Power Demand and Supply Management in Microgrids with Uncertainties of Renewable Energies

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Abstract

In operation of microgrids, an important task is to match the power generation and consumption profiles at a minimum cost. Since the highly fluctuant renewable energies constitute a significant portion of the power resources in microgrids, the microgrid system central controller (MGCC) faces the challenge of effectively utilizing the renewable energies while fullfilling the requirements of customers. In this paper, we propose a power demand and supply management framework to tackle the problem stated above. A flexible uncertainty model is developed to capture the randomness of renewable energy generation. Specifically, we introduce a reference distribution according to the past observations and empirical knowledge, and then define a distribution uncertainty set to confine the uncertainty of renewable energies. The new model allows the renewable energy to fluctuate around the reference distribution. An optimization problem is then formulated to determine the optimal power consumption and generation scheduling for minimizing the fuel cost. We present a two-stage optimization approach to first transform and then solve the prime problem. Numerical results indicate the properties of our problem formulation and provide some illuminations on the policy making for the MGCC. We also show that the proposed power demand and supply management mechanism can effectively reduce the energy cost.

Keywords: microgrids, demand and supply management, renewable energy uncertainties, reference distribution

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1. Introduction

The smart grid is the innovative future electric power system that will improve the conventional electrical grid network to be more clean, reliable, secure, cooperative, and efficient. The growth and evolution of the smart grid is expected to come with the plug-and-play integration of the basic structures called microgrids. Specifically, microgrids are small-scale low voltage power supply networks designed to supply electrical load for a small community such as a university campus, a commercial area and a trading estate, etc. Microgrids can autonomously coordinate local generations and demands in a dynamic manner. It can operate in either grid-connected mode or island mode [1]. There have been world wide deployments of pilot microgrids, such as those in US, Germany, Greece and Japan [2].

Microgrids are expected to be more robust and cost-effective than the traditional approach of centralized grids. However, to achieve a stable and economical operation, a number of technical and regulatory issues have to be resolved before microgrids can become a commonplace. One problem that would require due attention is the effective management of power supply and demand loads, which amounts to matching the power generation and consumption profiles [3][4]. Specifically, the power generators or microsources employed in microgrids are usually renewable or non-conventional distributed energy resources. While the incorporation of such renewable resources shall certainly bring great environmental benefits, it imposes new challenges as well: different from that in the traditional power systems with conventional controllable electric generators, generation scheduling in microgrids with fluctuant, climate-dependent renewable energy sources has to cope with the nontrivial uncertainties.

The microgrids may adopt hierarchical or decentralized demand control schemes [5] [6]. The decentralized control schemes facilitate distributed control and management of large complex systems. However, such control requires significant experiments before implementation and sophisticated coordination. Also it may introduce new security challenges. Hierarchical control is performed by a master controller which is responsible for matching the generation and load. When the demand resources are controlled upon the occurrence of disturbance, the strategy is often known as direct load control [7] [8]. In direct control program, based on an agreement between the central controller and customers, the controller can remotely control the operations of certain appliances in a household. This capability can be especically effecitve where there are electric devices allowing flexible usage time and/or energy storage, such as electric water heater (EWH) equipped with hot water storage tank and plug-in hybrid electric vehicles (PHEVs), etc. The Kyotango microgrid project in Japan is an example of hierarchically controlled microgrid [2].

In this paper, we tackle the basic problem faced by the microgrid system central controller (MGCC), namely to achieve a good match between power demand and supply subject to uncertainties of renewable energies. On the power demand side, we envision a scenario with real-time communication between the controller and energy consumer premises. Specifically, in each time period, the operator controller receives consumer power demands with different power level requirements, durations and time elasticity levels. The MGCC needs to minimize the electricity generation cost by optimally scheduling the operation of each appliance subject to the requirements set by the users. Here the generation cost is modeled as a convex function of instantaneous total power consumption.

On the power supply side, MGCC has to focus on how to effectively manage power generation so as to match the user load and maintain system reliability. We propose an uncertainty model to capture the fluctuant nature of renewable energies. Specifically, we extract an empirical distribution as a useful reference and allow the actual distribution of renewable energies to vary around it. To the best of our knowledge, this is the first time that the distribution uncertainty model is adopted to depict the indeterminacy property of renewable energy generation. The load balance constraint is aptly approximated using the chance constraint representation, where we can use a parameter to vary the conservation level of the solution. A tractable robust optimization approach is developed for transforming the chance constraints into linear constraints. Finally we investigate some of the desired properties of our problem formulation and provide some illuminations for the policy making of MGCC. It is also shown that the power demand and supply management scheme can greatly reduce the energy cost for the microgrid system.

The remainder of this paper is organized as follows. Section 2 provides a brief survey of the related work. In Section 3, we show the mathematical depiction of the power demand and supply management problem and the uncertainty model of the renewable energies. Section 4 presents the robust approach for handling the load balance constraint, and the problem decomposition process. We provide the simulation results and discussions in Section 5 and conclude the paper in Section 6.

2. Related Work

The problem we are tackling can be viewed as containing two different parts. On the power demand side, we try to build a hierarchical demand control scheme so as to achieve the economic consumption scheduling and fulfill the requirements set by energy users; on the power supply side, we need to properly model the randomness of renewable energy generation, which may account for a significant portion of power supply in microgrids. Note that load balance constraints act as the connection between power consumption and generation.

Demand control techniques can be categorized into either price based load control techniques, referred to as demand response methods, or direct load control, referred to as demand side management. Under price based load control scheme, users are encouraged to make energy consumption decisions individually according to the price information. Demand side management strategies, however, are usually applied directly by a central controller and require consumer subscription to an economic incentive program. Several representative works have studied demand control techniques in residential microgrids. A recent paper [9] developed a real-time pricing scheme that aims at reducing the peak to average load ratio (PAR) through demand response management in smart grid systems. A two-stage optimization problem was then proposed and solved. Fathi *et al.* developed a stochastic model of scheduling in a local area network with the objective of cost minimization and PAR minimization [10]. The work [11] presented a linear programming formulation for minimizing the energy cost through direct load control. In [12] an optimization model was presented to adjust the hourly load level of a given consumer in response to hourly electricity prices. The price uncertainty was modeled through robust optimization techniques. The uncertainties of renewable energies, however, are not considered in these studies. As such, the control schemes may not be readily optimal and applicable to microgrid scenario where renewable energies constitute a significant portion of power resources.

