \in \mathcal{B}(h)} x_b^h \leq E_m^h, \tag{5}$$

where E_m^h is the energy cap for the decentralized charging group. Obviously, the demand of the connected EVs are coupled through the above constraint. For the system controller, it tries to properly optimize the electricity price p_d^h such that the revenue for selling the energy is maximized. A lower electricity price means sacrificing revenues. However if the price is set too high, customers (EVs) may reduce their demand, amounting to losing profits. Thus a suitable p_d^h has to be decided to maximize the benefits of the charging system.

The interactions between system controller and EVs can be modeled as a leader-follower noncooperative Stackelberg game, in which there is a single leader (system controller) and multiple followers (EVs). The system controller chooses the total amount of energy it can provide to EVs in $\mathcal{B}(h)$ and the electricity price. Given these two parameters, EVs respond to the controller by properly choosing their own charging demands. The game can be defined in its strategic form as $\mathcal{S} = \{(\mathcal{B}(h) \cup \{\text{system controller}\}), \{x_b^h\}_{b \in \mathcal{B}(h)}, E_m^h, p_d^h, \{U_b^h\}_{b \in \mathcal{B}(h)}, U_{sc}^h\}$, where U_b^h and U_{sc}^h are utility functions of EV b and system controller, respectively.

Note that, the main task of the system controller is to properly coordinate the charging profiles of all the connected EVs (belonging to either the centralized charging group or the decentralized charging group) to minimize the cost of the whole system.

IV. Problem Formulation

A. Centralized Charging Control

1) Global Optimal Scheduling: To find the global optimal EV charging profiles during the scheduling horizon, we first make the following assumptions: (1) the arrival time and departure time of each EV in the set $\mathcal{A}$ are known; (2) the plug-in SOC and desired plug out SOC for each EV in the set $\mathcal{A}$ are known; (3) the local controller collects all the information and performs the scheduling optimization. Specifically, the local controller solves the following optimization problem to obtain the global optimal charging scheduling sequences:

$$\begin{aligned} \min_{\mathbf{X},\mathbf{Y}} \quad & \sum_{h=1}^{H} \mathcal{C}_m^h(l^h) \\ \text{s.t.} \quad & l^h = \sum_{a\in\mathcal{A}} x_a^h \\ & x_a^h \geq 0,\ (2)\ (3) \text{ and } (4) \\ & y_a^h \in \{0,1\} \text{ and } y_a^h = 0,\ \text{if } h \notin \mathcal{T}_a, \end{aligned} \tag{6}$$

where $\mathbf{X} = [\mathbf{x_1}, \mathbf{x_2}, ..., \mathbf{x_a}, ...]^{\mathrm{T}}$ and $\mathbf{Y} = [\mathbf{y_1}, \mathbf{y_2}, ..., \mathbf{y_a}, ...]^{\mathrm{T}}$ are matrices of decision vectors $\mathbf{x_a}$ and $\mathbf{y_a}$ for $a \in \mathcal{A}$, respectively; l^h represents the system load; $\mathcal{C}_m^h(\cdot)$ is the cost function of the charging system for generating or importing electricity, which is assumed to be an increasing convex function. The convex property reflects the fact that each additional unit of power needed to serve the demands is provided at a non-decreasing cost. Without loss of generality, we consider quadratic cost function $\mathcal{C}_m^h(l^h) = n^h l^h + m^h {l^h}^2$ throughout this paper, where m^h and n^h are two coefficients [15], [16]. The global scheduling optimization problem can be interpreted as to minimize the total cost of the centralized EV charging system during the scheduling horizon, by optimizing over the EV charging scheduling matrix $\mathbf{X}$ and state matrix $\mathbf{Y}$. Problem (6) is an MIQP problem, which can be effectively tackled by cutting plane method, branch and bounded method, etc [43], [44]. The solution to this problem provides the global (off-line) optimal EV charging scheduling sequences during the scheduling horizon. However this scheduling scheme is impractical since the EVs' arrival and departure patterns are unknown and so are their parameters (current SOCs and plug out SOCs). In the following subsection, we introduce a practical dynamic scheduling approach, which relaxes the assumptions adopted in the global optimal scheduling problem (6). The solution of this dynamic scheduling approach performs close to the global optimal scheduling scheme.

2) A Dynamic Scheduling Approach: One difficulty of the centralized charging lies in the fact that the system is dynamic with EVs coming and departing all the time. Thus it is not possible to have a stationary long-term scheduling profile. To tackle with the system condition dynamics, we adopt the MPC approach (also known as "receding horizon approach") [45] [46], of which the basic idea is to calculate the optimal control sequences yet implement only the first step of them. In other words, the centralized EV scheduling problem is solved at time $h = \tau$ ($\tau \in \mathcal{H}$ denotes the current time index) for the remaining horizon $[\tau, \tau+1, ..., W^\tau]$, yet only the solution for the current time slot τ is implemented (W^τ is the decision making horizon). In the next time slot, the local controller shall update the system information (e.g., the set of connected EVs currently, their current SOC and desired plug out SOC, etc.) and re-do the calculations. The time horizon for the decision making can be defined as the latest de-energize time of the EV in the current connected vehicle set, i.e., $W^\tau = \max_{a\in\mathcal{A}(\tau)} \lfloor t_a^f \rfloor$, where $\mathcal{A}(\tau)$ is the current connected vehicle set of centralized charging. The illustration of the current time horizon for the decision making is depicted in Fig. 3. The optimization problem at current time τ can be formulated as:

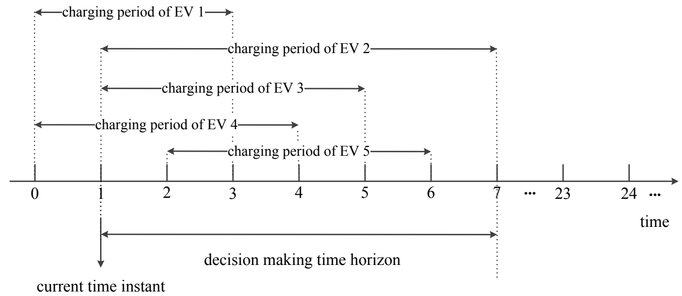


Fig. 3: The illustration of the current time horizon for the decision making of the charging scheduling, i.e., $W^\tau = \max_{a\in\mathcal{A}(\tau)} \lfloor t_a^f \rfloor$. The current connected EVs' set is $\mathcal{A}(\tau) = \{\text{EV 1, EV 2, EV 3, EV 4, EV 5}\}$.

