On Imperfect Node Protection in Complex Communication Networks

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Abstract — Motivated by recent research on complex networks, we study enhancing complex communication networks against *intentional attack* which takes down network nodes in a decreasing order of their degrees. Specifically, we evaluate an effect which has been largely ignored in existing studies: many real-life systems, especially communication systems, have protection mechanisms for their important components. Due to the existence of such protection, it is generally quite difficult to totally crash a protected node, though partially paralyzing it may still be feasible. Our analytical and simulation results show that such "imperfect" protections generally speaking still help significantly enhance network robustness. Such insight may be helpful for the future developments of efficient network attack and protection schemes.

Index Terms — Complex network, scale-free network, network robustness, intentional attack.

1. Introduction

Recent research results showed that many different real-life systems, including the Internet, world-wide web (WWW), airline transportation systems, food web, protein-protein reactions, co-authorship, and terrorist activities etc., when formulated into network models, share some stunning common features. Such interesting observations encourage even more research efforts on these network models, which have spawned a new research area named *complex networks* [1-7].

The most important complex network model is probably the *scale-free network* [1-2]. In such networks the fraction of nodes with a degree k, denoted as $P(k)$, is proportional to $k^{-\alpha}$, where α is the exponent which typically lies between 2 and 3 in real-life systems. With a *power-law* nodal-degree distribution, scale-free networks are very different from the well-known Erdős-Rényi random graph [1], with an easily visible feature of having a relatively large number of high-degree hub nodes.

Studies on complex networks shed new light which helps better understand complex systems and networks. One of the most important results is that while a scale-free network is robust against random failures, it is very fragile under *intentional attack* which crashes network nodes in a decreasing order of their nodal degrees [8].

Extensive research efforts have been made to study the robustness of scale-free networks [8-19]. Theoretical models for analyzing the robustness of scale-free networks under random failures and intentional attack were developed in [9] and [10], respectively. The applicable ranges of these theories were discussed in [11]. It was shown that, under either random failure or intentional attack, there exists a phase transition in the fraction of node loss, below which the network remain to function reasonably well [12]. A different model considered in [13] is that the probability that a node gets crashed is a function of its degree. To enhance network robustness against the intentional attack, different methods have been proposed including link insertion [14-15], link rewiring [16] and link recovery [17], etc. Though these methods appear to be effective, they may not be easily implemented in self-organized, extra-large networks, e.g., the Internet.

We evaluate an effect which has been largely ignored in existing studies: in real-life complex systems especially communication systems, usually there exist protection mechanisms for protecting critical components against hostile attack. Due to the existence of such mechanisms, it is generally quite difficult to totally crash a protected node though partially paralyzing it may still be feasible. In this paper, we term such protection as *imperfect node protection*. With theoretical analysis and extensive simulations, we show that such protection nevertheless helps significantly enhance network robustness.

The rest of the paper is organized as follows. A few different scenarios of imperfect node protection are defined in Section II. Theoretical analysis and simulation results on the effects of such protections in *random* scale-free networks (to be defined later) are presented in Section III and Section IV, respectively. Numerical evaluations of the proposed scenarios in a few real-life network models are presented in Section V. Finally, Section VI concludes the paper.

2. Imperfect node protection

We consider a few different scenarios of imperfect node protection where the protected nodes are partially paralyzed rather than totally crashed under intentional attack.

- Scenario I: all the nodes in the network are imperfectly protected. When a node is attacked, each of the links connected to it is blocked or removed at a probability of P_f.
- Scenario II: only high-degree nodes in the network have protection mechanism. Specifically, we assume that there exists a threshold value T. Each node with its degree no less than T is imperfectly protected. When such a node is attacked, each of the links connected to it is blocked or removed at a probability P_f. For any node with its degree lower than T, it is totally crashed once attacked.
- Scenario III: similar to the Scenario II, only that links between high-degree nodes are perfectly protected. Specifically, when nodes with degrees no less than T are attacked, they do not lose any links between them, meanwhile however the links between these nodes and those nodes with degrees lower than T will be removed at a probability of P_f. For nodes with degrees lower than T, they are totally crashed once attacked.

None of these three scenarios may easily happen in real-life systems since (i) there is hardly any system with equal protection over all its nodes; and (ii) strictly perfect protection probably does not exist at all. However, these scenarios nevertheless provide some useful benchmarks for estimating the effects of more realistic cases. Specifically, the three scenarios loosely resemble the real-life cases where (i) most nodes have some protection; (2) only important nodes (usually high-degree hubs) are heavily guarded; and (3) important nodes are protected and the links between them are also carefully safeguarded, respectively.

Note that the these three scenarios differentiate themselves from all the existing models such as the one where the probability that a node is crashed is a function of its degree [8], or imperfect protection over hub nodes for epidemic control [19], etc,

3. Analysis

The random network is defined as a network with random connections between different nodes, only subject to the given nodal-degree distribution $P(k)$ [18]. It is well known that a random network loses its global connectivity when

$$\kappa \equiv \frac{\langle k^2 \rangle}{\langle k \rangle} \leq 2, \tag{1}$$

where $\langle k \rangle$ denotes the average nodal degree and $\langle k^2 \rangle$ the average of the square of nodal degree [4]. The conclusion also holds in *correlated* random networks where network nodes are randomly connected subject to given distributions of $\{e_{ij}\}$; e_{ij} denotes the probability that degree-i nodes are connected to degree-j nodes [11]. In this paper, theoretical analysis is conducted on uncorrelated random networks with given $P(k)$, while analysis on different types of correlated networks is of our future research interest.

For a random scale-free network, denote its lowest nodal degree as m and its cutoff nodal degree as K. To facilitate the discussions, we term a node that has ever been attacked when a network is crashed as an *attacked* node; and an *un-attacked* node otherwise. Below we analyze the three different scenarios in random scale-free networks separately.

Scenario I:

Assume that the network is crashed after a fraction p_c of all the nodes has been attacked. Hereafter we term the value of p_c as the *crash threshold*. Also assume that when a node is attacked, each link connected to it has a probability of P_f to get removed. In this paper, we name P_f as the *link-removal probability*. Denote the lowest *original* degree of the attacked nodes as $\widetilde{K}$. We have [10]

$$p_c = \sum_{\widetilde{K}}^{K} p(k) = \frac{K^{1-\alpha} - \widetilde{K}^{1-\alpha}}{K^{1-\alpha} - m^{1-\alpha}}. \tag{2}$$

The fraction of links connected to all the attacked nodes, denoted as $\tilde{p}$, can be expressed as [10]

$$\tilde{p} = \sum_{\tilde{K}}^{K} \frac{kp(k)}{\langle k \rangle} = \frac{K^{2-\alpha} - \tilde{K}^{2-\alpha}}{K^{2-\alpha} - m^{2-\alpha}}. \tag{3}$$

