# Solving Channel Assignment Problems for Cellular Radio Networks Using Transiently Chaotic Neural Networks

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## ABSTRACT

The channel-assignment problem (CAP) in cellular radio networks is known to belong to the class of NP-complete optimization problems. Many heuristic techniques including Hopfield neural networks (HNN) have been devised for solving the CAP. However, HNNs often suffer from local minima. On the other hand, a recently proposed transiently chaotic neural network (TCNN) has been successfully used in solving travelling the salesman problems (TSP). The transient chaotic neurodynamics can prevent the network from being trapped in local minima and can search for the globally optima or near-optima efficiently. In this paper we show that the TCNN can also be a useful tool for tackling the CAP.

## 1. INTRODUCTION

With the growth in demand for cellular mobile services and the limited allocation of spectrum for this purpose, the problem of optimal assignment of frequency channels is becoming increasingly important. The channel assignment problem (CAP) is to assign the required number of channels to each region in such a way that interference is precluded and the frequency spectrum is used efficiently. Efficiently finding a valid channel assignment is known to be very difficult: the CAP belongs to the class of NP-complete problems [1]. On the other hand, many heuristics techniques including Hopfield neural networks (HNN) have been devised for solving the CAP [2-7]. However, the HNNs are often trapped in local minima [3,8].

Recently Chen and Aihara proposed a transiently chaotic neural networks (TCNN) by modifying a chaotic neural network which Aihara et al proposed earlier. Their experiments on the TSP showed that the TCNN can obtain quite good solutions much more easily [9].

## 2. CHANNEL ASSIGNMENT PROBLEM

Suppose there are $N$ cells (or regions) and the number of channels available is given by . The channel requirements for cell $i$ are given by $D_i$. The constraints specify the minimum distance in the frequency domain by which two channels must be separated in order to guarantee an acceptably low signal/interference ratio each region. These minimum distances are stored in a $N \times N$ symmetric *compatability matrix* $C$.

The channel assignment problem is then as follows:

minimize severity of interferences
subject to demand constraints

given $N$, $M$, $C$, $D$.

Assume $x_{j,k}$ to be the neuron output which represents cell $j$ is assigned to channel $k$, as shown in Figure 1. Suppose that $x_{j,k} = x_{i,l} = 1$, $(i, j = 1,..., N;$ $k, l = 1,..., M)$ means that calls in cells $j$ and $i$ have been assigned channels $k$ and $l$, respectively. One way to measure the degree of interference caused by such assignments is by an element in a cost tensor $P_{j,i,m+1}$, $(m = \|k - l\|$ is the distance in the channel

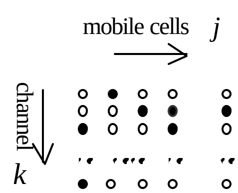



**Figure 1. Matrix of neurons to solve the CAP**

domain between channels $k$ and $l$) [7]. The interference cost should be at its maximum when $k = l$ and decreases until the two channels are far enough that no interference exists.

The cost tensor $P$ can be generated recursively as follows:

$$P_{j,i,m+1} = \max(0, P_{j,i,m} - 1) \;, \forall m = 1,...,M-1 \quad (1)$$

$$P_{j,i,1} = C_{ji} \;, \forall j, i \neq j \quad (2)$$

$$P_{j,j,1} = 0 \;, \forall j \quad (3)$$

where $c_{ij}$ is element of $N \times N$ symmetric *compatability matrix* $C$ .

Then the CAP can be formulated to minimize the following cost:

minimize

$$f(x) = \sum_{j=1}^{N} \sum_{k=1}^{M} x_{j,k} \sum_{i=1}^{N} \sum_{l=1}^{M} P_{j,i,(|k-l|+1)} x_{i,l} \tag{4}$$

subject to

$$\sum_{k=1}^{M} x_{j,k} = D_j, \ \forall j = 1,\ldots,N \tag{5}$$

$$x_{j,k} \in \{0,1\}, \quad \forall j = 1,\ldots,N$$
$$\forall k = 1,\ldots, M \tag{6}$$

where $D_j$ is the channel requirements for cell $j$ .

## 3. TRANSIENTLY CHAOTIC NEURAL NETWORKS

Since the optimization process of the TCNN is deterministically chaotic rather than stochastic, the TCNN is also called as chaotic simulated annealing (CSA), in contrast to the conventional stochastic simulated annealing (SSA). The CSA uses slow damping of negative self-feedback to produce successive bifurcations so that the neurodynamics eventually converges from strange attractors to a stable equilibrium point.