The research literature also includes some work related to renewable energy analysis. Zhang *et al.* considered a distributed economic dispatch problem for microgrid with high penetration of renewable energies [13]. The intrinsically stochastic and time-varying availability of renewable energy sources are captured by a polyhedral uncertainty set. Another work [14] defined stochastic upper and lower supply curves to capture a broad range of fluctuations in the power system. Energy generated by each power source was modeled as stochastic arrivals in the queuing model. In addition, hidden Markov models were adopted to characterize renewable energy generation [15] [16] [17]. Considering the fact that in practical scenarios, renewable energy may not follow Markov process or any simple distributions, Fang *et al.* used non-stochastic multi-armed bandit online learning technique to learn the evolution properties of renewable energy [18]. Our work jointly considers power demand and supply management, proposing a flexible way of analyzing the uncertainties of renewable energies. Furthermore, we propose an algorithm with different levels of safety guarantees while harvesting the renewable energies.

3. Formulation of the Microgrid Demand and Supply Management Problem

In this section, we provide a mathematical representation of the energy consumption and generation scheduling problem in an islanded microgrid with renewable energies. An MGCC is responsible for controlling the operations of the microgrid as well as performing optimization for minimizing the electricity generation cost for the microgrid system. We introduce the operations of the system and its mathematical depictions from the energy user side and energy generation side, respectively. The uncertainty model for describing the randomness of renewable energies is then demonstrated.

3.1. Energy Demand Side

Consider a group of energy consumers participating in this energy consumption scheduling program. We assume that there are two-way communication infrastructures (e.g., a local area network (LAN)) between MGCC and energy consumers. Let $\mathcal{A}$ denote the set of appliances belonging to these consumers, which may include PHEVs, dishwashers, cloth dryers, air conditioners, etc. We divide time into discrete time slots with equal length. For each appliance a that is switched on, the active power consumed during one unit of time slot is x_a. We also define an energy consumption scheduling vector $\mathbf{y_a}$ for each appliance a as follows:

$$\mathbf{y_a} = [y_a^1, ..., y_a^H] \tag{1}$$

where $H \geq 1$ is the scheduling horizon indicating the number of time slots ahead that are taken into account for decision making in the energy consumption scheduling. For each coming time slot $h \in \mathcal{H} = [1, 2, ..., H]$, a binary variable $y_a^h = 0/1$ denotes the state of appliance a (on/off). In this case, the actual energy consumption for appliance a at time slot h can be expressed as $x_a \cdot y_a^h$.

There is usually an upper limit on the total energy consumption in the microgrid in each time slot. Denoting this limit as E^{max}, we have:

$$\sum_{a\in\mathcal{A}} x_a \cdot y_a^h \leq E^{max}, \quad \forall h \in \mathcal{H}. \tag{2}$$

Next, assume that for each appliance $a \in \mathcal{A}$, the user indicates $\alpha_a, \beta_a \in \mathcal{H}$ as the beginning and end of a time interval in which the appliance a can be scheduled, respectively. Obviously, $\alpha_a < \beta_a$. For instance, the user may select $\alpha_a = 8$ PM and $\beta_a = 6$ AM (the next day) for his PHEV so that he could plug it in at night and get it fully charged before going to work the next day. Denote the minimum number of time slots needed for appliance a to finish its preset work as T_a. Given the predetermined parameters α_a, β_a, T_a, the appliance scheduling is subject to the following constraints:

$$\sum_{h=\alpha_a}^{\beta_a} y_a^h \geq T_a, \quad \forall a \in \mathcal{A}, \tag{3}$$

and

$$y_a^h = 0, \quad \forall a \in \mathcal{A}, \quad \forall h \in \mathcal{H} \setminus [\alpha_a, \beta_a]. \tag{4}$$

Constraint (3) shows that the time length $\beta_a - \alpha_a$ needs to be large enough to allow finishing the normal operation of appliance a. In addition, the energy user can choose proper α_a, β_a and T_a to indicate whether the operation of appliance a needs to be started immediately ($\beta_a - \alpha_a = T_a$) or can be deferred ($\beta_a - \alpha_a > T_a$).

To reveal the ramping down and ramping up limits on load levels of each time slot, we have:

$$\sum_{a\in\mathcal{A}} x_a \cdot y_a^h - \sum_{a\in\mathcal{A}} x_a \cdot y_a^{h+1} \leq r^D, \quad h \in [1, 2..., H-1], \tag{5}$$

$$\sum_{a\in\mathcal{A}} x_a \cdot y_a^{h+1} - \sum_{a\in\mathcal{A}} x_a \cdot y_a^h \leq r^U, \quad h \in [1, 2..., H-1]. \tag{6}$$

In this regard, we assume that each household participating in this energy consumption scheduling program is equipped with a smart meter, which is capable of detecting the electric power level of each appliance. The energy consumer announces to the MGCC his needs by selecting parameters α_a, β_a and T_a for each appliance $a \in \mathcal{A}$.

The above constraints (2) to (6) describe common characteristics of household appliances. However, there exist some appliances of which the operation cannot be interrupted. We call such kind of loads as uninterruptible loads. Discussions on how we may handle such loads are presented below.

Operation of Uninterruptible Loads: Some loads are interruptible, such as PHEV, which means that it is possible to charge the battery for some time, stopping charging for some time and then switching on the charging process again. Some other loads, however, are not interruptible, e.g., microwave oven. Appliances generating such loads, once started, has to be finished in one go. For each uninterruptible appliance $a \in \mathcal{A}'$, where $\mathcal{A}'$ represents the set of uninterruptible appliances, and each time slot h, let z_a^h denote an auxiliary binary variable such that $z_a^h \triangleq 1$ if appliance a starts operation at time slot h and $z_a^h \triangleq 0$ otherwise. We have

$$\sum_{h=\alpha_a}^{\beta_a - T_a + 1} z_a^h = 1 \tag{7}$$

and

$$z_a^h = 0, \quad \forall h \in \mathcal{H} \backslash [\alpha_a, \beta_a - T_a + 1]. \tag{8}$$

Then we relate start time vector $\mathbf{z_a^h}$ with decision variable vector $\mathbf{y_a^h}$ as follows:

$$y_a^h \geq z_a^h, y_a^{h+1} \geq z_a^h, ..., y_a^{h+T_a-1} \geq z_a^h. \tag{9}$$

From (9), if $z_a^h = 1$, then $y_a^h = y_a^{h+1} = ... = y_a^{h+T_a-1} = 1$.