For a link connecting an attacked node and an un-attacked node in the original network, it has a probability of P_f to be removed; for a link connecting two attacked nodes in the original network, since both of its two ends are attacked, it has a probability of $\left[1-(1-P_f)^2\right]$ to be removed. Therefore, denoting the link loss probability of an attacked node and an un-attacked node as $\tilde{p}^+$ and $\tilde{p}^-$ respectively, we have

$$\begin{cases} \tilde{p}^+ = P_f(1-\tilde{p}) + \tilde{p}\left[1-(1-P_f)^2\right] \\ \tilde{p}^- = P_f \cdot \tilde{p} \end{cases}. \tag{4}$$

After attack, the network nodal-degree distribution is changed from $p(k)$ to $\tilde{p}(k)$ where

$$\tilde{p}(k)=\begin{cases}\sum_{k_0=\tilde{K}}^{K} p(k_0)\binom{k_0}{k}\left(1-\tilde{p}^+\right)^k\left(\tilde{p}^+\right)^{k_0-k}, & k\ge\tilde{K}\\ \sum_{k_0=\tilde{K}}^{K} p(k_0)\binom{k_0}{k}\left(1-\tilde{p}^+\right)^k\left(\tilde{p}^+\right)^{k_0-k}+\sum_{k_0=m}^{\tilde{K}} p(k_0)\binom{k_0}{k}\left(1-\tilde{p}^-\right)^k\tilde{p}^{-k_0-k}, & k<\tilde{K}\end{cases} \tag{5}$$

For the attacked nodes, their average nodal degree in the original network, denoted as $\langle k\rangle^+$, is

$$\langle k\rangle^+=\frac{\sum_{\tilde{K}}^{K} kp(k)}{p_c}=\frac{\langle k\rangle\cdot\tilde{p}}{p_c}. \tag{6}$$

After attack, their average nodal degree is reduced to

$$\langle k'\rangle^+=\frac{(1-\tilde{p}^+)\cdot\langle k\rangle\cdot\tilde{p}}{p_c}. \tag{7}$$

As to the average square of nodal degree of the attacked nodes, before attack it is

$$\langle k^2\rangle^+=\frac{\sum_{\tilde{K}}^{K} k^2 p(k)}{p_c}=\frac{C}{p_c(3-\alpha)}(K^{3-\alpha}-\tilde{K}^{3-\alpha}) . \tag{8}$$

After attack, it becomes

$$\begin{aligned}\langle k'^2\rangle^+&=(1-\tilde{p}^+)^2\langle k^2\rangle^+ +\langle k\rangle^+\cdot\tilde{p}^+\cdot(1-\tilde{p}^+)\\ &=\frac{1-\tilde{p}^+}{p_c}\left[\frac{(1-\tilde{p}^+)C}{3-\alpha}(K^{3-\alpha}-\tilde{K}^{3-\alpha})+\tilde{p}\cdot\langle k\rangle\cdot\tilde{p}^+\right].\end{aligned} \tag{9}$$

For the un-attacked nodes, before attack, their average nodal degree is

$$\langle k\rangle^-=\frac{\langle k\rangle(1-\tilde{p})}{1-p_c} . \tag{10}$$

After attack, it becomes

$$\langle k'\rangle^-=\frac{(1-\tilde{p}^-)\cdot\langle k\rangle\cdot(1-\tilde{p})}{(1-p_c)}. \tag{11}$$

Before attack, the average square of nodal degree of the un-attacked nodes is

$$\langle k^2\rangle^-=\frac{\sum_{m}^{\tilde{K}} k^2 p(k)}{1-p_c}=\frac{C}{(1-p_c)(3-\alpha)}(\tilde{K}^{3-\alpha}-m^{3-\alpha}). \tag{12}$$

After attack, it becomes

$$\begin{aligned}\langle k'^2\rangle^- &= (1-\tilde{p}^-)^2\langle k^2\rangle^- + \langle k\rangle^- \cdot \tilde{p}^- \cdot (1-\tilde{p}^-) \\ &= \frac{1-\tilde{p}^-}{1-p_c}\left[\frac{(1-\tilde{p}^-)C}{3-\alpha}(\tilde{K}^{3-\alpha}-m^{3-\alpha}) + \langle k\rangle(1-\tilde{p})\cdot\tilde{p}^-\right].\end{aligned} \tag{13}$$

From Eqs. (6)-(13), we have that after attack the value of $\tilde{\kappa}$ is

$$\tilde{\kappa} = \frac{\langle k'^2\rangle}{\langle k'\rangle} = \frac{p_c\langle k'^2\rangle^+ + (1-p_c)\langle k'^2\rangle^-}{p_c\langle k'\rangle^+ + (1-p_c)\langle k'\rangle^-} \ . \tag{14}$$

From Eqs. (1)-(4), (7), (9), (11), (13) and (14), the crash threshold p_c can be numerically obtained.

Consider the extreme case where all the nodes in the network are attacked. We have

$$\begin{aligned}\tilde{\kappa} &= \frac{(1-P_f)^4\langle k^2\rangle + [1-(1-P_f)^2](1-P_f)^2\langle k\rangle}{(1-P_f)^2\langle k\rangle} \\ &= (1-P_f)^2\kappa + 1 - (1-P_f)^2.\end{aligned} \tag{15}$$

It shows that when $P_f < 1-\sqrt{\frac{1}{\kappa-1}}$, or in other words if each node can protect at least a fraction of $\sqrt{\frac{1}{\kappa-1}}$ of all the links connected to it, in Scenario I a random scale-free network will never be totally crashed by the intentional attack.

Scenario II:

In such scenario, only nodes with degrees no less than T have protection. Denote the crash threshold under such case as p_c' and the lowest original degree of the attacked nodes as $\tilde{K}'$. When $T \le \tilde{K}'$, the analysis remains the same as that for Scenario I. Hereafter we only consider the case where $T > \tilde{K}'$. Denote the fraction of links connected to the attacked nodes as $\tilde{p}'$. We have

$$\begin{cases} p_c' = \sum_{\tilde{K}'}^{K} kp(k) = \dfrac{K^{1-\alpha}-\tilde{K}'^{1-\alpha}}{K^{1-\alpha}-m^{1-\alpha}} \ , \\ \tilde{p}' = \sum_{\tilde{K}'}^{K} \dfrac{kp(k)}{\langle k\rangle} == \dfrac{K^{2-\alpha}-\tilde{K}'^{2-\alpha}}{K^{2-\alpha}-m^{2-\alpha}} \ . \end{cases} \tag{16}$$

Denote the fraction of nodes with degrees no less T as p_t, and the fraction of links connected to these nodes as $\tilde{p}_t$. They can be expressed as

$$\begin{cases} p_t = \sum_{T}^{K} kp(k) = \dfrac{K^{1-\alpha} - T^{1-\alpha}}{K^{1-\alpha} - m^{1-\alpha}} , \\ \tilde{p}_t = \sum_{T}^{K} \dfrac{kp(k)}{\langle k \rangle} = \dfrac{K^{2-\alpha} - T^{2-\alpha}}{K^{2-\alpha} - m^{2-\alpha}} . \end{cases} \tag{17}$$