$$\begin{aligned} \min_{x_a^h, y_a^h} \quad & \sum_{h=\tau}^{W^\tau} \mathcal{C}_m^h(l^h) \\ \text{s.t.} \quad & l^h = \sum_{a\in\mathcal{A}} x_a^h \\ & x_a^h \geq 0,\ (2)\ (3) \text{ and } (4) \\ & y_a^h \in \{0,1\} \text{ and } y_a^h = 0,\ \text{if } h \notin \mathcal{T}_a \\ & a \in \mathcal{A}(\tau),\ h \in [\tau, \tau+1, ..., W^\tau]. \end{aligned} \tag{7}$$

The dynamic EV charging scheduling problem (7) at the beginning of time slot τ is still an MIQP problem which can be solved efficiently by many commercial optimization softwares including CPLEX, Mosek, FortMP and Gurobi, etc. By solving (7), we obtain charging scheduling sequences x_a^h, $a \in \mathcal{A}(\tau)$, $h \in [\tau, \tau+1, ..., W^\tau]$, among which only the charging scheduling sequences x_a^h, $a \in \mathcal{A}(\tau)$, $h = \tau$ are executed, and other scheduling sequences x_a^m, $a \in \mathcal{A}(\tau)$, $m \in [\tau+1, ..., W^\tau]$ are discarded, which will be finally updated at the beginning of time slot m.

B. Game Formulation of Decentralized Charging

In this paper, we formulate the interactions between system controller and EVs into a leader-follower noncooperative Stackelberg game, where the system controller acts as the leader and the EVs as followers. At any time slot h, two principle components of the game $\mathcal{S} = \{(\mathcal{B}(h) \cup \{\text{system controller}\}), \{x_b^h\}_{b\in\mathcal{B}(h)}, E_m^h, p_d^h, \{U_b^h\}_{b\in\mathcal{B}(h)}, U_{sc}^h\}$ are the utility functions of the leader (system controller) U_{sc}^h and the followers (EVs) U_b^h, $b \in \mathcal{B}(h)$, respectively. We have detailed discussions as follows:

1) Utility Functions of EVs: EV's utility function captures the benefit it obtains for consuming the demand energy. The utility function $U_b^h(x_b^h, \mathbf{x}_{-b}^h, \alpha_b^h, \beta_b^h, p_d^h)$ of EV b is defined as a function of the energy it charges. Here, x_b^h is the requested charging energy of EV b from the charging station. $\mathbf{x}_{-b}^h$ is the vector formed of all EVs' decision variables except the one of EV b, i.e., $\mathbf{x}_{-b}^h = (x_1^h, x_2^h, ..., x_{b-1}^h, x_{b+1}^h, ...)$. $\alpha_b^h > 0$ and $\beta_b^h > 0$ are parameters measuring the charging habit of EV b. The value of α_b^h and β_b^h may depend on the current SOC, the battery capacity and the travel plan of the EV b. In addition, the price of electricity p_d^h also influences the charging benefit of an EV. Mathematically, we have the following assumptions on the properties of the utility function of EV b:

1) Assumption 1: the utility functions U_b^h is non-decreasing with respect to the amount of energy the EV charges. In other words, each EV tends to charge more if possible until it reaches its maximum battery level, i.e.,

$$\frac{\partial U_b^h(x_b^h, \mathbf{x}_{-b}^h, \alpha_b^h, \beta_b^h, p_d^h)}{\partial x_b^h} \geq 0. \tag{8}$$

2) Assumption 2: An EV has a non-increasing marginal benefit with respect to the charging amount. This statement can be interpreted from these two aspects: 1) the marginal charging time (i.e., drivers' waiting time) increases since the charging rate slows down when the battery gets drenched; 2) the satisfaction level of an EV gradually gets saturated when more and more energy is charged; i.e.,

$$\frac{\partial^2 U_b^h(x_b^h, \mathbf{x}_{-b}^h, \alpha_b^h, \beta_b^h, p_d^h)}{\partial {x_b^h}^2} \leq 0. \tag{9}$$

3) Assumption 3: an EV's benefit gets lower when the electricity price increases, i.e.,

$$\frac{\partial U_b^h(x_b^h, \mathbf{x}_{-b}^h, \alpha_b^h, \beta_b^h, p_d^h)}{\partial p_d^h} < 0. \tag{10}$$

Without loss of generality, the quadratic utility function is defined as:

$$\begin{aligned} & U_b^h(x_b^h, \mathbf{x}_{-b}^h, \alpha_b^h, \beta_b^h, p_d^h) \\ = \; & -\frac{1}{2}\alpha_b^h (x_b^h)^2 + \beta_b^h \cdot x_b^h - p_d^h \cdot x_b^h. \end{aligned} \tag{11}$$

Note that the game formulation we proposed in this paper is a general methodology which is not restricted to the current quadratic utility function. As long as the utility function is differentiable and satisfies the above assumptions, the proposed method can be applied with virtually no change.

2) Utility Function of the System Controller: The objective of the system controller is to maximize the revenue for selling the electricity to EVs, thus the utility function of the system controller is defined mathematically as:

$$\begin{aligned} U_{sc}^h = \; & p_c \cdot \sum_{a \in \mathcal{A}(h)} x_a^h + p_d^h \cdot \sum_{b \in \mathcal{B}(h)} x_b^h \\ & - \mathcal{C}_m^h \left(\sum_{a \in \mathcal{A}(h)} x_a^h + \sum_{b \in \mathcal{B}(h)} x_b^h \right), \end{aligned} \tag{12}$$

where $\mathcal{C}_m^h$ is the cost function of the charging system. Utility function U_{sc}^h captures the revenue for selling the energy (fist two terms) and the cost for generating or buying the energy (the last term). In the proposed game, the system controller can control the price for selling the energy p_d^h and total energy cap E_m^h. The EVs respond to this price and choose the amount of energy to charge x_b^h to maximize their utilities and simultaneously they have to ensure that their total charging demand should not exceed the energy cap E_m^h. Note that, the centralized charging scheduling sequences x_a^h, $a \in \mathcal{A}(h)$ are determined by the centralized charging scheme (local controller). In this regard, for a fixed electricity price p_d^h, an EV b solves the following optimization problem:

$$\max_{x_b^h} \quad U_b^h(x_b^h, \mathbf{x}_{-b}^h, \alpha_b^h, \beta_b^h, p_d^h) \tag{13}$$

$$\text{s.t.} \quad \sum_{b \in \mathcal{B}(h)} x_b^h \leq E_m^h. \tag{14}$$

Obviously, the charging strategy of EV b depends on not only its own utility function, but also other EVs' charging strategies through constraint (14); and this constraint is shared by all the players (i.e., EVs). This game is a jointly convex generalized Nash equilibrium problem[3 4 5] (GNEP) due to the same shared "coupled constraint" (14) and the max-concave (i.e., min-convex) objective functions of EVs [47]. Then, after all the EVs' charging amount reach the generalized Nash equilibrium (GNE), the system controller optimizes the energy price p_d^h to maximize the revenue of the system. Given the GNE charging amount of EVs $(x_b^h, \mathbf{x}_{-b}^h)$, the system controller solves the following problem:

$$\max_{p_d^h} U_{sc}^h \tag{15}$$

to maximize the system revenue. The solution of the formulated non-cooperative leader-follower generalized Stackelberg game (GSG) is the GSE in which the leader finds its optimal price and the followers reach their equilibrium states. At this equilibrium, no player (i.e., both the leader and the followers) can increase his utility by changing unilaterally his strategy to any other feasible point. Here we term the formulated game as generalized Stackelberg game (GSG) rather than Stackelberg game because of the coupled constraint (14) for the followers. Since the followers' strategies are coupled, they need to seek a GNE instead of a traditional Nash equilibrium (NE).

