CONSTRAINED PREDICTIVE CONTROL OF MULTI-ZONE AIR-CONDITIONING SYSTEMS

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INTRODUCTION

Variable-air-volume (VAV) air-conditioning systems control the air temperatures inside a multi-zone building by varying the amount of air flowing into the zones. Fig. 1 shows a typical VAV system used in commercial building which requires only cooling to meet the thermal loads resulting from occupancy, lighting and equipment. As the zone temperatures rise, the dampers in the terminal boxes open to allow more cold air into the zones, so as to maintain the zone temperatures at their set-points. When the dampers in the terminal boxes open, the static pressure in the supply duct drops, and the control system increases the speed of the supply fan in order to maintain the pressure. The opposite occurs when terminal boxes are closed.

May (3) describes a number of control algorithms which have been proposed for different types of VAV systems. In conventional VAV systems, both the chilled water supplied to the cooling-coil and the temperature of the cold air delivered by the central air-handling plant are maintained at constant temperatures. In more sophisticated systems, a load analyser is employed to monitor the load variations in the zones, and vary the set-point of the supply air temperature to satisfy the zone with the greatest cooling demand. The design of the load analyser is often based on heuristics and is usually in the form of min-max selector logic. Norford et al (5) examine the effects of varying the supply-air temperature on the energy used by a VAV system.

Mutammara et al (4) compare the energy consumption when using different control strategies. The cost of operating the VAV system is the sum of the cost of providing sufficient air flow and the cost of lowering the supply-air temperature. Normally, the cooling-coil's valve is used to adjust the flow rate of chilled-water through the coil to maintain the desired supply-air temperature (pay-cooling). Alternatively, when the outdoor temperature is cooler than the temperature of the air returned from the building, the mixing-box is used to adjust the proportions of the outdoor and return air streams to achieve the required temperature (free-cooling). The cost of using mixed-air is negligible compared to the cost of operating the fan and the chiller which generates the cold water for the cooling-coil. The total cost of operating the VAV system must be considered when some pay-cooling is needed in addition to the available free-cooling. There is an optimal combination of supply-air temperature and supply-air flow rate that will minimise the cost of operating the VAV plant for a given cooling demand in the building.

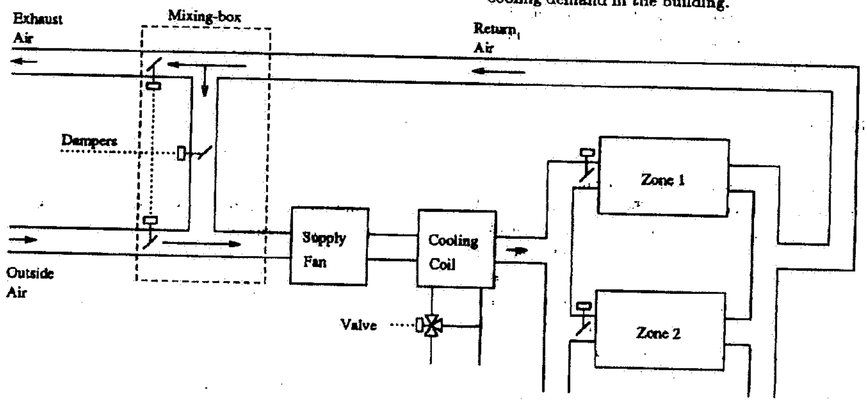

Figure 1: A multi-zone variable-air volume system.

CONTROL '94, 21–24 March 1994. Conference Publication No. 389, © IEE 1994.

This paper explains how a constrained predictive control algorithm, that was developed by Ling (1) to control multi-input plant, can be used to minimise the cost of operating a multi-zone VAV system.

CONSTRAINED PREDICTIVE CONTROL

Plant modelling

Although the behaviour of the plant is nonlinear, previous work by Ling and Dexter (2) has shown that (i) a linearised process model can be used in the design of the controller if safety factors are introduced to account for the nonlinearities (ii) the short term dynamic response of the zone temperatures to changes in the control variables can be approximated by that of a first order plus time-delay transfer function. For the two-zone system considered in this paper, the following locally linearised transfer function model of the plant is used:

$$\begin{bmatrix} T_{z1} \\ T_{z2} \end{bmatrix} = \frac{e^{-sL}}{1+Ts} \begin{bmatrix} G_c & G_d & G_{z1} & 0 \\ G_c & G_d & 0 & G_{z2} \end{bmatrix} \begin{bmatrix} u_c \\ u_d \\ \dot{m}_1 \\ \dot{m}_2 \end{bmatrix}$$

where G_{z1} and G_{z2} are the incremental gains of the zone terminal boxes and G_c and G_d are the incremental gains of the cooling-coil and mixing-box in the central air-handling plant respectively.