Now we calculate link loss probabilities of different nodes with different degrees. For those nodes with degrees between $\tilde{K}'$ and T, since they have no protection, they will be totally crashed. Their link loss probability is therefore 100%. Denote the link loss probability of the nodes with degrees no less than T as $\tilde{p}'^{+}$, and the link loss probability of un-attached nodes as $\tilde{p}'^{-}$. It can be derived that

$$\begin{cases} \tilde{p}'^{+} = P_f(1-\tilde{p}') + \left[1-(1-P_f)^2\right] \cdot \tilde{p}_t + \left(\tilde{p}' - \tilde{p}_t\right) , \\ \tilde{p}'^{-} = \tilde{p}' - (1-P_f)\tilde{p}_t . \end{cases} \tag{18}$$

Note that the first equation of (18) contains three different parts, calculating the link-loss probabilities between high-degree (no less than T) and low-degree (lower than $\tilde{K}'$) nodes, between high-degree and high-degree nodes, and connected to the in-between nodes, respectively.

After attack, the network degree distribution is changed to

$$\tilde{p}(k) = \begin{cases} \sum_{k_0=T}^{K} p(k_0) \binom{k_0}{k} \left(1-\tilde{p}'^{+}\right)^k \tilde{p}'^{+k_0-k}, & k \ge \tilde{K}' \\ \sum_{k_0=T}^{K} p(k_0) \binom{k_0}{k} \left(1-\tilde{p}'^{+}\right)^k \tilde{p}'^{+k_0-k} + \sum_{k_0=m}^{\tilde{K}'} p(k_0) \binom{k_0}{k} \left(1-\tilde{p}'^{-}\right)^k \tilde{p}'^{-k_0-k}, & k < \tilde{K}' \end{cases} \tag{19}$$

For the nodes with original degrees no less than T, after attack their average nodal degree, denoted as $\langle k'' \rangle^{+}$, becomes

$$\langle k'' \rangle^{+} = \frac{(1-\tilde{p}'^{+}) \cdot \langle k \rangle \cdot \tilde{p}_t}{p_t} . \tag{20}$$

And their average square of nodal degree is

$$\langle k''^2 \rangle^{+} = \frac{1-\tilde{p}'^{+}}{p_t} \left[\frac{(1-\tilde{p}'^{+})C}{3-\alpha} (K^{3-\alpha} - T^{3-\alpha}) + \tilde{p}_t \cdot \langle k \rangle \cdot \tilde{p}'^{+} \right]. \tag{21}$$

For the un-attacked nodes, after attack, their average nodal degree, denoted as $\langle k'' \rangle^{-}$, is

$$\langle k'' \rangle^{-} = \frac{(1-\tilde{p}'^{-}) \cdot \langle k \rangle \cdot (1-\tilde{p}')}{(1-p'_c)} ; \tag{22}$$

and their average square of nodal degree is

$$\left\langle k''^2 \right\rangle^- = \frac{1-\tilde{p}'^-}{1-p_c'}\left[\frac{(1-\tilde{p}'^-)C}{3-\alpha}(\tilde{K}'^{3-\alpha} - m^{3-\alpha}) + \left\langle k \right\rangle (1-\tilde{p}')\cdot \tilde{p}'\right]. \tag{23}$$

Hence,

$$\tilde{\kappa} = \frac{\left\langle k''^2 \right\rangle}{\left\langle k'' \right\rangle} = \frac{p_t \left\langle k''^2 \right\rangle^+ + (1-p_c')\left\langle k''^2 \right\rangle^-}{p_t \left\langle k'' \right\rangle^+ + (1-p_c')\left\langle k'' \right\rangle^-}. \tag{24}$$

From Eqs. (16)-(18), (20)-(24), the crash threshold p_c' can be numerically solved.

Scenario III:

Similar to that in Scenario II, only those nodes with degrees no less than T have protection. The difference is that the links between these high-degree nodes will never be removed. Denote the crash threshold for such case as p_c'', and the lowest degree of the attacked nodes as $\tilde{K}''$. Similar to that for Scenario II, we only need to consider the case where $T > \tilde{K}''$.

Denote the fraction of the links connected to the attacked nodes as $\tilde{p}''$. The derivations of all the equations keep very similar to those for Scenario II. The only difference lies in the calculations of the link loss probability of those nodes with degrees no less than T. Denoting such probability as $\tilde{p}''^+$, we have

$$\tilde{p}''^+ = P_f (1-\tilde{p}'') + \tilde{p}'' - \tilde{p}_t. \tag{25}$$

Replace $\tilde{p}'^+$ with $\tilde{p}''^+$ in Eqs. (20) and (21), p_c'' can be numerically solved.

In addition to the crash threshold, another parameter useful for measuring the robustness of complex networks, especially complex communication networks, is their *cluster diameter*, defined as the average hop length of the shortest paths between all the source-destination node pairs in the largest connected component [10]. A network with a large cluster diameter, though may be still connected, cannot efficiently support communication applications. Accurate analysis on the cluster diameter, however, largely remains as an open issue. The best existing result, as far as we know, is that approximately the cluster diameter $d = \log P_\infty N / \log(\kappa - 1)$, where P_∞ denotes the probability of an arbitrary node belonging to the largest connected component in the network and $\kappa = \left\langle k^2 \right\rangle / \left\langle k \right\rangle$ [12]. In this paper, we evaluate cluster diameter by numerical simulations.

4. Simulation results on random scale-free networks and discussion

In our simulations, random scale-free network are generated by adopting the algorithm proposed in [14], with exponent ranging from 2 to 3. The network size N is 10,000. The minimum degree is 1 and the cutoff degree is 100, which equals to $\sqrt{N}$. Unless otherwise specified, the simulation results come from 50 independent realizations.

All the three scenarios have been simulated. We adopt the definition that a network is crashed once $\langle k^2 \rangle / \langle k \rangle \leq 2$ is achieved [9, 16].

As discussed in Section III, for Scenario I there exists a threshold value of P_f , below which the intentional attack can never completely crash a random scale-free network. Fig. 1 shows this threshold for networks with different exponents. The simulation results are calculated by the trial-and-error method with a step length of 1%. As we can see, subject to the same minimum and cutoff nodal degrees, networks with smaller exponents have higher average nodal degrees and consequently higher crash thresholds. A higher average nodal degree also tends to make imperfect node protection more effective in enhancing network robustness. That explains our observation in Fig. 1 that networks with smaller exponents have higher values of κ and consequently higher thresholds of link-removal probability.

The influences of link-removal probability on network robustness are further evaluated for all the three different scenarios. The results for the case where $\alpha = 2.5$ are plotted in Fig. 2. For scenarios II and III, we let the top 0.5% highest-degree nodes be protected. An interesting observation is that there exists a phase transition in the network crash threshold: once the link-removal probability is low enough, the threshold quickly jumps up from a rather low value to 1. This can be explained: for both scenarios II and III, similar to that of Scenario I, there exists a threshold of the link-removal probability below which the network will never be crashed. The network remains to be fragile under intentional attack until the link-removal probability gets close to the threshold. The threshold value therefore may set a target for network protection side to achieve.