V. Solutions and Algorithms

A. Existence of GSE

We first specify the definition of GSE and then discuss in detail the existence and the properties of it.

Definition 1: For the GSG formulation $\mathcal{S} = \{(\mathcal{B}(h) \cup \{\text{system controller}\}), \{x_b^h\}_{b\in\mathcal{B}(h)}, E_m^h, p_d^h, \{U_b^h\}_{b\in\mathcal{B}(h)}, U_{sc}^h\}$ defined in IV-B, where U_{sc}^h and U_b^h, $b \in \mathcal{B}(h)$ are utility functions of the leader and followers given by (12) and (11), respectively. A strategy set $(\mathbf{x}^{h^*}, p_d^{h^*})$ constitutes the GSE of the game, if and only if the following inequalities are

[3]The generalized Nash equilibrium problem (GNEP) is a noncooperative game in which each player's admissible strategy set depends on the other players' strategies.

[4]In a non-cooperative game, if the players' actions are coupled solely through the constraints, then this game is a special class of game whose solution is a generalized Nash equilibrium (GNE).

[5]The objective functions of EVs are all min-convex (max-concave) functions, and the strategy set which is constrained by a single linear function is closed and convex with respect to all variables, then we have that this formulated GNEP is jointly convex [47]. Detailed discussions will be presented in the next section.

satisfied:

$$U_b^h({x_b^h}^*, {\mathbf{x}_{-b}^h}^*, \alpha_b^h, \beta_b^h, {p_d^h}^*) \geq U_b^h(x_b^h, {\mathbf{x}_{-b}^h}^*, \alpha_b^h, \beta_b^h, {p_d^h}^*)$$
$$\forall {x_b^h}^* \in {\mathbf{x}^h}^*, b \in \mathcal{B}(h), \sum_{b\in\mathcal{B}(h)} x_b^h \leq E_m^h$$

and

$$U_{sc}^h({p_d^h}^*, {\mathbf{x}^h}^*) \geq U_{sc}^h(p_d^h, {\mathbf{x}^h}^*). \tag{16}$$

In other words, no EV can increase its utility by deviating from its GSE charging amount ${\mathbf{x}^h}^*$ and no price other than the GSE price ${p_d^h}^*$ can improve the utility of the charging system.

Typically in non-cooperative games, the existence of NE is not guaranteed. For the followers' game, to investigate the existence of GNE in response to a price p_d^h, we first propose the following definitions and theorems:

Definition 2: We say a game satisfies the convexity assumption if the following condition holds: for every player $v \in \mathcal{N}$ and every strategy $x_v \in \mathbb{R}^{n_v}$, where $\mathcal{N}$ is the set of players, the objective function $U_v(\cdot, x_v, \mathbf{x}_{-v})$ is min-convex (max-concave) and the strategy set $X_v(\mathbf{x}_{-v})$ is closed and convex. Note that we use $X_v(\mathbf{x}_{-v})$ to represent the strategy set of player v since his strategy set is dependent on other players' strategies.

Obviously, in the proposed followers' game, for each player, the objective function U_b^h is max-concave and the strategy set which is merely confined by constraint (14) is closed and convex. Thus the followers' game satisfies convexity assumption.

Definition 3: Let a GNEP be given, which satisfies convexity assumption, this GNEP is jointly convex if for some closed convex set $\mathbf{X} \subseteq \mathbb{R}^n$ $(n = n_1 + n_2 + ... + n_N)$ and all $v \in \mathcal{N}$, we have

$$X_v(\mathbf{x}_{-v}) = \{x_v \in \mathbb{R}^{n_v} : (x_v, \mathbf{x}_{-v}) \in \mathbf{X}\}. \tag{17}$$

For the proposed followers' game, it is easy to check that the strategy set of EV b is:

$$X_b(\mathbf{x}_{-b}^h) = \left\{ x_b^h \in \mathbb{R}_0^+, \sum_{b\in\mathcal{B}(h)} x_b^h \leq E_m^h \right\}. \tag{18}$$

Obviously, this game satisfies the jointly convex condition. Based on the previous definitions, the following theorem is proposed.

Theorem 1: In a jointly convex GNEP, the utility function of each player U_v is continuously differentiable, then every solution of the variational inequality (VI) problem VI$(\mathbf{X}, \mathbf{F})$[6] is also a solution of GNEP, where $\mathbf{X}$ is as defined in the definition of jointly convex (Definition 3) and $\mathbf{F} = [\frac{\partial U_v}{\partial x_v}]_{v=1}^N$.

The proof for this theorem can be found in [47]. Note that Theorem 1 does not indicate that any solution of a jointly convex GNEP is also a solution of the VI$(\mathbf{X}, \mathbf{F})$ and some solutions may be lost. We further have the definition of the variational equilibrium (VE) as follows.

[6]The variational inequality (VI) problem VI$(\mathbf{X}, \mathbf{F}(\mathbf{x}))$ consists in finding a vector $\bar{\mathbf{x}} \in \mathbf{X}$ such that $(\mathbf{y} - \bar{\mathbf{x}})^T \cdot \mathbf{F}(\bar{\mathbf{x}}) \geq 0$ for all $\mathbf{y} \in \mathbf{X}$.

Definition 4: In a jointly convex GNEP, the utility function of each player U_v is continuously differentiable, we call a solution of the GNEP that is also a solution of VI$(\mathbf{X}, \mathbf{F})$ a VE.

In a GNEP, the existence of VE is of particular interest since a VE is more socially stable than other GNE (if there exists any), and thus it is a desirable equilibrium state [48]. Next, we will prove the existence and uniqueness of VE in our proposed followers' game.

Theorem 2: If $\mathbf{X}$ is a compact convex set and $\mathbf{F}(\mathbf{x})$ is continuous on $\mathbf{X}$, then the VI problem admits at least one solution $\mathbf{x}^*$.