The TCNN is defined below:

$$x_{jk}(t) = \frac{1}{1+e^{-y_{jk}(t)/e}} \tag{7}$$

$$y_{jk}(t+1) = k y_{jk}(t) + a\left(\sum_{i=1, i\neq j}^{N} \sum_{l=1, l\neq k}^{M} w_{jkil} + I_{jk}\right) - z_{jk}(t)(x_{jk}(t) - I_0) \tag{8}$$

$$z_{jk}(t+1) = (1-b) z_{jk}(t) \qquad (i = 1,\ldots, n) \tag{9}$$

where

$x_{jk}$ = output of neuron $j,k$ ;

$y_{jk}$ = input for neuron $j,k$ ;

$w_{jkil} = w_{iljk}$ ; $w_{jkjk} = 0$; $\sum_{i=1,i\neq j}^{N} \sum_{l=1,l\neq k}^{M} w_{jkil} x_{jk} + I_{jk} = -\partial E / \partial x_{jk}$ connection weight from neuron $j,k$ to neuron $i,l$ ;

$I_{jk}$ = input bias of neuron $j,k$ ;

$k$ = damping factor of nerve membrane ( $0 \le k \le 1$);

$a$ = positive scaling parameter for inputs;

$z_{jk}(t)$ = self-feedback connection weight or refractory strength $(z(t) \ge 0)$ ;

$b$ = damping factor of the time dependent ($0 \le b \le 1$);

$I_0$ = positive parameter;

$e$ = steepness parameter of the output function ($e$ >0);

$E$ = energy function.

Considering (4) and (5), we can define a computational energy as a sum of the total interferences and constraints:

$$E = \frac{W_1}{2} \sum_{j=1}^{N} \left(\sum_{k=1}^{M} x_{jk} - D_j\right)^2 + \frac{W_2}{2} \sum_{j=1}^{N} \sum_{k=1}^{M} x_{jk} \sum_{i=1}^{N} \sum_{l=1}^{M} P_{j,i,(|k-l|+1)} x_{il} \tag{10}$$

$W_1$ and $W_2$ are the coupling parameters corresponding to the constraints and severity of interferences, respectively, where $D_j$ is the channel requirements for cell $j$ . $P$ is the cost tensor.

Connection weight $W_{jkil}$ can be obtained similarly as in the HNN, the network dynamics of the TCNN for the CAP is as follows:

$$y_{jk}(t+1) = k y_{jk}(t) - z_{jk}(t)(x_{jk}(t) - I_0)$$
$$+ a\left\{ -W_1 \sum_{q\neq k}^{M} x_{jq} + W_1 D_j - W_2 \sum_{p=1, p\neq j}^{N} \sum_{q=1, q\neq k}^{M} P_{j,p,|k-q|+1} x_{pq} \right\} \tag{11}$$

## 4. SIMULATION RESULTS

In this section, we use the data set EX1 and EX2 to test the performance of TCNN on solving the CAP. These data sets have been used by many researchers previously [7].

EX1:

$$N = 4, M = 11, D^T = (1,1,1,3)$$

and the compatability matrix is given by:

$$C = \begin{pmatrix} 5 & 4 & 0 & 0 \\ 4 & 5 & 0 & 1 \\ 0 & 0 & 5 & 2 \\ 0 & 1 & 2 & 5 \end{pmatrix}$$

EX2:

$$N = 5, M = 17, D^T = (2,2,2,4,3)$$

the compatability matrix is as follows:

$$C = \begin{pmatrix} 5 & 4 & 0 & 0 & 1 \\ 4 & 5 & 0 & 1 & 0 \\ 0 & 0 & 5 & 2 & 1 \\ 0 & 1 & 2 & 5 & 2 \\ 1 & 0 & 1 & 2 & 5 \end{pmatrix}$$

In our simulation, the updating scheme is cyclic and asynchronous. It means that all neurons are cyclically updated in a fixed order. Once the state of a neuron is updated, the new state information is immediately available to the other neurons in the network (asynchronous). The initial neuron inputs are generated randomly between [-1,1].