Fig. 3 makes the comparisons between analytical and simulation results. The link-removal probability is set as 80% for all the three scenarios. For scenarios II and III, we still let the top 0.5% hub nodes be protected. It can be observed that the analytical results match well with the simulation results. Also we can see that even by protecting only 20% of the links connected to an attacked node, the network crash threshold can be significantly increased.

The comparisons between the effects of the three scenarios are presented in Fig. 4. We adopt the same parameters of link-removal probability and node protection percentage as those in Fig. 3. Since simulation results match well with theoretical analysis results, to avoid making the figure too crowded, only the analytical results are plotted. We see that Scenario I slightly outperforms the other two scenarios when the exponent α has a high value. When the exponent value is low, however, Scenario III leads to a much larger crash threshold. In fact, as we can easily observe, the networks will never be crashed when $\alpha \leq 2.4$, which can be explained as follows: in Scenario III, the sub-network composed of a small number of hub nodes and the connections between them are strictly protected. The survival of this sub-network helps keep $\langle k^2 \rangle / \langle k \rangle > 2$ in networks under attack. In networks with smaller threshold values, the numbers of moderate-degree nodes are relatively larger. The protected sub-network together with these moderate-degree nodes make a network extremely difficult to be totally crashed. Note that Scenario III puts only a small number of links between hub nodes under perfect protection, which nevertheless significantly enhances network robustness. For example, for the case where $\alpha = 2.1$, a total of 168 links are perfectly protected, which enhances the network to be almost never crashed. For another case where $\alpha = 2.9$, an even smaller number of 46 links between hubs are protected, The enhanced network can tolerate additional removals of about 300 links before it is crashed. It is efficient to enhance network robustness, if measured by crash threshold, by protecting links between hub nodes.

Fig. 5 shows in Scenarios II and III the improvements to network robustness when more hub nodes are protected. The simulation results are for the case where $\alpha = 2.5$ and the link-removal probability is 80%. We see that protecting a larger number of hub nodes only gradually increases network crash threshold in Scenario II, whereas in Scenario III since the links between the protected hubs are also protected, the threshold value boosts up with the number of protected hubs. In other words, protecting more hub nodes may not be effective unless the links between these hubs are also protected.

Figs. 4 and 5 may leave the impression that Scenario III is clearly the winner over the other two in enhancing network robustness. While this may be true in measuring the crash threshold, especially in random scale-free networks with small exponents, Fig. 6 shows that the situation is actually not that simple. In this figure, we plot *the largest cluster size*, defined as the number of nodes in the biggest connected component versus the number of nodes in the original network during the procedure of attack [3]. Due to space limitation, we show only the results with $\alpha = 2.1$, 2.5 and 2.9, respectively. The link-removal probability is 80% and the top 0.5% hub nodes are protected in Scenarios II and III. It can be observed that though Scenario III pushes the crash threshold to be very high when $\alpha = 2.1$ and 2.5, most of time during the procedure of intentional attack, it does not make the largest connected component be much larger than that in Scenario I. The small set of protected hubs makes the network to stay with $\langle k^2 \rangle / \langle k \rangle > 2$ even when the connected component size is already quite small.

Finally we show in Fig. 7 the cluster diameters in all the three different scenarios. All the parameters remain the same as those in Fig. 6. The basic observation is that all the three scenarios help reduce the cluster diameter under attack. Among them, the most effective one is Scenario III. The existence of the links between hubs not only keeps network connected but also effectively lowers the cluster diameter in the largest connected component. Scenario III may indeed become the favourable option when keeping a small cluster diameter is of high importance.

5. Simulations on real-life network models

To evaluate the effects of the proposed node protection schemes in communication networks, we carry out simulations on two different models as follows:

- A real-life Internet model on the AS-level as measured by the Applied Network Research (NLANR) Project on January 2, 2000 [22], which contains 6470 inter-connected nodes and 12,566 links. We have verified that it is indeed a scale-free network.
- A real-life Internet model on the router level as measured by the Cooperative Association for Internet Data Analysis (CAIDA), which contains 192,244 nodes and 609,066 links [23].

As that in most existing studies (e.g., [8-11]), we evaluate during the procedure of intentional attack the largest cluster size and the cluster diameter.

The earlier definition of crash threshold p_c given by $\langle k^2 \rangle / \langle k \rangle \leq 2$ does not apply to these two networks since neither of them is a random network. To differentiate, we evaluate the *threshold of crash* (TOC), defined as the percentage of network nodes that has to be removed to reduce the largest cluster size to be no more than 5%. It is shown in Fig. 8 and Fig. 9 that without any protection, the two real-life models have their TOC values at about 3.2% and 14% respectively.

We still consider all the three different scenarios. For each scenario, we consider different cases where the link removal probability is 50%, 70% and 80%, respectively. For Scenarios II and III, we still let the top 0.5% of hub nodes be protected. For all the cases, the presented simulation results for the AS-level model come from average values of 100 independent realizations. For the router-level model, due to its extra-large size, it is prohibitively time-consuming to carry out extensive realizations. Therefore only 5 realizations have been conducted.

The simulation results on the AS-level model are shown in Fig. 8. The three subfigures are for the three different cases where link removal probabilities are 50%, 70% and 80%, respectively. We observe that Scenario I steadily leads to the best performance among the three. In fact, since the threshold value of link removal probability to ensure that the network never be crashed in Scenario I is 78%, the network never gets crashed when the link removal probabilities are 50% and 70% respectively. Even when the link removal probability is 80%, TOC remains to be quite high at 60.3%.

Scenarios III performs only slightly better than Scenario II. Their corresponding TOC values are 75.1% vs. 77.9%, 58.4% vs. 64%, and 46.4% vs. 55.6% in the three different cases with different link removal probabilities, respectively. As we have observed in the last section, the extra protection over the links between a small number of hubs does not significantly increase the largest cluster size though it does decrease the cluster diameter. The TOC value therefore may not be significantly increased by such extra protection unless the number of the protected hubs is large enough to increase the largest cluster size to be higher than the threshold value (5% in our simulation) with the existence of the links between them.

Fig. 9 shows the simulation results in the router-level models where all the parameter values remain the same as those in Fig. 8. Once again we can easily observe that Scenario I leads to the best performance. The threshold value of the link removal probability to crash the network in Scenario I is 75%. When the link-removal probability is 80%, the TOC value of Scenario I is 48.9%.