3.2. Energy Supply Side

We now turn to the energy supply side to consider the load balance constraint in the microgrid. The microgrid may be considered as a graph consisting of three nodes as illustrated in Fig. 1. The first node represents the renewable energy generation sources such as wind turbines, solar panels and fuel cells. At time slot h, denote the total energy generated in this node as ξ^h, where ξ^h is a random variable of which the probability density function may not be known. Node 2 in Fig. 1 represents the load connected through the transmission line to node 1 and node 3. The load at time h, denoted as l^h, is dependent on the energy consumption from the user side which, from the above analysis, can be expressed as:

$$l^h = \sum_{a \in \mathcal{A}} x_a \cdot y_a^h \tag{10}$$

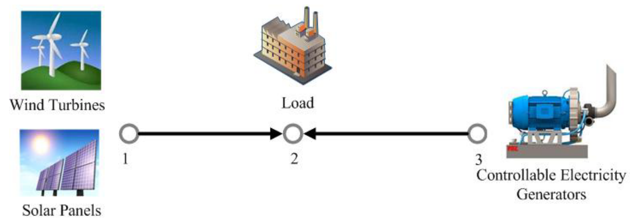

Figure 1: The microgrid's graph.

Finally the third node includes a group of controllable electricity generators, which has a total amount of generation P_{cg}^h as commanded by MGCC. A key requirement to the MGCC is to set the generation source power such that the supply could meet the demand. This statement can be mathematically described as

$$\xi^h + P_{cg}^h \geq l^h. \tag{11}$$

3.3. Problem Formulation

The objective function of MGCC can be defined in terms of minimizing the energy cost of the whole microgrid system. Hence we have the optimal energy consumption scheduling problem formulated as follows:

$$\begin{aligned} \min \quad & \sum_{h=1}^{H} \mathcal{C}_h(P_{cg}^h) \\ \text{s.t.} \quad & (2) \quad \text{to} \quad (11) \end{aligned} \tag{12}$$

where $\mathcal{C}_h(\cdot)$ is the cost function of electricity plant in the microgrid, which is assumed to be an increasing convex function. The convex property reflects the fact that each additional unit of power needed to serve the demands is provided at a higher cost. Example cases include the quadratic cost function [19] [20] and the piecewise linear cost function [21] [3], etc. Without loss of generality, we consider quadratic cost function $\mathcal{C}_h(P_{cg}^h) = a_h {P_{cg}^h}^2 + b_h P_{cg}^h + c_h$ throughout this paper, where $a_h > 0$, $b_h \geq 0$ and $c_h \geq 0$ are known parameters for each time slot h. In practice, the coefficient of the quadratic term is usually small. Therefore, the quadratic cost function can be reduced to a linear cost function. We also assume that the marginal cost of renewable energy generation is 0, leading to its omission in the objective function [22]. The main difficulty in solving problem (12) is the indeterminacy of renewable energy generation ξ^h that exists in constraint (11). Note that to optimize over the space defined by (11) amounts to solving an optimization problem with potentially large or even infinite number of constraints. Obviously, this realization of uncertainty is an intractable problem. Next, we develop a practical and flexible model to capture the uncertainty of ξ^h.

3.4. Probability Distribution Measure of Renewable Energies

It is generally difficult to characterize the renewable energy generation. In our optimization models, operations on the random variable ξ^h is cumbersome and computationally intractable. Moreover, in practice, we may not know the precise distribution of ξ^h. Solutions based on assumed distributions hence may not be justified. We usually measure the variability of a random variable using its variance or second moments which, however, may not provide sufficient details in describing the random variable. In this paper, we extract a reference distribution, rather than moment statistics, from historical data that will capture the distribution properties. Since renewable energy generation distribution is fluctuating over time and hard to be described in a closed-form expression, we may adopt empirical distribution as a useful reference and allow the actual distribution to fluctuate around it. For example, we may assume that the renewable energy generation distribution $f_0(\xi^h)$ is shifting around a known Gaussian distribution (or other distribution) $g_h(\xi^h)$, which can be obtained based on long-term field measurement.

The discrepancy between $f_0(\xi^h)$ and its reference $g_h(\xi^h)$ can be described by a probabilistic distance measure, for example the Kullback-Leibler (KL) divergence [23], which is a non-symmetric measure of the difference between two probability distributions. Name these two distributions as $f(\xi^h)$ and $g(\xi^h)$, respectively. Generally, one of the distributions, say, $f(\xi^h)$, represents the real distribution through precise modeling, while the reference $g(\xi^h)$ is a closed-form approximation based on the theoretic assumptions and simplifications. The definition of the KL divergence between two continuous distributions is given as follows:

$$D_{KL}(f(\xi^h), g(\xi^h)) = \int_{\xi^h \in S} [\ln f(\xi^h) - \ln g(\xi^h)] f(\xi^h) d\xi^h \tag{13}$$

where S is the integral domain. When distributions $f(\xi^h)$ and $g(\xi^h)$ are close to each other, the distance measure is close to zero. Adopting the KL divergence, we define the distribution uncertainty set as follows:

$$\boldsymbol{U}_r(g(\xi^h), D_0) = \{f(\xi^h) \mid \mathbb{E}_f[\ln f(\xi^h) - \ln g(\xi^h)] \leq D_0\} \tag{14}$$

where $D_0 > 0$ represents a distance limit and is obtained from empirical data or real-time measurement. It indicates energy generation's variation level. If the energy generation is very volatile, we have less confidence on the reference distribution and thus may set a larger distance limit.

Considering the renewable energy generation distribution $f_0(\xi^h)$ with reference distribution $g_h(\xi^h)$ and distance limit D_h, we have the following constraints for renewable energy generation distribution $f_0(\xi^h)$:

$$\mathbb{E}_{f_0}[\ln f_0(\xi^h) - \ln g_h(\xi^h)] \leq D_h \tag{15}$$

$$\mathbb{E}_{f_0}[1] = 1 \tag{16}$$

Given (15) and (16), we are now ready to transform the load balance constraint (11) to allow efficient solution of problem (12).

4. Optimization Algorithms

In this section, we present the optimization algorithms for solving the prime problem (12). We first present a robust approach for handling the load balance constraint, and then decompose the prime problem into a subproblem and a main problem to allow easier solution.