The variables T_z, w, $\dot{m}$, u_c and u_d represent the zone air temperature, the set-point, the air flow rate into the zones and the control signal to the cooling-coil and mixing-box respectively. The air flow rates into the individual zones can be used as control inputs because most VAV systems have pressure independent terminal boxes which are equipped with local flow controllers.

Controller design

The cost function of the controller includes terms which relate to the power consumption of the different components of the air-conditioning plant and to the cost of unsatisfactory regulation of the zone temperatures. The cost of operating the chiller is a monotonically increasing function of the water flow rate through the coil and therefore of the control signal to the valve's actuator. The cost of operating the fan is similarly a monotonically increasing function of the total air flow rate through the fan. The cost of operating the mixing-box is negligible. For the two zone system, the following cost function is used by the predictive controller:

$$\begin{aligned} J \;=\; & \sum_{j=1}^{N_2} (w_1(t+j) - T_{z1}(t+j))^2 \\ & + \sum_{j=1}^{N_2} (w_2(t+j) - T_{z2}(t+j))^2 \\ & + \lambda_1 \sum_{j=1}^{N_u} (u_c(t+j-1))^2 \\ & + \lambda_2 \sum_{j=1}^{N_u} (\dot{m}_1(t+j-1) + \dot{m}_2(t+j-1))^2 \end{aligned}$$

The weights λ_1 and λ_2 can be chosen to reflect the different costs of operating the chiller and the fan. The magnitudes of the weights must be small if significant offset errors are to be avoided and their ratio should depend on the operating conditions. For example, if the outdoor temperature is lower than the indoor temperature, a small value of $\frac{\lambda_2}{\lambda_1}$ could be selected so that the controller will minimise the cost of operating the chiller by maximising the use of free cooling. Conversely, if the outdoor temperature is higher than the indoor temperature, a large value of $\frac{\lambda_2}{\lambda_1}$ could be used to minimise the cost of operating the fan.

Operating constraints

There are several constraints that have to be met while operating the central air-handling unit and the terminal boxes. A minimum air flow is required in each zone to ensure adequate air distribution and ventilation, and the sum of the air flow in each zone must not exceed the capacity of the supply fan. The motor driven actuators attached to the valves and dampers also have amplitude and rate limits. The operating constraints considered in this example are

- Minimum and design air flow in each zone:

$$0.145 \leq \dot{m}_1 \leq 0.289 \quad \text{kg/s}$$

$$0.145 \leq \dot{m}_2 \leq 0.289 \quad \text{kg/s}$$

- Supply-fan capacity limit:

$$\dot{m}_1 + \dot{m}_2 \leq 0.462 \quad \text{kg/s}$$

- Amplitude and rate limits on cooling-coil and mixing-box control signals:

$$0.0 \leq u_c \leq 1.0$$

$$0.1 \leq u_d \leq 1.0$$

$$-h/65 \leq \Delta u_c \leq h/65$$

$$-h/100 \leq \Delta u_d \leq h/100$$

where the controller sampling interval h is chosen to be 30 seconds.

The predictive controller uses Fletcher's active set algorithm to determine optimal values for the control signals. The control horizon N_u and the prediction horizon N_2 are set at the default values of 1 and 10 respectively to provide robust and stable control (2).

The algorithm is applied to a detailed simulation of a two-zone system to evaluate the validity of the approach. A simulation program developed by Park et al (6) is used to simulate the behaviour of the two-zone system. Ideal control of fan static pressure is assumed and the two zones are subjected to different thermal loads based on weather data collected on a summer day in the United Kingdom. The temperature set-points for both of the zones are 22°C.

Effect of control weightings on operating cost

The correct ratio $\frac{\lambda_2}{\lambda_1}$ can only be calculated if the true cost of operating the fan and the chiller are known. However, the operating costs are a complex function of the plant design and will vary with the operating conditions. Table 1 shows the effect of using different values of $\frac{\lambda_2}{\lambda_1}$ on the cost of operating the plant when the outdoor temperature is held constant at 18°C. The results show that the total energy consumed by the plant is almost constant for both very small and very large values of $\frac{\lambda_2}{\lambda_1}$, and increases as the value of $\frac{\lambda_2}{\lambda_1}$ increases.

$\frac{\lambda_2}{\lambda_1}$	Normalised Fan Energy	Normalised Chiller Energy	Normalised Total Energy
0.01	1.00	1.00	1.00
0.1	0.96	1.06	1.02
0.5	0.75	1.40	1.11
1.0	0.70	1.49	1.13
5.0	0.67	1.54	1.14
10.0	0.67	1.54	1.14

Table 1: Effect of different weightings on operating cost when the zone temperature is greater than the outside temperature.