Similar to that in the AS-level model, Scenario III only slightly outperforms Scenario II. The TOC values under different link removal probabilities are 28.1% vs. 28.1%, 22.9% vs. 25%, and 19.8% vs. 22.9%, respectively. The most visible difference between Fig. 8 and Fig. 9 is probably that Scenarios II and III become much less effective in enhancing network robustness in the router-level model than that in the AS-level model. This difference mainly comes from the degrees of their hub nodes: The biggest hub in the AS-level modal has a degree of 1458. The top 0.5% hubs combined together have a total degree of 6937. Putting these nodes under protection therefore strongly enhances network robustness even when such protection is imperfect. In the router-level model, on the contrary, the biggest hub has a degree of only 1071, a small value compared to the network size. The top 0.5%

hubs have total degrees of only 129,681, still moderate considering the extra-large network size. The protection effects therefore become less significant.

Finally, simulation results on the cluster diameters are presented in Fig. 10. We present only the results for the case with a link removal probability of 80% while the conclusion holds for all the other cases we have simulated: all the three different scenarios, especially Scenario III, help significantly reduce the cluster diameter. Protecting the links between a small number of hubs effectively helps shorten the paths between different node pairs.

Overall, we have the conclusion that protecting a small set of hubs may be an effective strategy in networks with high-degree hubs, e.g., the random scale-free network models and the AS-level Internet model. For networks with moderate-size hub nodes such as the router-level Internet model, putting more nodes under protection may be necessary in addition to carefully protecting hubs. The protection covering a large number of node can be highly effective even when it is rather weak. For both cases, protecting the links between hubs helps significantly reduces the cluster diameter.

6. Conclusion

In this paper, we evaluated the network robustness against intentional attack where some or all of the network nodes are with some imperfect protection. It is found that overall speaking even a rather weak protection helps significantly enhance the chance that the network survives the most hostile attack. While protecting a small number of big hubs or a large number of moderate-size nodes perform differently in different networks, protecting the links between hubs may be worth the efforts if maintaining a short cluster diameter is of priority. Such insights shall be helpful for the future developments of more efficient network attack and protection schemes.

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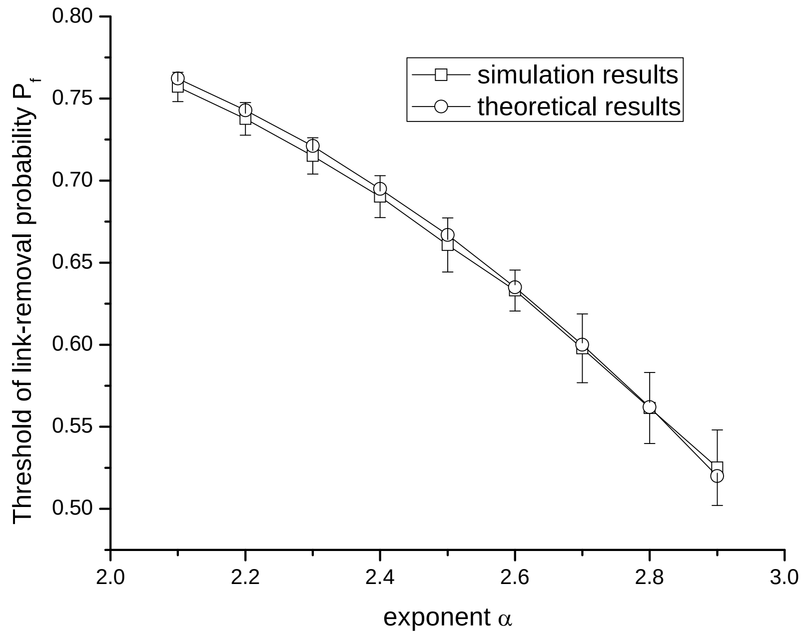
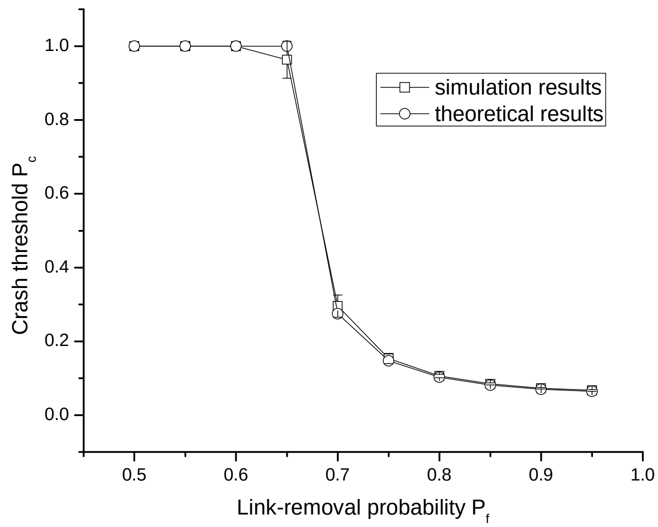
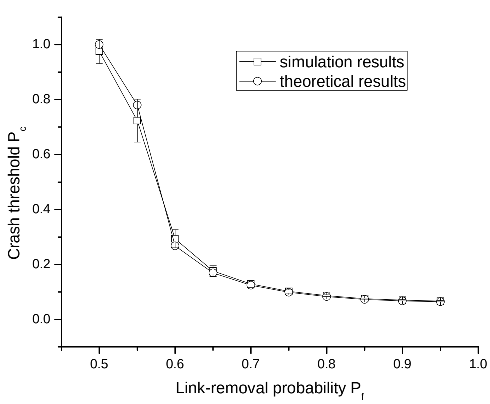
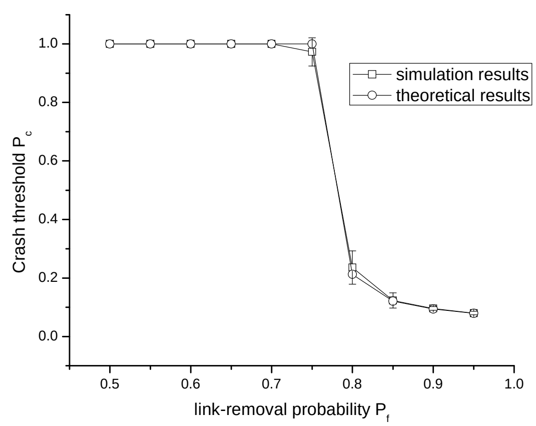


Fig. 1. The thresholds of link-removal probability to ensure that the networks never be totally crashed by intentional attack in random scale-free networks.