The proof for this theorem is lengthy and can be found in [49]. Considering the proposed followers' game, the strategy set of EVs

$$\mathbf{X}^h = \Big\{ (x_1^h, x_2^h, ..., x_b^h, ...) : \tag{19}$$
$$\forall b \in \mathcal{B}(h),\ x_b^h \geq 0, \sum_{b\in\mathcal{B}(h)} x_b^h \leq E_m^h \Big\}$$

is a Polyhedron, which is compact and convex. For the corresponding

$$\mathbf{F}^h = -\left[\frac{\partial U_b^h}{\partial x_b^h}\right]_{b=1}^{B(h)} = \begin{bmatrix} \alpha_1^h x_1^h + p_d^h - \beta_1^h \\ \alpha_2^h x_2^h + p_d^h - \beta_2^h \\ \vdots \\ \alpha_{B(h)}^h x_{B(h)}^h + p_d^h - \beta_{B(h)}^h \end{bmatrix} \tag{20}$$

is obviously continuous (linear), therefore we claim there exists VE in the followers' game. Note that compared with that in **Theorem 1**, we add a minus sign in the definition of $\mathbf{F}^h$. This is because the objective of an EV is to maximize its utility, while in **Theorem 1**, the default objective of the problem is minimizing the cost. Thus a minus sign is added here to keep the definition of $\mathbf{F}^h$ consistent. To investigate the uniqueness of VE, we propose the following theorem.

Theorem 3: In a variation inequality problem VI$(\mathbf{X}, \mathbf{F})$, if $\mathbf{F}(\mathbf{x})$ is strictly monotone on $\mathbf{X}$. Then the solution is unique, if one exists.

The proof for this theorem is presented in the Appendix. Now turn to the definition of $\mathbf{F}^h$, we have that the Jacobian of $\mathbf{F}^h$ is

$$\mathbf{JF}^h = \begin{bmatrix} \alpha_1^h & 0 & \dots & 0 \\ \cdot & \alpha_2^h & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ 0 & 0 & \dots & \alpha_{B(h)}^h \end{bmatrix}, \tag{21}$$

which is a diagonal matrix with all the diagonal elements being positive. In other words, $\mathbf{JF}^h$ is a positive definite matrix so $\mathbf{F}^h$ is strictly monotone on $\mathbf{X}^h$. Therefore, given an electricity price p_d^h, there exists GNE and more precisely, an unique VE for the followers' GNEP.

Theorem 4: For a fixed electricity price p_d^h, the unique VE is the socially optimal solution of the proposed followers' GNEP between EVs.

The proof for Theorem 4 is presented in the Appendix. This theorem states that by solving the VI$(\mathbf{X}^h, \mathbf{F}^h)$, where $\mathbf{X}^h$

and $\mathbf{F}^h$ are defined by (19) and (20) respectively, the socially optimal solution of the followers' GNEP can be obtained. As a result, when the system controller sets its optimal price in response to the VE demand of the EVs, the GSG reaches its GSE, which represents the socially optimal solution.

B. Algorithms to Find GSE

1) VE for the Followers' GNEP: For the decentralized charging scheme proposed in this paper, the GNEP among the EVs is transformed into a strictly monotone VI problem whose solution leads to the socially optimal VE. Numerous methods have been proposed to solve the VI problem, including projection method, relaxation method, decomposition method, etc. In this paper, we adopt Solodov and Svaiter (S-S) method to solve the VI problem [50], [51]. The S-S method is a kind of extragradient method (a sub-class of the projection method) which can solve the VI problem efficiently. The S-S method works briefly as follows: suppose $\mathbf{x}^k \in \mathbf{X}$ be the current approximation of the solution of VI$(\mathbf{X}, \mathbf{F})$; first, we compute the point $P_X(\mathbf{x}^k - \mu^k \mathbf{F}(\mathbf{x}^k))$, where $P_X(\cdot)$ denotes the orthogonal projection map onto $\mathbf{X}$ and μ^k is a judiciously chosen positive steplength. Here, $P_X(\mathbf{x}^k - \mu^k \mathbf{F}(\mathbf{x}^k))$ is the solution of the following quadratic programming problem

$$\min_{\mathbf{x}\in\mathbf{X}} \frac{1}{2}\mathbf{x}^T\mathbf{x} - (\mathbf{x}^k - \mu^k \mathbf{F}(\mathbf{x}^k))^T\mathbf{x}; \tag{22}$$

next, the line segment between $\mathbf{x}^k$ and $P_X(\mathbf{x}^k - \mu^k \mathbf{F}(\mathbf{x}^k))$ is searched for a point $\mathbf{z}^i$ such that the hyperplane

$$\partial H_k = \{\mathbf{x} \in \mathbb{R}^n | < \mathbf{F}(\mathbf{z}^k), \mathbf{x} - \mathbf{z}^k >= 0\} \tag{23}$$

strictly separates $\mathbf{x}^k$ from the solution of the VI$(\mathbf{X}, \mathbf{F})$ $\mathbf{x}^*$, where $< \cdot, \cdot >$ is the usual inner product in $\mathbb{R}^n$. To compute $\mathbf{z}^k$, an Armijo-type procedure is adopted, i.e., $\mathbf{z}^k = \mathbf{x}^k - \eta^k r(\mathbf{x}^k, \mu^k)$ where $\eta^k = \gamma^{\bar{i}}\mu^k$ with $\bar{i}$ being the smallest nonnegative integer i satisfying

$$\begin{aligned} &< \mathbf{F}(\mathbf{x}^k - \gamma^i \mu^k r(\mathbf{x}^k, \mu^k)), r(\mathbf{x}^k, \mu^k) > \\ &\qquad \geq \frac{\sigma}{\mu^k}\|r(\mathbf{x}^k, \mu^k)\|^2 \end{aligned} \tag{24}$$

where $r(\mathbf{x}^k, \mu^k) = \mathbf{x}^k - P_X(\mathbf{x}^k - \mu^k \mathbf{F}(\mathbf{x}^k))$ is the projected residual function; after the hyperplane ∂H_k is constructed, in the next iteration $\mathbf{x}^{k+1}$ is computing by projecting $\mathbf{x}^k$ onto the intersection between the feasible set $\mathbf{X}$ with the halfspace $\partial H_k = \{\mathbf{x} \in \mathbb{R}^n | < \mathbf{F}(\mathbf{z}^k), \mathbf{x} - \mathbf{z}^k >\leq 0\}$ which contains the solution set $\mathbf{X}^*$. The details of the S-S method is shown in Algorithm 1. Upon solving the VI$(\mathbf{X}, \mathbf{F})$, the VE demand of each EV can be obtained. Next, we show how to optimize the electricity price by the system controller given the VE demand of the EVs.