Operation of the plant at low outdoor temperatures

Figure 2 shows the behaviour of the plant on a day when the outdoor temperature is always at least 1°C lower than the indoor temperature. The value of $\frac{\lambda_2}{\lambda_1} = 0.1$ is chosen so that the controller will maximise the total air flow rate into the zones while maintaining the zone temperatures close to their set-points. The results show that the mixing-box is always providing 100% outdoor air and the fan is operating at its maximum capacity for most of the day. The controller successfully adjusts both the supply-air temperature and air flow rates into the zones to meet the cooling demands without violating the operating constraints.

Minimising operating cost according to plant operating conditions

Figure 3 illustrates the behaviour of the plant on a day when the outdoor temperature rises above the indoor temperature for the latter part of the day. The controller monitors the indoor and outdoor temperatures and automatically switches the mixing-box to minimum outside air when the outdoor temperature exceeds the indoor temperature. At the same time, the controller changes the value of $\frac{\lambda_2}{\lambda_1}$ from 0.1 to 10 so as to minimise the total air flow rate supplied by the fan. After the switch has occurred, the controller reduces the air flow into the zone which has the lowest cooling demand to its lower limit, decreases the supply-air temperature, and maximises the air flow rate into the zone which has the greatest cooling demand.

$\frac{\lambda_2}{\lambda_1}$	Normalised Fan Energy	Normalised Chiller Energy	Normalised Total Energy
Switched	1.00	1.00	1.00
Constant	1.47	1.07	1.10

Table 2: Effect of minimising air flow rate when outdoor temperature is higher than indoor temperature.

Table 2 compares the energy consumed using this strategy with that when the value of $\frac{\lambda_2}{\lambda_1} = 0.1$ is used throughout the course of the day. Again, the values presented in the table have been normalised for ease of comparison. In this example, savings of about 10% have been made by varying the value of $\frac{\lambda_2}{\lambda_1}$.

CONCLUSIONS

A constrained predictive control algorithm has been successfully applied to a simulated, two-zone, VAV air-conditioning system. The design of the controller requires minimal modelling effort and is based on available prior knowledge. The controller varies both the supply-air temperature and the air flow rates into the zones to maintain the temperatures of the zones at their set-points. Depending on the operating conditions, the weights in the cost function are adjusted automatically so as to reduce the total cost of operating the plant. Ad hoc supervisory control logic is avoided and constraints arising when operating the plant are included in a systematic manner.

A distributed implementation of the control scheme is now being developed that could be used on air-conditioning systems in buildings with a large number of zones. The use of a rule-based supervisor (2), which monitors the performance of the control system and adjusts the zone temperature set-points to further reduce the operating costs, is also under investigation.

Acknowledgements

The authors would like to thank Dr P. Haves, Department of Civil Engineering, Loughborough University of Technology for his help and advice on simulating the air-conditioning system. K.V. Ling acknowledges the financial support of Trend Control Systems Ltd., the Building Research Establishment, BICC plc. and the Committee of Vice-Chancellors and Principals (Overseas Research Students Awards Scheme).