4.1. Robust Approach for the Load Balance Constraint

As shown in (11), the load balance constraint is $\xi^h + P_{cg}^h \geq l^h$. In practice, a decision criterion is to set ξ^h in such a way that we are confident that the load balance constraint is achieved. To achieve that, we may introduce a small value ϵ to control the degree of conservatism and change the above expression into a chance constraint:

$$\mathbf{P}(\xi^h \leq l^h - P_{cg}^h) \leq \epsilon \tag{17}$$

where ϵ is the fault tolerance limit of the power grid, representing the acceptable probability that the desirable power supply is not attained. Then we can have its robust expression:

$$\max_{f_0(\xi^h)\in \boldsymbol{U}_r(g_h, D_h)} \mathbf{P}(\xi^h \leq l^h - P_{cg}^h) \leq \epsilon \tag{18}$$

which is equivalent to:

$$\max_{f_0(\xi^h)\in \boldsymbol{U}_r(g_h, D_h)} \int_0^{l^h - P_{cg}^h} f_0(\xi^h) \mathrm{d}\xi^h \leq \epsilon. \tag{19}$$

Defining $\delta^h = l^h - P_{cg}^h$ as the robust renewable energy usage (REU) decision, which equals the amount of energy dispatched to renewable energy plants at time slot h, we can introduce an auxiliary function as follows:

$$h(\xi^h, \delta^h) = \begin{cases} 1, & \xi^h \le \delta^h; \\ 0, & \xi^h > \delta^h. \end{cases} \tag{20}$$

The left part of inequality (19) then can be formulated into an optimization problem:

$$\begin{aligned} \max_{f_0(\xi^h)} \quad & \int_0^{+\infty} h(\xi^h, \delta^h) \cdot f_0(\xi^h) \mathrm{d}\xi^h \\ \text{s.t.} \quad & \mathbb{E}_{f_0}[\ln f_0(\xi^h) - \ln g_h(\xi^h)] \le D_h \\ & \mathbb{E}_{f_0}[1] = 1 \end{aligned} \tag{21}$$

Define $Q_f^h(\delta^h) = \max_{f_0(\xi^h) \in \boldsymbol{U}_r(g_h, D_h)} \int_0^{+\infty} h(\xi^h, \delta^h) \cdot f_0(\xi^h) \mathrm{d}\xi^h$ as the worst-case fault probability. We can then get a worst-case mapping $\mathcal{M}_{ws}^h$ which maps robust REU decision δ^h to $Q_f^h(\delta^h)$:

$$\mathcal{M}_{wc}^h : \quad \delta^h \longrightarrow Q_f^h(\delta^h). \tag{22}$$

4.2. Sub-Problem: Determine the Robust REU Decision Threshold

Since there exists a random variable ξ^h in the constraints, we cannot solve energy generation and consumption scheduling problem (12) directly. As aforementioned, we decompose the problem into a subproblem and a main problem. The goal of our sub-problem is to determine the robust REU decision threshold δ^{h^*} so that the load balance constraint can be transformed into a solvable form.

Proposition 1: Problem (21) is a convex optimization problem.

Proof: Rewrite (21) as follows:

$$\max_{f_0(\xi^h)} \quad \int_0^{+\infty} h(\xi^h, \delta^h) \cdot f_0(\xi^h) \mathrm{d}\xi^h \tag{23}$$

$$\text{s.t.} \quad \int_0^{+\infty} [\ln f_0(\xi^h) - \ln g_h(\xi^h)] f_0(\xi^h) \mathrm{d}\xi^h \le D_h \tag{24}$$

$$\int_0^{+\infty} f_0(\xi^h) \mathrm{d}\xi^h = 1. \tag{25}$$

We can see that the objective function (23) and equality constraint function (25) are affine with respect to $f_0(\xi^h)$. Next we show that the inequality constraint function (24) is convex.

Lemma 1: If $f : \mathbf{R^n} \longrightarrow \mathbf{R}$ is convex, then the perspective of f, which is denoted as a function $g : \mathbf{R^{n+1}} \longrightarrow \mathbf{R}$ that

$$g(x,t) = tf(x/t), \tag{26}$$

with domain

$$\mathbf{dom}\ g = \{(x,t)|x/t \in \mathbf{dom}\ f,\ t > 0\} \tag{27}$$

preserves convexity.

That is to say, if f is a convex function, so is its perspective function g. Similarly, if f is concave, so is g. This can be proved in several ways, e.g., by direct verification of the defining inequality or using epigraphs and the perspective mapping on $\mathbf{R^{n+1}}$. Readers can refer to [24] for more detailed discussions.

We consider the convex function $f(x) = -\ln x$ on $\mathbf{R}_{++}$. Its perspective is

$$g(x,t) = -t\ \ln(x/t) = t\ \ln(t/x) = t(\ln t - \ln x) \tag{28}$$

and it is convex on $\mathbf{R}^{\mathbf{2}}_{++}$. The function g is called the relative entropy of t and x. Then we have that the KL divergence $\int_{x\in S}[\ln f(x) - \ln g(x)]f(x)\mathrm{d}x$ between distribution $f(x)$ and $g(x)$ is convex in $f(x)$ (and $g(x)$ as well). In this case, we claim that the inequality constraint (24) is convex in distribution $f_0(\xi^h)$.

Through Slater's condition, we can see that strong duality holds for problem (23)-(25). Using the Lagrangian method, we can obtain the worst-case fault probability $Q_f^h(\delta^h)$ as follows:

$$Q_f^h(\delta^h) = \min_{\tau,\eta} \max_{f_0(\xi^h)} \mathbb{E}_{f_0}\left[h(\xi^h,\delta^h) - \eta - \tau\ln\frac{f_0(\xi^h)}{g_h(\xi^h)}\right] + \tau D_h + \eta$$

where $\tau \geq 0$ and η are Lagrangian multipliers associated with constraints (24) and (25), respectively. Let $\mathcal{P}(\delta^h, f_0, \tau, \eta) = \mathbb{E}_{f_0}\left[h(\xi^h,\delta^h) - \eta - \tau\ln\frac{f_0(\xi^h)}{g_h(\xi^h)}\right]$, the derivative of $\mathcal{P}(\delta^h, f_0, \tau, \eta)$ with respect to f_0 can be derived as

$$\frac{\partial \mathcal{P}}{\partial f_0} = \int_0^{+\infty}\left(h(\xi^h,\delta^h) - \tau\ln\frac{f_0(\xi^h)}{g_h(\xi^h)} - \eta - \tau\right)\mathrm{d}\xi^h. \tag{29}$$