2) Electricity Price Optimization: To investigate the electricity price optimization, we first consider the Karush-Kuhn-Tucker (KKT) optimal condition system of the VI problem, which is given by

$$\mathbf{F}^h + \nabla^h_{\mathbf{x}}\left(\sum_{b\in\mathcal{B}(h)} x^h_b - E^h_m\right)\cdot\lambda = 0, \tag{25}$$

$$\lambda\left(\sum_{b\in\mathcal{B}(h)} x^h_b - E^h_m\right) = 0, \tag{26}$$

for some multiplier $\lambda \geq 0$. Note that if

$$\sum_{b\in\mathcal{B}(h)} x^h_b < E^h_m, \tag{27}$$

then some EVs are able to increase their charging demands to gain higher utilities. This constraint hence becomes an equality at the VE,

$$\sum_{b\in\mathcal{B}(h)} x^h_b = E^h_m, \tag{28}$$

i.e., for a fixed electricity price, the sum of demands of all the EVs at the VE is equal to the available energy cap E^h_m. Note that in the game formulation, energy is assumed to be a scarce resource. The energy cap E^h_m should be lower than the total energy consumption capacity of the connected EVs. This avoids the trivial case where all the EVs get the energy allocation equaling to their maximum capacities. From (25) we have that:

$$\lambda + \alpha^h_b {x^h_b}^* + p^h_d - \beta^h_b = 0, \tag{29}$$

for any $b \in \mathcal{B}(h)$. Thus the electricity price should satisfy

$$p^h_d = \beta^h_b - \lambda - \alpha^h_b {x^h_b}^*. \tag{30}$$

Considering the utility function of the system controller U^h_{sc} from (12), obviously when p^h_d reaches its maximum the system can obtain the maximum utility, therefore the optimal price of the proposed game is:

$${p^h_d}^* = \beta^h_b - \alpha^h_b {x^h_b}^*, \tag{31}$$

i.e., $\lambda = 0$ when the GSG reaches the GNE. The requested charging amount of each vehicle should be

$${x^h_b}^* = \frac{\beta^h_b - {p^h_d}^*}{\alpha^h_b} \tag{32}$$

with the optimal electricity price ${p^h_d}^*$. This is the equilibrium state of the game.

3) Algorithm Description: In order to reach the equilibrium, the system controller and EVs have to communicate with one another to make their choices. Upon any EV b is plugged in, the system controller receives its utility parameters α^h_b and β^h_b via communication channels (e.g., V2G). The algorithm starts with the setting of energy cap E^h_m. Given the fixed amount E^h_m, the system controller solves VI$(\mathbf{X}^h, \mathbf{F}^h)$ to obtain the optimal charging strategy vector $\mathbf{x}^{h*}$ using the S-S method. The system controller then gets the optimal electricity price p^{h*}_d adopting (31). p^{h*}_d is broadcast to EVs through communication channels and EVs determine their charging demands by solving (13), which is actually given by (32). The details of the S-S scheme to find GNE and the proposed algorithm to reach GSE are depicted in Algorithm 1 and Algorithm 2, respectively. Note that a different algorithm was proposed in [26], which also adopts the game theory framework to model the interactions between EVs and the system controller. The algorithm proposed in this paper shows its advantages over that in reference [26] from the following aspects: 1) Algorithm 2 is implemented in a distributed fashion (each EV chooses its own charging demand) and EVs undertake very low computational burden since the VI problem is solved by the system controller; while in [26], EVs have to participate into the problem solving of the VI problem. 2) By adopting the proposed algorithm, the communication traffic between the EVs and the system controller is very low. In each time slot, only one round-trip communication is implemented (i.e., the EVs submit their utility functions and the system controller broadcasts the optimal electricity price); while in [26], dozens of round-trip messages have to be exchanged before the game reaches its GSE. Particularly when the communication channel is poor, our approach can easily overcome the unstable channel by retransmission. While in [26], the game are difficult to reach GSE in such scenario. This would become more challenging when the EV fleet is of a large size.

Algorithm 1 Solodov and Svaiter (S-S) method [50] [51]

Input: The matrix $\mathbf{F}^h$ and the strategy set $\mathbf{X}^h$ which are given in (19) and (20), respectively; Initial electricity price p_d^{h0}; Final tolerance ε.
Output: Optimal charging strategy vector $\mathbf{x}_b^{h*}$.

1: **Begin**
2: choose $\mathbf{x}^0 \in \mathbf{X}^h$, $\eta_{-1} > 0$, $\gamma \in (0,1)$, $\sigma \in (0,1)$, $\theta > 1$, $k = 0$, gap $= e$, where e is a vector with entries being equal to 1;
3: **if** $\|\text{gap}\| < \varepsilon$
4: **then** stop;
5: **else**
6: compute $\mu^k = \min\{\theta \cdot \eta_{k-1}, 1\}$;
7: **if** $r(\mathbf{x}^k, \mu^k) = \mathbf{x}^k - P_X(\mathbf{x}^k - \mu^k \mathbf{F}(\mathbf{x}^k))$
8: **then** $\mathbf{x}^k \in \mathbf{X}^{h*}$, stop;
9: **else**
10: compute $\bar{i} = \arg\min_{i \in Z^+} \{< \mathbf{F}^h(\mathbf{x}^k - \gamma^i \mu^k r(\mathbf{x}^k, \mu^k)), r(\mathbf{x}^k, \mu^k) > \geq \frac{\sigma}{\mu^k} \|r(\mathbf{x}^k, \mu^k)\|^2\}$, where $\eta_k = \gamma^{\bar{i}} \mu^k$;
11: compute $\mathbf{z}^k = \mathbf{x}^k - \eta_k r(\mathbf{x}^k, \mu^k)$;
12: compute the halfspace $\partial H_k = \{\mathbf{x} \in \mathbb{R}^n | < \mathbf{F}(\mathbf{z}^k), \mathbf{x} - \mathbf{z}^k > \leq 0\}$;
13: compute $\mathbf{x}^{k+1} = P_{\mathbf{X}^h \cap H_k}(\mathbf{x}^k)$;
gap $= \mathbf{x}^{k+1} - \mathbf{x}^k$;
$k = k + 1$;
go to 3;
14: **end if**
15: **end if**
16: **End**

C. Algorithm to Determine a Proper E_m^h

In the decentralized charging scheme, EVs compete with each other for a fair allocation of the scarce energy. Intuitively, when the energy cap E_m^h is low, the competition between EVs becomes fierce and the optimal energy price p_d^h gets high. In contrast, if the E_m^h is high, then p_d^h is low. Under both cases, the total revenue of the system U_{sc}^h is poor. Hence we may assume that U_{sc}^h will first increase and then decline with respect to E_m^h (i.e., quasi-concave) and a proper E_m^h exists which maximizes U_{sc}^h (such assumptions will be validated in the following simulation part). Various algorithms can be adopted to search the optimal E_m^h, including Genetic Algorithm (EA), Newton-Raphson method, Gradient Descent method, etc. In this paper, we assume U_{sc}^h is derivable with respect to E_m^h and propose Algorithm 3 based on Gradient Descent method to search the optimal E_m^h. The effectiveness of the proposed algorithm will be verified in the following section.