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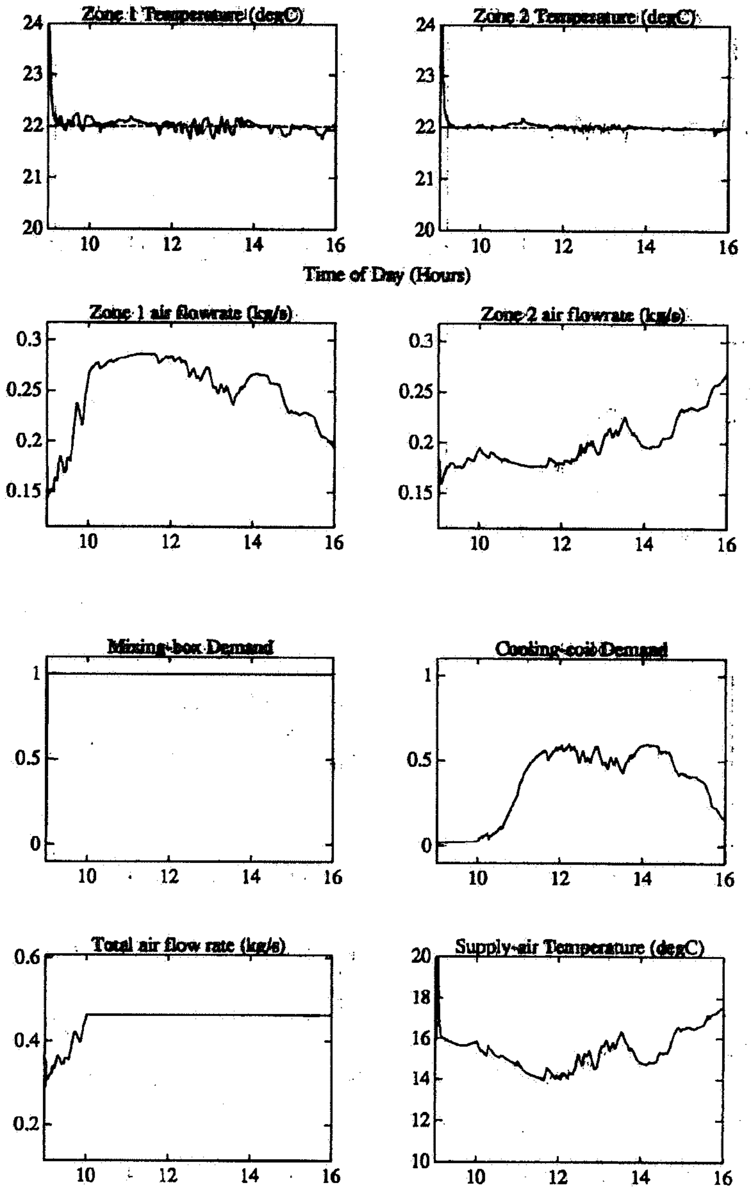

Figure 2: Constrained predictive control of a two zone building when the outdoor temperature remains below the zone temperature.

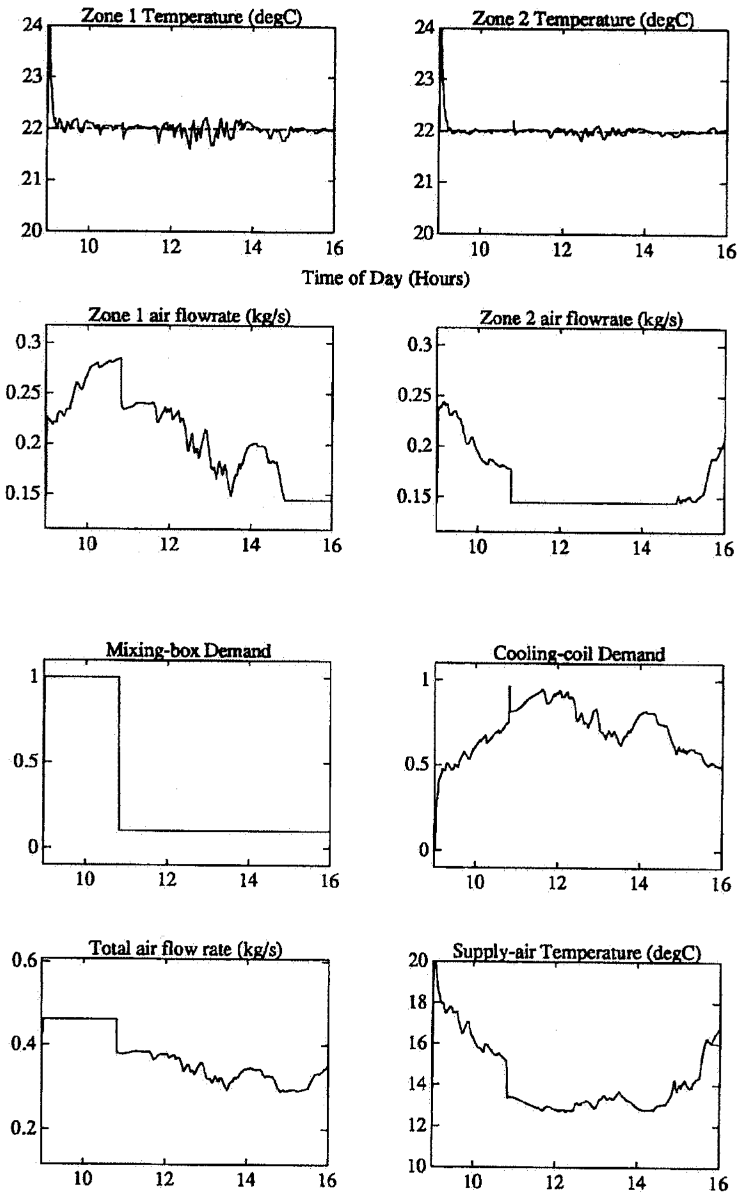

Figure 3: Control performance when the outdoor temperature rises above the indoor temperature during the latter part of the day.