(a) Scenario I

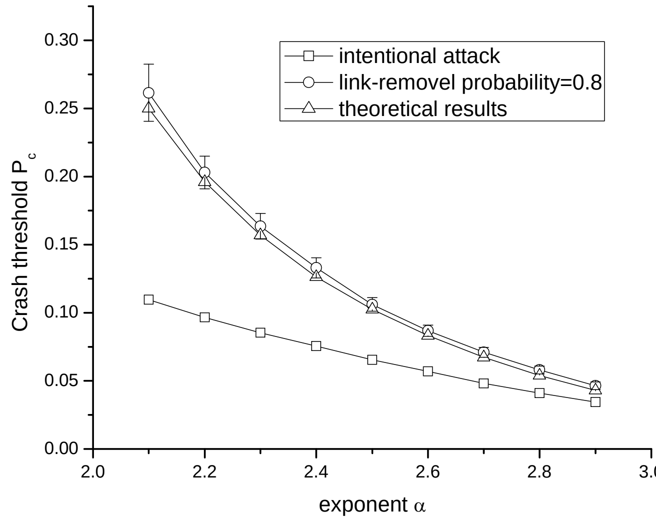




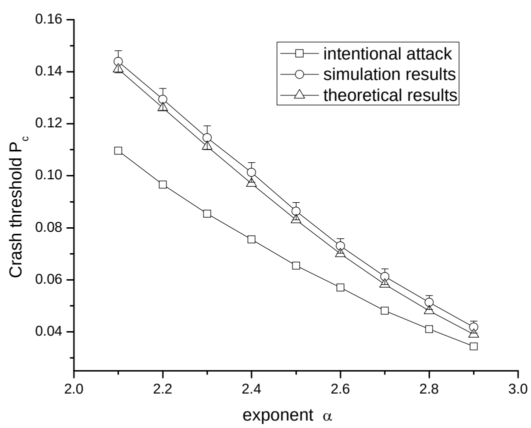
(b) Scenario II

(c) Scenario III

Fig. 2. The effects of link-removal probability on network crash threshold in the random scale-free network. We let α=2.5. For Scenarios II and III, the top 0.5% highest-degree nodes are protected.



(a) Scenario I



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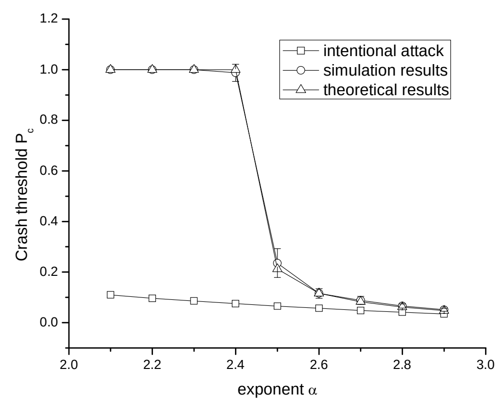


(c) Scenario III

Fig. 3. Comparisons between theoretical and simulation results of crash thresholds in random scale-free networks. The link-removal probability is 80% for all the three scenarios. For Scenarios II and III, the top 0.5% hub nodes are protected.

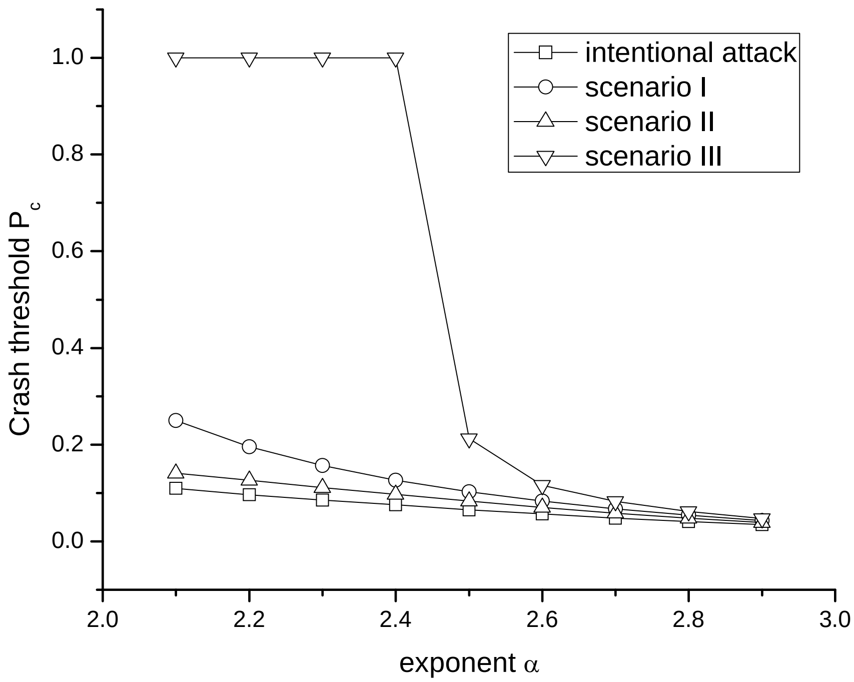
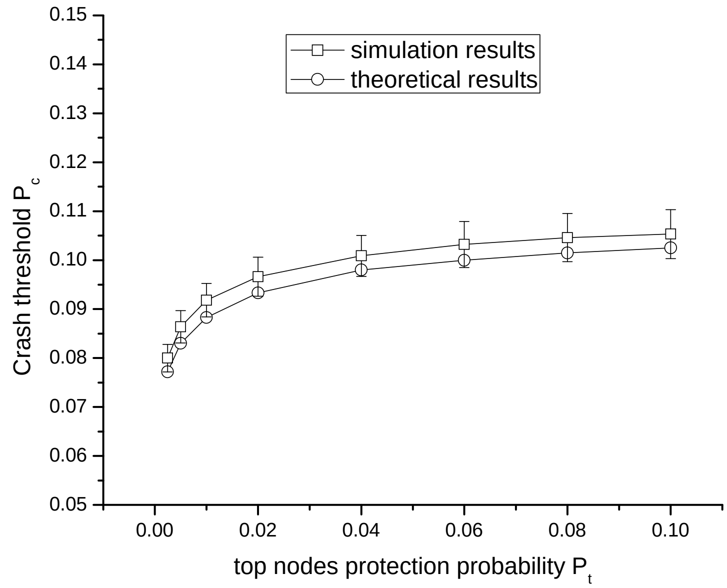
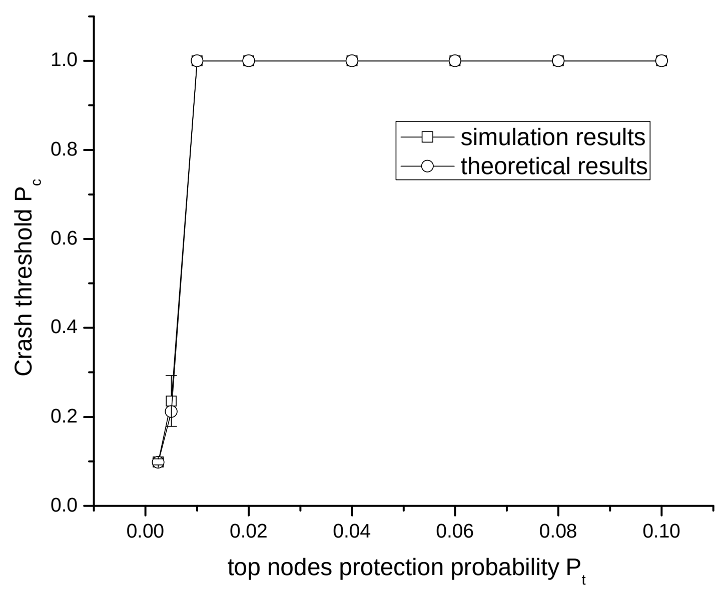


Fig. 4. The crash thresholds of random scale-free networks in different scenarios. The link removal probability is 80% for all the three scenarios. For Scenarios II and III, the top 0.5% hub nodes are protected.