Algorithm 2 Algorithm to reach GSE

Input: Utility function U_b^h for each vehicle $b \in \mathcal{B}(h)$.
Output: Optimal electricity price p_d^{h*}; optimal charging strategy x_b^{h*} selected by each vehicle $b \in \mathcal{B}(h)$.

1: **Begin**
2: Each EV $b \in \mathcal{B}(h)$ submits its utility function parameters α_b^h and β_b^h;
3: The system controller determines the energy cap E_m^h for the uncontrolled EVs;
4: The system controller solves VI$(\mathbf{X}^h, \mathbf{F}^h)$ by adopting Algorithm 1 and obtains the optimal charging strategy vector $\mathbf{x}^{h*}$;
5: The system controller computes the optimal electricity price $p_d^{h^*}$ based on (31);
6: The system controller broadcasts the electricity price $p_d^{h^*}$ to all the EVs in the decentralized charging group.
7: Each vehicle chooses its charging demand by solving problem (13) and obtains the optimal charging strategy x_b^{h*}, $b \in \mathcal{B}(h)$;
8: **End**

VI. Experimental Evaluation

In this section, we present simulation results for assessing the performance of the proposed HCD EV charging scheme and evaluate the effects of different parameters.

A. Simulation Setting

The parameters concerning EVs' charging rates and battery capacities are obtained from [40]. The units of the electricity price, the cost functions and the utility functions are US cent ¢/KWh. For the centralized charging, the scheduling horizon is 8 hours with time evenly divided into 32 time slots, i.e., the length of each time slot is 15 mins. The number of available charging plugs is 100 unless otherwise stated. The arrival time of EV is uniformly distributed and the arrival rate is 25 vehicles per hour by default. The plug out time is uniformly distributed between 1 time slot and 32 time slots. The amounts of energy needed for EVs are evenly distributed between 8 KWh and 64 KWh. The maximum allowable charging rate of an EV is 28 KW and the minimum

Algorithm 3 Algorithm to search an optimal E_m^h

Input: Starting point E_m^{h0}, tolerance ε;
Output: Optimal energy cap E_m^{h*}.

```
Begin
  x_0 = E_m^{h0}, k = 0;
  compute y_0 = U_sc^h(x_0) based on Algorithm 2;
  while (true)
      compute ∇U_sc(x_k) = (U_sc(x_k + ϵ) − U_sc(x_k)) / ϵ,
      where ϵ is a small number;
      x_k = x_k + α · ∇U_sc(x_k);
      y_1 = U_sc^h(x_k);
      if ||y_1 − y_0|| ≤ ε
          break;
      end if
      y_0 = y_1;
      k = k + 1;
  end while
  E_m^{h*} = x_k;
End
```

charging rate of an EV is 0 KW. For the cost function of the electricity acquisition $\mathcal{C}_m^h(\cdot)$, we set $m^h = 1 \times 10^{-3}$ ¢/KWh^2 and $n^h = 1.6$ ¢/KWh by default. To solve the optimization problem (7), interior point method is adopted, which can solve the convex optimization problem efficiently. For the decentralized charging, 100 vehicles participate in this scheme. Unless otherwise stated, their utility function parameters α_b^h and β_b^h are chosen randomly in the range of $[0.75, 1.25]$ and $[13, 15]$, respectively. The energy cap E_m^h is set as 700 KWh by default. Note that all statistical results are averaged over all possible random values of the EVs' parameters using 500 independent simulation results.

B. Results and Discussions

We first investigate how the optimal charging price p_d^{h*} varies with respect to energy cap E_m^h. The energy cap E_m^h is linearly varied from 675 KWh to 725 KWh for different EV numbers $B(h) = 95$, 100 and 105. Adopting the Algorithm 2, we compute the corresponding optimal electricity price p_d^{h*}. The results are depicted in Fig. 4. It is shown that the optimal price decreases with the energy cap. This is due to the fact that when the total available capacity of the charging system increases, the grid has more energy to sell, thus the competition between EVs gets weaker and price declines. In other words, as the available energy increases, the system controller has to reduce the energy price to encourage the vehicles to charge more energy. Meanwhile in Fig. 5, the effect of the number of connected EVs on the optimal electricity price is presented. It appears that a growing vehicle number leads to an increasing optimal price. The reason is that a larger vehicle number leads to a larger electricity demands. Hence, the system controller can set a higher electricity price to induce EVs to charge less.

The impacts of EVs' utility function parameters α_b^h and β_b^h on the optimal electricity price is illustrated in Figs. 6 and 7, respectively. To do the test, we vary the value

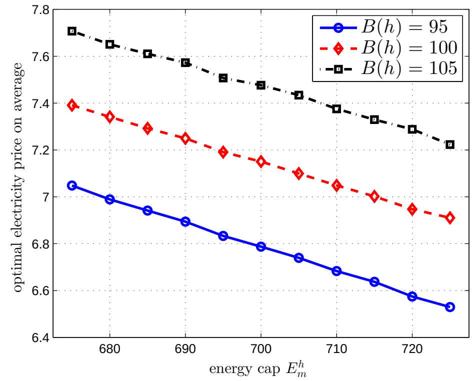


Fig. 4: The variation of optimal electricity price for decentralized controlled vehicles p_d^{h*} with respect to their energy cap E_m^h.

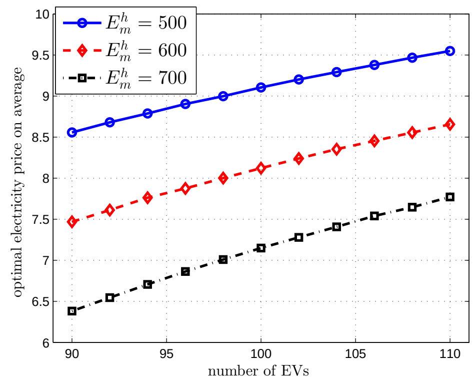


Fig. 5: The variation of optimal electricity price for decentralized controlled vehicles p_d^{h*} with respect to their number.

of α_b^h for different ranges of $\beta_b^h \in [10, 12]$, $[12, 14]$ and $[14, 16]$, respectively. The test for assessing the impact of β_b^h is conducted in a similar way, i.e., β_b^h is increased for various ranges of $\alpha \in [0.5, 0.8]$, $[0.8, 1.1]$ and $[1.1, 1.4]$, respectively. We observe that the optimal price is a decreasing function of α_b^h. In contrast, when β_b^h increases, the optimal price grows. The reason is that a higher α_b^h indicates that the EV's marginal utility declines. Thus EVs are prone to charge less and the corresponding electricity price decreases. While on opposite, an increment on β_n^h implies a rise of the marginal utility of the vehicle, therefore leading to a brisker energy demand and a higher electricity price. These results also verify the theoretical analysis results presented in Section V-B.