Using the Karush-Kuhn-Tucker (KKT) optimality conditions, we thus have

$$h(\xi^h, \delta^h) - \tau \ln \frac{f_0(\xi^h)}{g_h(\xi^h)} - \eta - \tau = 0 \tag{30}$$

$$\int_0^{+\infty} f_0(\xi^h) \mathrm{d}\xi^h = 1 \tag{31}$$

$$\mathbb{E}\left[\ln \frac{f_0(\xi^h)}{g_h(\xi^h)}\right] - D_h \leq 0 \tag{32}$$

$$\tau \cdot \left(D_h - \mathbb{E}\left[\ln \frac{f_0(\xi^h)}{g_h(\xi^h)}\right]\right) = 0 \tag{33}$$

$$\tau \geq 0 \tag{34}$$

From (30), the optimal distribution function can be expressed as follows:

$$f_0^*(\xi^h) = g_h(\xi^h) \exp\left(\frac{h(\xi^h, \delta^h) - \eta}{\tau} - 1\right). \tag{35}$$

The dual variables (τ, η) in (35) should be chosen properly such that conditions (31)-(34) are satisfied. Specifically, we have the following results:

Proposition 2: The choice of (τ, η) is a solution of the following nonlinear equations.

$$H_1(\tau, \eta) = R(\delta^h)e^{-\eta/\tau} + S(\delta^h)e^{(1-\eta)/\tau} - 1 = 0 \tag{36}$$

$$H_2(\tau, \eta) = S(\delta^h)e^{(1-\eta)/\tau} - \eta - \tau(1 + D_h) = 0, \tag{37}$$

where $S(\delta^h) = (1 - G_h(\delta^h)) \exp(-1)$, $R(\delta^h) = G_h(\delta^h) \exp(-1)$, and $G_h(\delta^h) = \int_{\xi^h \geq \delta^h} g_h(\xi^h) \mathrm{d}\xi^h$ denotes the complementary cumulative distribution function of reference distribution $g_h(\xi^h)$.

Proof: By substituting the optimal distribution $f_0^*(\xi^h)$ back into (31) and $f_0^*(\xi^h)$, (30) into (33), we have

$$\int_0^{+\infty} g_h(\xi^h) \exp\left(\frac{h(\xi^h, \delta^h) - \eta}{\tau} - 1\right) \mathrm{d}\xi^h = 1 \tag{38}$$

$$\int_0^{+\infty} \left(h(\xi^h, \delta^h) - \eta - \tau\right) g_h(\xi^h) \tag{39}$$

$$\cdot \exp\left(\frac{h(\xi^h, \delta^h) - \eta}{\tau} - 1\right) \mathrm{d}\xi^h - D_h \cdot \tau = 0,$$

which are equivalent to:

$$\exp\left(-1-\frac{\eta}{\tau}\right)\cdot\int_{\delta^h}^{+\infty} g(\xi^h)\mathrm{d}\xi^h \tag{40}$$
$$+\exp\left(-1+\frac{1-\eta}{\tau}\right)\cdot\int_0^{\delta^h} g(\xi^h)\mathrm{d}\xi^h - 1 = 0$$
$$(1-\eta-\tau)\exp\left(-1+\frac{1-\eta}{\tau}\right)\cdot\int_0^{\delta^h} g(\xi^h)\mathrm{d}\xi^h \tag{41}$$
$$+(-\eta-\tau)\exp\left(-1-\frac{\eta}{\tau}\right)\int_{\xi^h}^{\infty} g(\xi^h)\mathrm{d}\xi^h - \tau D_h = 0.$$

Equation (36) can be easily obtained from (40) by introducing $S(\delta^h)$ and $R(\delta^h)$. Through (40), (41) can be transformed into:

$$(1-\eta-\tau)\exp\left(-1+\frac{1-\eta}{\tau}\right)\cdot\int_0^{\delta^h} g(\xi^h)\mathrm{d}\xi^h +$$
$$(-\eta-\tau)\left[1-\exp\left(-1+\frac{1-\eta}{\tau}\right)\cdot\int_0^{\delta^h} g(\xi^h)\mathrm{d}\xi^h\right] - \tau D_h = 0.$$

Then we have

$$\exp\left(-1+\frac{1-\eta}{\tau}\right)\cdot\int_0^{\delta^h} g(\xi^h)\mathrm{d}\xi^h - \eta - \tau - \tau D_h = 0, \tag{42}$$

which is equivalent to (37). Hence, we prove **Proposition 2**.

It is, however, still rather difficult to obtain an explicit solution from (36) and (37). Hence we propose the Newton iterations as detailed in Algorithm 1.

Once we determine the solutions for (36) and (37) in Proposition 2, we can obtain the worst-case fault probability from (30) and (33) as follows:

$$Q_f^h(\delta^h) = \mathbb{E}_{f_0^*}[h(\xi^h,\delta^h)] = (1+D_h)\tau + \eta \tag{43}$$

Our next step is then to find the robust REU decision threshold δ^{h*} such that $Q_f^h(\delta^{h*}) = \epsilon$, which involves the calculation of inverse function of $Q_f^h(\delta^h)$ and it is not directly possible from (43). The following property of function $Q_f^h(\delta^h)$, however, may help us design such a search method.

Proposition 3: The worst-case fault probability $Q_f^h(\delta^h)$ is non-decreasing with respect to the REU decision δ^h.

The conclusion in Proposition 3 is straightforward since we have $\mathrm{d}Q_f^h(\delta^h)/\mathrm{d}\delta^h = \mathrm{d}\mathbb{E}_{f_0^*}[h(\xi^h, \delta^h)]/\mathrm{d}\delta^h = f_0^*(\delta^h) \geq 0$. Though direct solution is not available, the monotonicity of $Q_f^h(\delta)$ enlightens us a bisection method to search for the solution for $Q_f^h(\delta^h) = \epsilon$. The main idea is to perform the search within an interval of $[0, \rho]$, where ρ is an empirical constant such that $Q_f^h(\rho) > \epsilon$.

Details of the algorithm for searching the robust REU decision threshold are presented in Algorithm 1. Note that, from the 3rd to the 11th lines of the algorithm, we use Newton iteration to solve the equation in Proposition 2 and obtain the worst-case probability with fixed robust REU decision threshold. Then we compare the worst-case probability at ${\delta^h}_-$ and ${\delta^h}^-$ with the fault tolerant limit ϵ, respectively. The comparison results help shrink the search region as shown in lines 12-14.