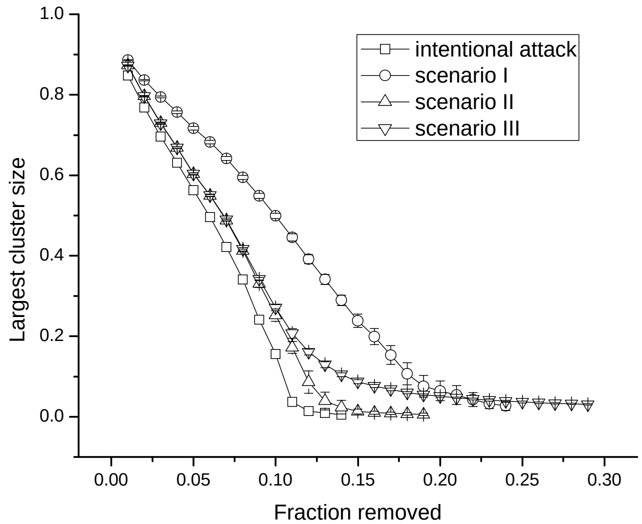


(a) Scenario II

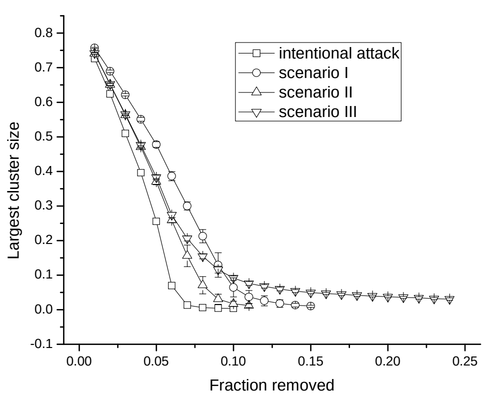


(b) Scenario III

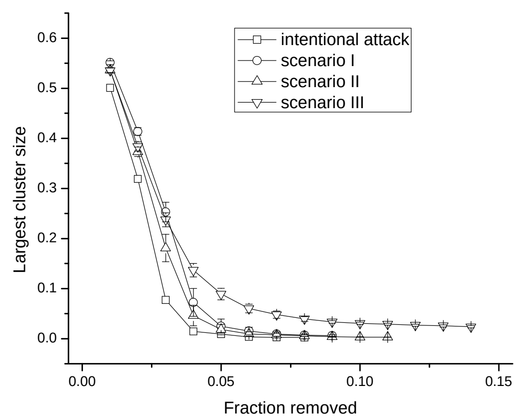
Fig. 5. The relationship between the crash threshold and the portion of hub nodes being protected in random scale-free network. We used α=2.5 and $P_f = 0.8$.



(a) α=2.1

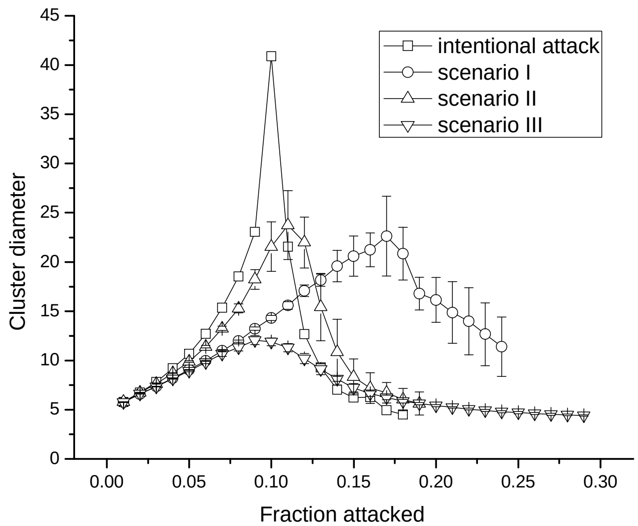


(b) α=2.5

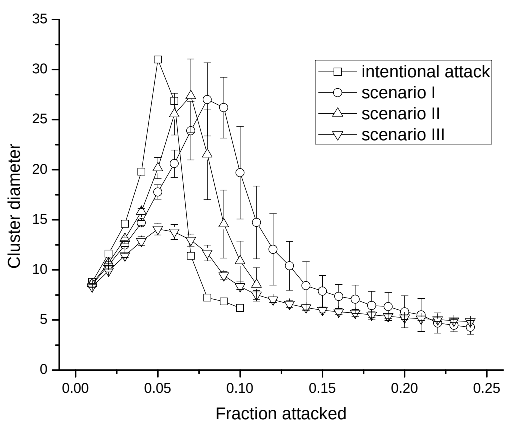


(c) α=2.9

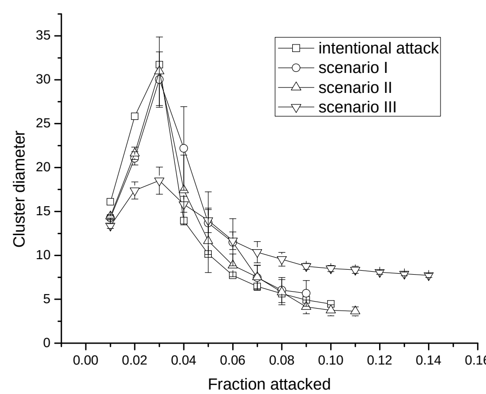
Fig. 6. The largest cluster sizes in random scale-free networks during the procedure of attack. The link removal probability is 80% for all the three scenarios. For Scenarios II and III, the top 0.5% hub nodes are protected.