In Fig. 8, we present the total utilities of the charging system as a function of the energy cap of the distributed charging scheme E_m^h. To do the test, 100 vehicles are centrally controlled and the other 100 vehicles choose their charging profiles by their own. The energy cap of the latter group E_m^h increases from 300 KWh to 700 KWh and we compare the utilities of the whole system. It appears that the utility first shows an upper trend and then declines. There exists an optimal E_m^h which maximizes system's utility. By adopting Algorithm 3 proposed in the previous section, the system controller can properly determine an optimal E_m^h to maximize

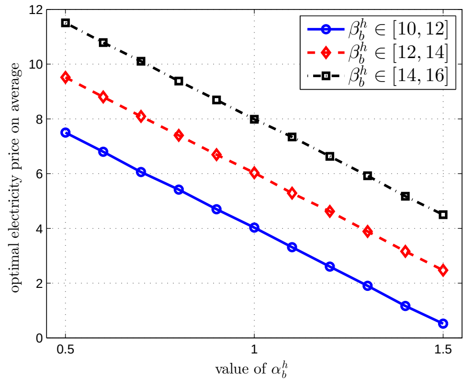


Fig. 6: The variation of optimal electricity price for decentralized controlled vehicles p_d^{h*} with respect to the users' utility parameter α_b^h.

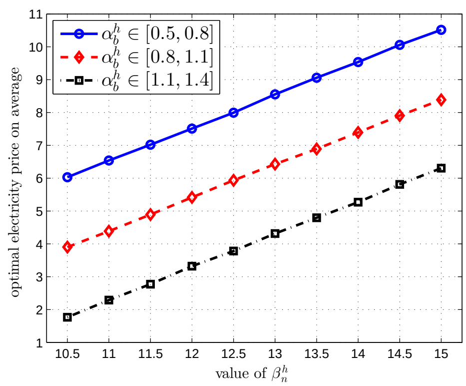


Fig. 7: The variation of optimal electricity price for decentralized controlled vehicles p_d^{h*} with respect to the users' utility parameter β_b^h.

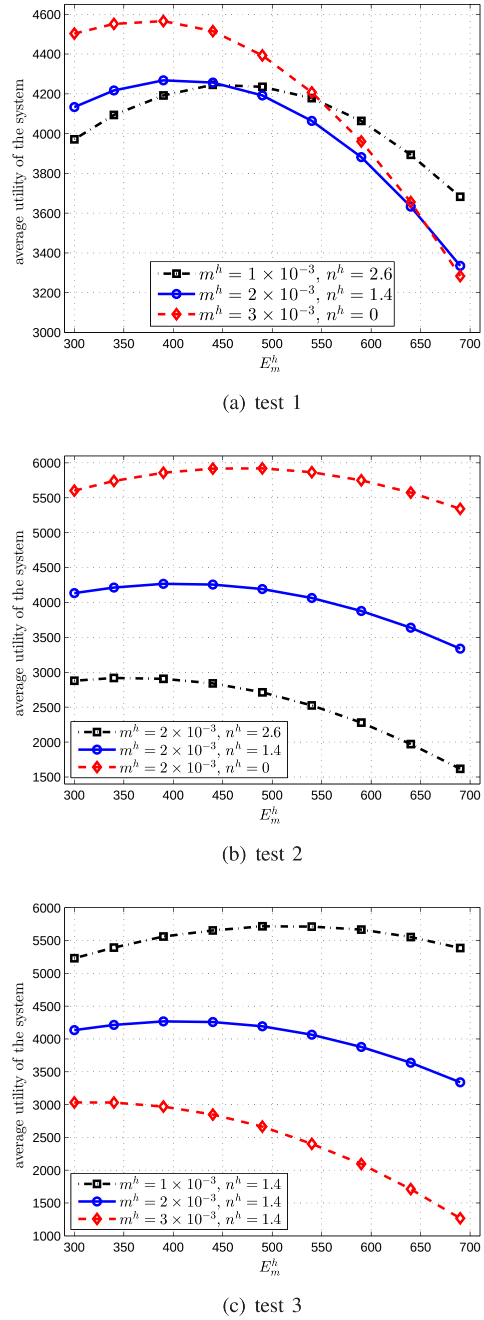


Fig. 8: The utility of the system with respect to decentralized controlled EVs' energy cap E_m^h under different cost functions.

its revenue given the system condition. The cost function $\mathcal{C}_m^h$ is further altered to investigate its impact. We observe that if the cost for acquiring energy grows, i.e., parameters m^h or n^h increases, both system utility and optimal energy cap E_m^{h*} decline. For the centralized controlled EV group, since the charging requirements have to be satisfied, the adjustments conducted by the system controller are relatively limited. Therefore, as the energy cost increases, the system controller is prone to cut down the proportion of energy allocated to decentralized controlled EVs so that it can curtail the energy expenses.

In Fig. 9, we evaluate how the vehicle numbers in both charging groups impact the system utility. To conduct this test, the total connected vehicles are fixed to 200, we vary the proportion of centralized controlled vehicles (equivalently, the proportion of decentralized controlled EVs) and compute the system utilities. Specifically, the number of centralized controlled vehicle varies from 80 to 120. We compare the cases where the centralized charging price for one unit of electricity p_c is low, medium and high. It is shown (see Fig. 9(a)) that when p_c is low, the total system utility drops as the centralized controlled EVs' proportion increases. This fact indicates that at this price stage, the incremental revenue due to the number growth of the centralized controlled EVs is less than the loss caused by the departure of decentralized controlled vehicles. Then, we increase the values of p_c to a medium level and recompute the corresponding system revenues. We note in Fig. 9(b) that the system utility first increases and then declines as the number of centralized controlled EVs varies from 80 to 120. If p_c is further increased, the system revenue will show a growing tendency as depicted in Fig. 9(c). This is because when the price p_c is high, the increasing earning from centralized controlled EVs has surpassed the loss due to the decreasing number of decentralized controlled vehicles.