4.3. Main-Problem: Determine the Optimal Energy Consumption and Generation Scheduling

Once the robust REU decision threshold ${\delta^h}^*$ for the robust load balance constraint (19) is obtained, we can reformulate the energy generation and consumption management problem. Specifically, we can tackle the following optimization problem rather than the original Eq. (12)

$$\begin{aligned} \min \quad & \sum_{h=1}^{H} \mathcal{C}_h(P_{cg}^h) \\ \text{s.t.} \quad & \sum_{a\in\mathcal{A}} x_a \cdot y_a^h - P_{cg}^h = {\delta^h}^*, \quad h \in \mathcal{H}, \quad a \in \mathcal{A} \\ & \text{and} \quad (2) \quad \text{to} \quad (10), \end{aligned} \tag{44}$$

where the optimization variables include the controllable energy generation variable P_{cg}^h for all time slots $h \in \mathcal{H}$, and the energy consumption scheduling vector $\mathbf{y_a}$ for all appliances $a \in \mathcal{A}$. The objective function aims at minimizing the overall energy cost in microgrid over the whole time horizon.

We can see that all the constraints of (44) are linear and the objective function is quadratic. This problem is a mixed integer quadratic programming problem. Algorithms that can be used to tackle this kind of problem include cutting plane method and the branch and bound method. The problem can also be effectively solved by some commercial optimization softwares including CPLEX, Mosek, FortMP and Gurobi, etc.

Algorithm 1 Search for robust REU decision threshold δ^{h^*}

Input: Reference distribution $g_h(\xi^h)$;
Distance limit D_h;
Search radius ρ;
Load balance fault tolerant limit ϵ;
Tolerance ε.

Output: Robust REU decision threshold such that $Q_f^h(\delta^{h^*}) = \epsilon$;

Begin
initialize $\delta^h{}_- = 0$, $\delta^{h^-} = \rho$, and set $\mathbf{H}(\tau, \eta) = [H_1(\tau, \eta), H_2(\tau, \eta)]^T$
while $|\delta^h{}_- - \delta^{h^-}| > \varepsilon$
  set $\bar{\delta^h} = \frac{\delta^h{}_- + \delta^{h^-}}{2}$ and initiate the time iteration $k = 1$
  while $\mathbf{H}(\tau, \eta) > \varepsilon$
    evaluate $\mathbf{H}(\tau, \eta)$ and Jacobian matrix $\mathbf{J}(\tau, \eta)$
    solve $\mathbf{J}(\tau, \eta)\Delta\mathbf{x_k} = -\mathbf{H}(\tau, \eta)$
    update $\tau_{k+1} = [\tau_k + \Delta\tau_k]^+$, $\eta_{k+1} = \eta_k + \Delta\eta_k$
    update $Q_f^h(\bar{\delta^h}) = (1 + D_h)\tau_{k+1} + \eta_{k+1}$
    set $k = k + 1$
  end while
  if $\left(Q_f^h(\bar{\delta^h}) - \epsilon\right)\left(Q_f^h(\delta^{h^-}) - \epsilon\right) < 0$
    then set $\delta^h{}_- = \bar{\delta^h}$ **else** set $\delta^{h^-} = \bar{\delta^h}$ **end if**
  if $|Q_f^h(\bar{\delta^h}) - \epsilon| < \varepsilon$ break **end if**
end while
set $\delta^{h^*} = \bar{\delta^h}$
End

5. Simulation results and Discussions

In this section, we present simulation results for assessing the performance of the proposed power demand and supply management scheme and evaluating the effects of different system parameters. Here, we make an assumption on top of paper [25] [26], where Gaussian random process has been utilized to describe the renewable energy generation. Specifically, we assume that the reference distribution is a Gaussian distribution $g_h(\xi^h)$ with mean $\overline{m}_h$ and standard deviation σ_h. In addition, we set the parameters of the cost function in (44) for each time slot as $a_h > 0$, $b_h = 0$, and $c_h = 0$.

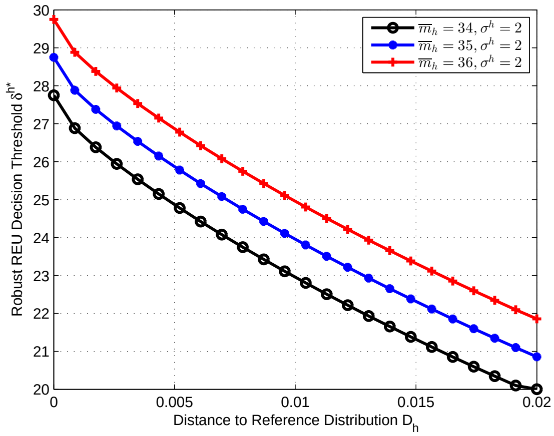

Figure 2: Robust REU Decision Threshold δ^{h^*} with Distance Limit D_h for different $\overline{m}_h$

5.1. The Impacts of Distribution Uncertainty Set

We first set the fault tolerant limit $\epsilon = 10^{-3}$ and try to investigate the relations between robust REU decision threshold δ^{h^*} and distance limit D_h for different values of $\overline{m}_h$ and σ_h. The results are plotted in Fig. 2 and Fig. 3. It is shown that the robust REU decision threshold decreases with the increase of the distance limit. This observation is intuitive since a larger distance limit defines a larger distribution set which allows the renewable energy output to fluctuate more intensively. Given the required fault tolerant limit, REU decision threshold has to be set at a lower value so as to rely less on the more uncertain renewable energy and keep the system reliable. Note that when $D_h = 0$, the renewable energy follows exactly the reference distribution $g_h(\xi^h)$. In this special case, renewable energy generation is a random variable with determinate distribution $g_h(\xi^h)$. While $D_h > 0$, our reference model considers a more general case which allows discrepancy between actual distribution and its reference. The discrepancy however is limited and confined by a probabilistic distance measure. Simply put, the reference model allows the actual renewable energy generation to follow a different distribution function from the reference distribution, but not be too disparate based on historical data or empirical knowledge.