(a) α=2.1

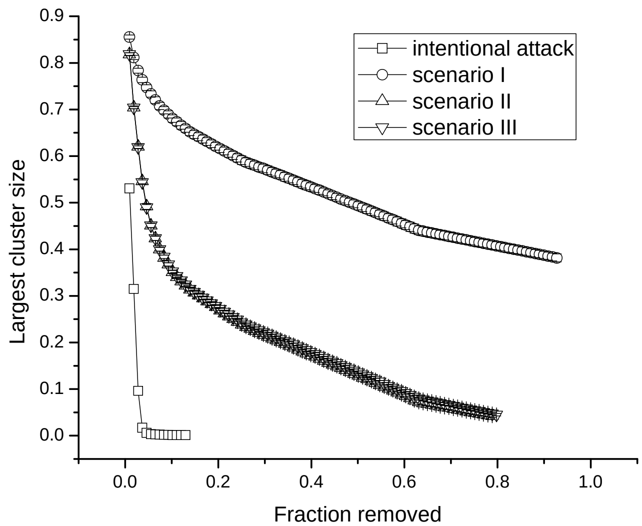


(b) α=2.5

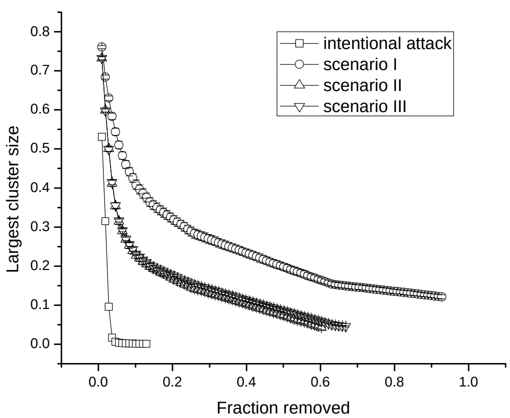


(c) α=2.9

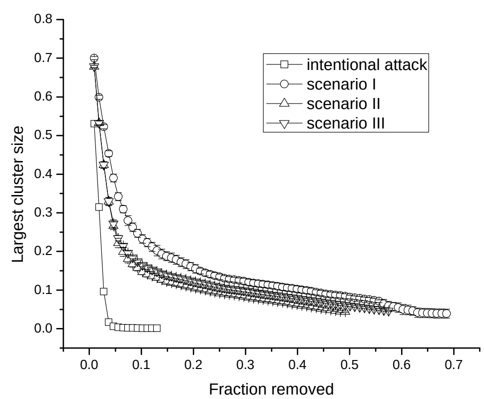
Fig. 7. The cluster diameters of random scale-free networks under attack. The link removal probability is 80% for all the three scenarios. For Scenarios II and III, the top 0.5% hub nodes are protected.



(a) Link-removal probability = 50%



(b) Link-removal probability = 70%



(c) Link-removal probability = 80%

Fig. 8. The largest cluster sizes of the AS-level model under attack with different link removal probabilities. For Scenarios II and III, the top 0.5% hub nodes are protected.

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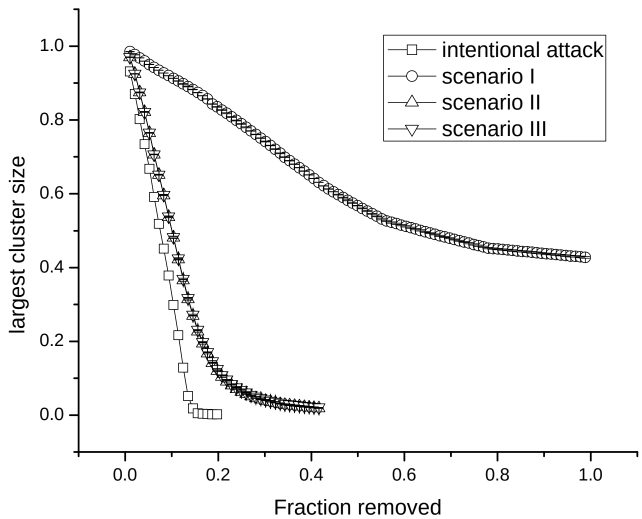
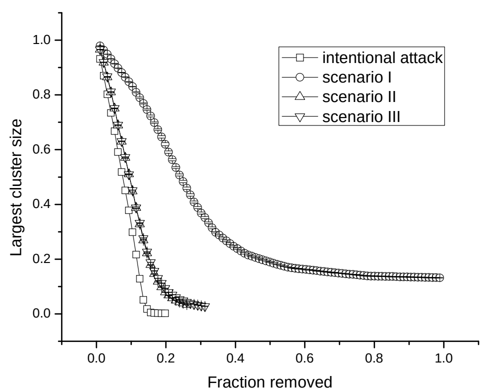
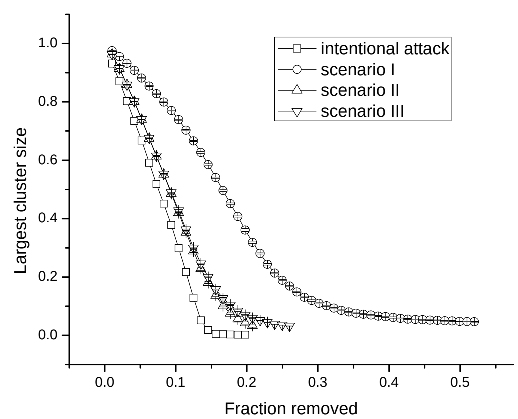


(a) Link-removal probability = 50%

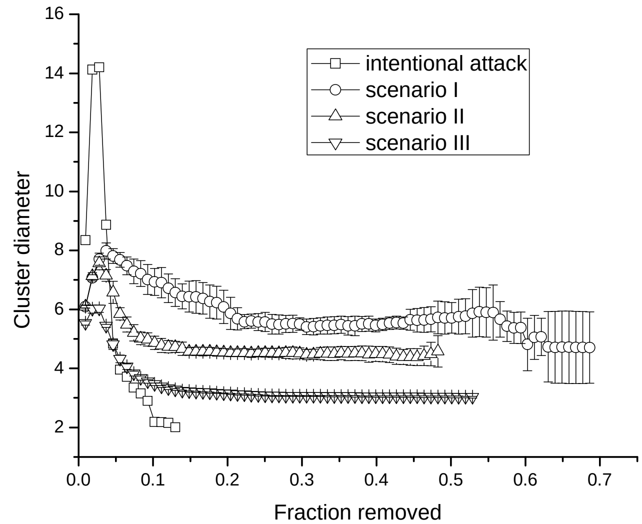




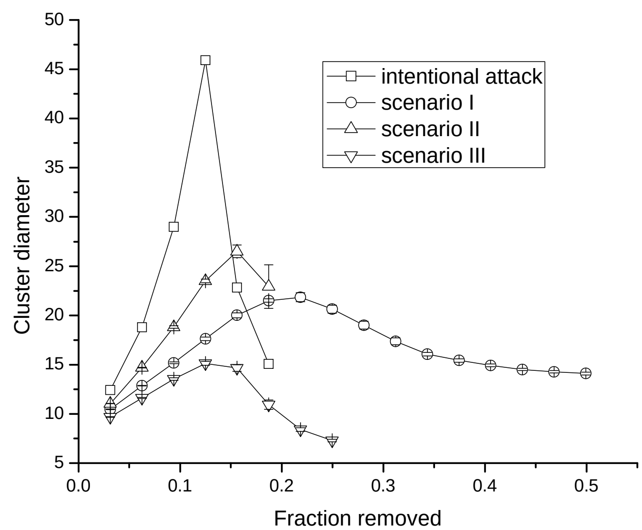
(b) Link-removal probability = 70%

(c) Link-removal probability = 80%

Fig. 9. The largest cluster sizes of the router-level model under attack with different link removal probabilities. For Scenarios II and III, the top 0.5% hub nodes are protected.



(a) AS-level model



(b) Router-level model

Fig. 10. The cluster diameters of the AS-level and router-level Internet models under attack. The link removal probability is 80%. The top 0.5% nodes are protected in Scenarios II and III.