We may see that given a price p_c, there exists an optimal ratio between the number of centralized controlled EVs and decentralized controlled EVs which maximizes the system utility. Therefore, based on the EVs' information and cost functions, the system controller can properly choose the energy cap E_m^h, electricity price p_c and other parameters such that the number ratio of these two EV groups is stimulated to its best value. Proper parameter selections can be obtained by various methods, typically involving large scale simulations and analyzing a large amount of historical data. In addition, given the best number ratio of these two charging groups, the charging park can properly determine the scales of centralized charging facilities and decentralized charging facilities so that the long-term expected revenue is maximized. Hence our research may provide some illuminations on the investment policy makings of the charging parks. Note that as all the EVs charging requirements and the operational charging constraints (e.g., the minimum and maximum charging rate) have been satisfied by our scheme, we do not show such results in the paper.

VII. Conclusion

In this paper, we investigated the coordination of EV charging at a charging park considering the EV owners' various charging preferences. An HCD charging control scheme was designed to determine the charging rates and demands of EVs. Specifically, at the centralized charging side, based on the EVs' arrival/departure patterns, a cost minimization problem was formulated and solved to obtain an offline global optimal scheduling. Considering the fact that the charging station is dynamic with unpredictable EVs' patterns, an MPC based adaptive charging approach was developed to determine the near-optimal EV charging profiles in real-time. On the decentralized charging side, to model the interactions between EVs and the charging system, a leader-follower noncooperative Stackelberg game based approach was proposed, where the system controller acts as the leader and EVs act as the followers. We prove the existence and optimality of the equilibrium state. It is also shown that the communication burden between EVs and the system controller is low and our decentralized charging scheme is robust to poor communication channels. We further studied how the size of these two charging groups influences the system utility. Simulation results investigated the impacts of different parameters. It is indicated that an optimal charging cap exists for the decentralized charing group which maximizes the revenues of the whole charging system. Our research may shed some illuminations on the investment policy making for charging park.

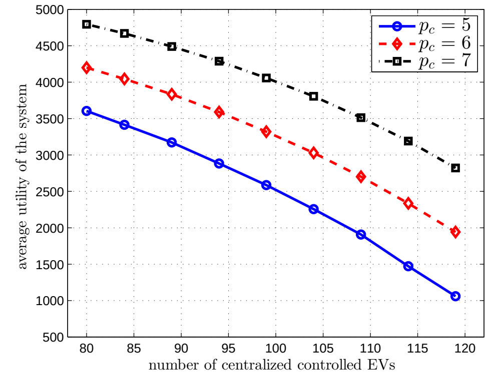
Appendix

A. Proof of Theorem 3

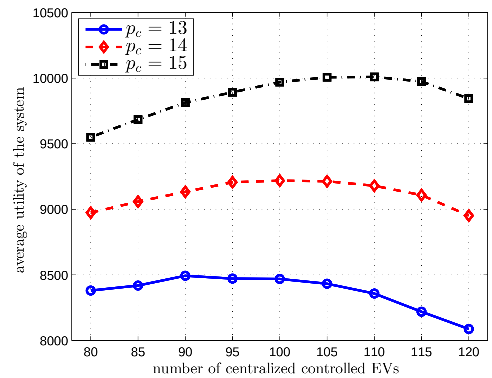
Suppose that $\mathbf{x}^1$ and $\mathbf{x}^*$ are both solutions and $\mathbf{x}^1 \neq \mathbf{x}^*$. Then since both $\mathbf{x}^1$ and $\mathbf{x}^*$ are solutions, they must satisfy:

$$\mathbf{F}(\mathbf{x}^1)^T \cdot (\mathbf{x}^{'} - \mathbf{x}^1) \geq 0, \;\; \forall \mathbf{x}^{'} \in \mathbf{X}, \tag{33}$$

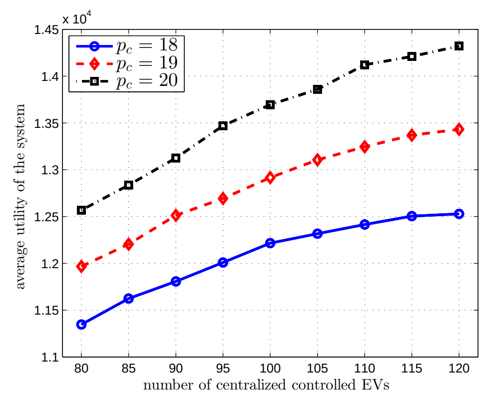
$$\mathbf{F}(\mathbf{x}^*)^T \cdot (\mathbf{x}^{'} - \mathbf{x}^*) \geq 0, \;\; \forall \mathbf{x}^{'} \in \mathbf{X}. \tag{34}$$



(a) p_c is low



(b) p_c is medium



(c) p_c is high

Fig. 9: The utility of the system with respect to the number of vehicles that are centralized controlled (the total number of EVs is 200).

After substituting $\mathbf{x}^*$ for $\mathbf{x}^{'}$ in (33) and $\mathbf{x}^1$ for $\mathbf{x}^{'}$ in (34) and adding the resulting inequalities, we obtain:

$$\mathbf{F}(\mathbf{x}^1 - \mathbf{x}^*)^T \cdot (\mathbf{x}^* - \mathbf{x}^1) \geq 0. \tag{35}$$

But inequality (35) is in contradiction to the definition of strict monotonicity. Hence, $\mathbf{x}^1 = \mathbf{x}^*$.

B. Proof of Theorem 4

Given a fixed electricity price p_d^h, to find the socially optimal solution of the proposed followers' GNEP, one has to solve the following optimization problem:

$$\max_{x_b^h} \quad \sum_{b\in\mathcal{B}(h)} U_b^h(x_b^h, \mathbf{x}_{-b}^h, \alpha_b^h, \beta_b^h, p_d^h) \tag{36}$$

$$\text{s.t.} \quad \sum_{b\in\mathcal{B}(h)} x_b^h \le E_m^h, \tag{37}$$

which is a quadratic programming problem. The Karush-Kuhn-Tucker (KKT) optimal conditions for this problem are:

$$\mathbf{F}^h + \nabla_{\mathbf{x}}^h \left(\sum_{b\in\mathcal{B}(h)} x_b^h - E_m^h \right) \cdot \lambda = 0, \tag{38}$$

$$\lambda \left(\sum_{b\in\mathcal{B}(h)} x_b^h - E_m^h \right) = 0, \tag{39}$$

which are exactly the same to the KKT conditions of the VI($\mathbf{X}^h, \mathbf{F}^h$) problem, i.e., (25) and (26). Since the Slater's condition holds, the KKT conditions provide sufficient and necessary conditions for optimality. Thus, the unique VE is the socially optimal solution of the proposed followers' GNEP.

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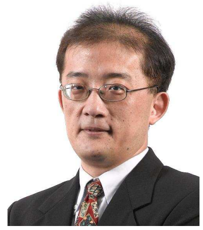

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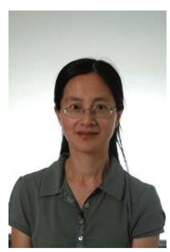

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