From Fig. 2 and Fig. 3, we also can achieve the following statement: when the reference distribution is Gaussian, the robust REU decision threshold δ^{h^*} linearly increases with the mean of reference distribution $\overline{m}_h$ and linearly decreases

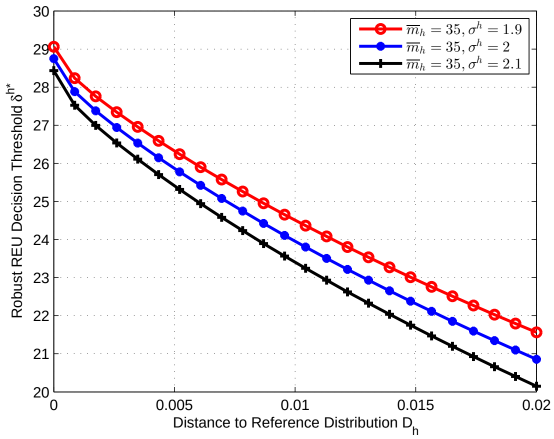

Figure 3: Robust REU Decision Threshold δ^{h^*} with Distance Limit D_h for different σ^h

with the standard deviation of reference distribution σ_h. This statement can be explained analytically as follows. We first transform $G_h(\delta^h)$ in (36) and (37) into

$$G_h(\delta^h) = \int_{\xi^h \geq \delta^h} g_h(\xi^h) \mathrm{d}\xi^h = \int_{\frac{\delta^h - \overline{m}_h}{\sigma_h}}^{+\infty} n(x) \mathrm{d}x, \tag{45}$$

where $n(x)$ is the probability density function of the standard Gaussian distribution. Since fault tolerant limit ϵ is of a relatively small value, we have that δ^{h^*} is less than $\overline{m}_h$. As $\overline{m}_h$ and σ_h vary, in order to preserve the same worst-case probability $Q_f^h(\delta^{h^*})$, the solutions (η, τ) of the equations (36) and (37) need to remain unchanged, indicating that $S(\delta^{h^*})$, $R(\delta^{h^*})$ and $G(\delta^{h^*})$ also need to be constants. In this case, we have:

$$\frac{\delta^{h^*} - \overline{m}_h}{\sigma_h} = C \quad \Longrightarrow \quad \delta^{h^*} = C\sigma_h + \overline{m}_h,$$

where C is a negative constant. Thus, δ^{h^*} linearly increases with $\overline{m}_h$ and linearly decreases with σ_h.

5.2. Effects of Fault Tolerant Limit ϵ

We set $\overline{m}_h = 36$ and $\sigma_h = 2$ and investigate how the robust REU decision threshold varies when fault tolerance limit increases. Figure 4 plots the mapping

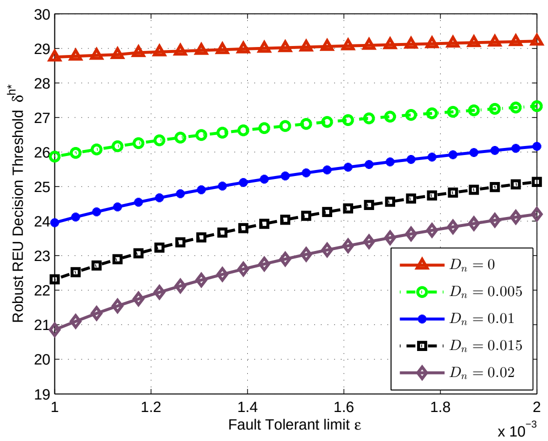

Figure 4: Robust REU Decision Threshold δ^{h^*} with Fault Tolerant Limit ϵ for different D_h

from fault tolerant limit ϵ to robust decision threshold δ^{h^*} under different values of the distant limit D_h. The figure indicates that a larger fault tolerant limit permits a higher reliance on renewable energy (a larger robust REU decision threshold), which is straightforward to understand. We also note that the worst-case fault probability is an increasing function of the REU decision threshold. Thus we are justified to use the bisection method as presented in **Algorithm 1** to search for the REU decision threshold which satisfies the fault tolerant limit requirement. Note that in this figure, the red triangle line is the special case where renewable energy follows reference distribution exactly. We also observe that robust REU threshold δ^{h^*} is more sensitive to ϵ when D_h increases.

5.3. The Impacts of Uninterruptible Loads

For the experiment studied in this part, we set the power consumption scheduling horizon $|\mathcal{H}| = 12$ h. That is, the MGCC solves optimization problem (44) to decide on the operations of each appliance for the next 12 hours. In this paper, 30 household appliances including electric cookers (EC), air conditioners (AC), electric water heaters (EWH), cloth dryers (CD), dish washers (DW) and plug-in hybrid electric vehicles (PHEVs) are considered to study the optimal power consumption scheduling with a mixed integer quadratic programming approach. The detailed operation data we use is modified based on the information from [27] [28] [29] [30] [31], and is shown in Table 1. Note that we also introduce a user elasticity index

$$\gamma_a = \frac{T_a}{\beta_a - \alpha_a + 1}$$

to describe the scheduling flexibility of appliance a. Obviously, $\gamma_a \in (0, 1]$, and a larger γ_a implies a more inflexible arrangement property. The operation window $[\alpha_a, \beta_a]$ of each appliance is chosen according to the preferences of different users. In this paper, we do not enumerate the values of α_a and β_a one by one due to limited space, instead, we list the range of γ_a for each kind of appliance, which is presented in the last column of Table 1.

Table 1: Operation Data for Appliances in the microgrid

Type of Appliance	Power Level (KW)	T_a	γ_a
EC	2	1	0.3-0.4
AC	3.5	10	0.9-1.0
EWH	4.5	3	0.5-0.7
CD	5	3	0.3-0.4
DW	0.85	2	0.3-0.4
PHEV	7.3	7	0.6-0.7

The mean $\overline{m}_h$ and standard deviation σ_h of the reference distribution, together with the distance limits for the next 12 time slots are given in Table 2. Based on these data and adopting Algorithm 1, we obtain the robust REU threshold δ^{h^*} for each time slot, representing the amount of energy dispatched to renewable energy plants. The results are demonstrated in the last column of Table 2 and are used to solve the main problem (44). Our experiments utilize MOSEK optimization toolbox 6.0 on an Intel-P4 2.4-GHz personal computer. To investigate the impacts of uninterruptible loads, we study the following cases:

- Case 1: Only electric cookers are classified into the uninterruptible appliance set $\mathcal{A}'$, i.e., $\mathcal{A}' = \{\text{EC}\}$.
- Case 2: On top of Case 1, air conditioners are added to the uninterruptible appliance set $\mathcal{A}'$, i.e., $\mathcal{A}' = \{\text{EC}, \text{AC}\}$.
- Case 3: On top of Case 2, electric water heaters are added to the uninterruptible appliance set $\mathcal{A}'$, i.e., $\mathcal{A}' = \{\text{EC}, \text{AC}, \text{EWH}\}$.

Table 2: Parameters of Distribution Uncertainty Set and Corresponding Robust REU Threshold

Time Slot	$\overline{m}_h$	σ_h	D_h	δ^{h*}
1	14.678	0.9571	0.0162	8.419
2	14.757	0.4853	0.0181	11.453
3	14.743	0.8002	0.0025	11.531
4	14.392	0.1418	0.0182	13.423
5	14.655	0.4217	0.0126	12.137
6	14.171	0.9157	0.0019	10.630
7	14.706	0.7922	0.0055	10.995
8	14.031	0.9594	0.0109	8.577
9	14.276	0.6557	0.0191	9.716
10	14.046	0.0357	0.0192	13.797
11	14.097	0.8491	0.0031	10.565
12	14.823	0.9339	0.0194	8.294

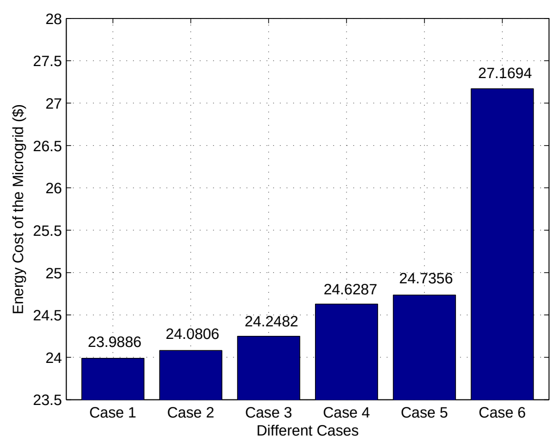

Figure 5: The impacts of uninterruptible loads on the energy cost of the microgrid

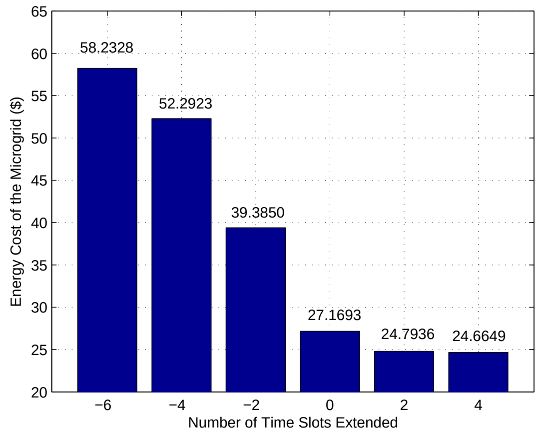

Figure 6: The Impact of User Elasticity on the Energy Cost of the Microgrid

- Case 4: On top of Case 3, cloth dryers are added to the uninterruptible appliance set $\mathcal{A}'$, i.e., $\mathcal{A}' = \{\text{EC}, \text{AC}, \text{EWH}, \text{CD}\}$.
- Case 5: On top of Case 4, dish washers are added to the uninterruptible appliance set $\mathcal{A}'$, i.e., $\mathcal{A}' = \{\text{EC}, \text{AC}, \text{EWH}, \text{CD}, \text{DW}\}$.
- Case 6: On top of Case 5, PHEVs are added to the uninterruptible appliance set $\mathcal{A}'$, i.e., $\mathcal{A}' = \{\text{EC}, \text{AC}, \text{EWH}, \text{CD}, \text{DW}, \text{PHEV}\}$.

Figure 5 demonstrates the energy cost for each case. Obviously, the energy cost goes up when we scale up the uninterruptible appliance set $\mathcal{A}'$. We compare the costs of adjacent cases, and the difference between these costs is called cost gap. The largest cost gap is shown between Case 5 and Case 6 due to PHEVs' high electric power consumption ($P = 7.3$ KW) and relatively considerable scheduling elasticity ($\gamma_a = 0.6 - 0.7$).

5.4. The Price of User Elasticity

In this section, we explore the effects of user elasticity on the energy cost of the microgrid system. First we assume that all the appliances are uninterruptible. Since the minimum running time of each appliance T_a is fixed, we extend or shrink the operation window $[\alpha_a, \beta_a]$ to relax or tighten the user elasticity. Note that, at one time, the operation window $[\alpha_a, \beta_a]$ of each appliance a will expand or shrink one unit of time slot from both sides, i.e., the operation window will scale up or down two time slots. If one side of the operation window cannot be extended due to the finite length of time horizon, the operation window will only scale up on one side until it covers the whole time horizon. We keep on extending or shrinking the operation window of each appliance until all the operation windows cannot be changed. Then, we demonstrate how energy cost changes when operation windows vary. We select the case when operation windows are shrunk 6, 4, 2 time slots and extended 0, 2, 4 time slots respectively. The results are presented in Fig. 6. In this figure, we find that when user elasticities are tightened, energy cost increases rapidly. We can interpret that the user elasticity can make a significant impact on the energy cost of the microgrid system. Compared with the the effects of interruptibility property, user elasticity has stronger influences on the expenditure of the whole system. This result may give MGCC an inspiration that it is worthy to provide more rewards to customers who agree to have more time flexibility than those who allow interruptions to some appliances. Moreover, we find that when the operation windows are shrunk by 6 time slots, nearly all the appliances' elasticity reaches 1. This approximates the case when all the appliances operate at their desired time with no flexibility. Compared with this benchmark case, we observe that the proposed power consumption management scheme can reduce the energy cost significantly.

6. Conclusion

In this paper, we studied a fundamental problem of using a microgrid system central controller to optimally schedule the demand and supply profiles so as to minimize the fuel consumption costs during the whole time horizon. We focused on a scenario where the control of customer appliances is delegated to the grid operator. To tackle the randomness of renewable energy, we introduced a reference distribution and then defined a distribution uncertainty set to confine the uncertainty. Such a new model allows convenient handling of fluctuating renewable generation as long as the renewable energy generation profile is not too drastically different from the past observation or empirical knowledge. An optimization formulation of the problem was proposed and a two-stage algorithm approach was developed, to first transform and then solve the problem. Numerical results indicated that the proposed energy consumption management scheme can significantly cut down energy expenses. Effects of a few factors, including the reference distribution, the fault tolerant limit, the types and amount of uninterruptible loads, and the user elasticity etc. were carefully evaluated. Such evaluations, as we believe, help provide some useful insights for MGCC to develop more effective payback policies for their customers.

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