The results presented in Table 1, also includes results given in ref.[7], i.e., GAMS/MINO-5 (labeled GAMS), the traditional heuristics of steepest descent (SD), simulated annealing (SA), HNN, hill-climbing Hopfield network (HCHN) and self-organizing neural network (SONN).

| | | EX1 | EX2 |
|---|---|---|---|
| **GAMS** | Minimum | 2 | 3 |
| **SD** | Average<br>Minimum | 0.6<br>0 | 1.1<br>0 |
| **SA** | Average<br>Minimum | 0.0<br>0 | 0.1<br>0 |
| **HNN** | Average<br>Minimum | 0.2<br>0 | 1.8<br>0 |
| **HCHN** | Average<br>Minimum | 0.0<br>0 | 0.8<br>0 |
| **SONN** | Average<br>Minimum | 0.4<br>0 | 2.4<br>0 |
| **TCNN** | Average<br>Minimum | 0.0<br>0 | 0.0<br>0 |

**Table 1. Results of TCNN and other techniques**

The results of TCNN in Table 1 are obtained with a set of network parameters as follows:

$$k = 0.9; e = 1/250; I_0 = 0.50; Z(0) = 0.10;$$
$$\boldsymbol{a} = 0.0045; \boldsymbol{b} = 0.001; W_1 = 1, W_2 = 0.02$$

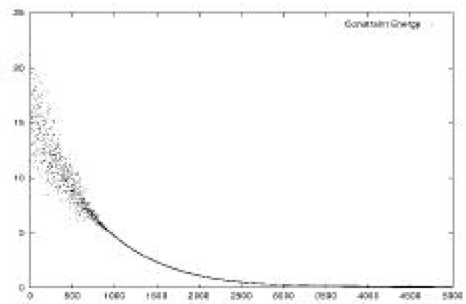
Table 1 shows that all techniques except for GAMS/MINO-5 are able to find interference free assignments (minimum=0). However, TCNN can obtain interference free assignments every case both in EX1 and EX2, although SA and HCHN are quite competitive with these. The averages of TCNN listed in the above table are calculated after 1000 runs with randomly generated initial states.

For EX2, we plot the constraint energy term $E_{Constr} = \frac{W_1}{2}\sum_{j=1}^{N}(\sum_{k=1}^{M} x_{jk} - D_j)^2$ and the optimization (interference) energy term $E_{OPT\_CAP} = \frac{W_2}{2}\sum_{j=1}^{N}\sum_{k=1}^{M} x_{jk}\sum_{i=1}^{N}\sum_{l=1}^{M} P_{j,i,(|k-l|+1)} x_{ij}$ in Figure 2 (eq. 10). For comparison, we also plot the corresponding constraint energy term optimization (distance) energy term

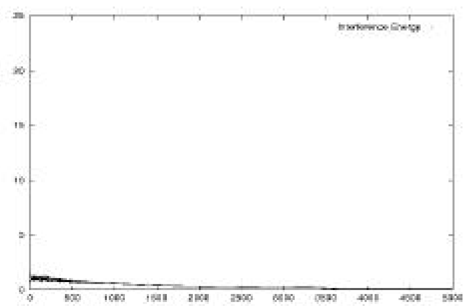
$$E_{OPT\_TSP} = \frac{W_2}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}(x_{kj+1} + x_{kj-1}) x_{ij} d_{ik}$$

in the 4-city TSP [9,16] in Figure 3.

Furthermore, we show the three input terms for EX2 respectively in Figure 4 (eq. 11): the single neuron term $y_{jk}(t+1) = ky_{jk}(t) - z_{jk}(t)(x_{jk}(t) - I_0)$, the constraint term

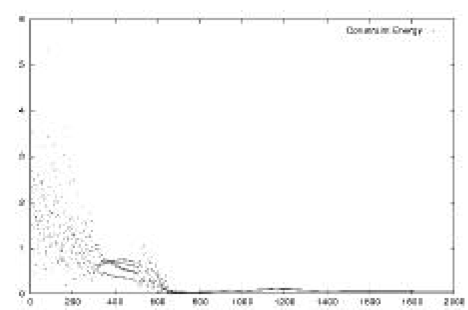



**(a) Constraint energy term**

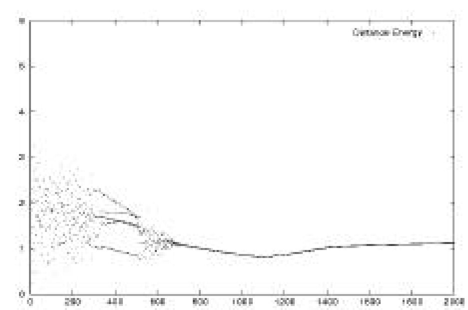



**(b) Optimization (interference) energy term**

**Figure 2. The two energy terms for CAP**




**(a) Constraint energy term**




**(b) Optimization (distance) energy term**

**Figure 3. The two energy terms for 4-city TSP**

$y_{jk}(t+1) = a\left\{-W_1\sum_{q\neq k}^{M} x_{jq} + W_1 D_j\right\}$ and the interference term $y_{jk}(t+1) = a\left\{-W_2\sum_{p=1,p\neq j}^{N}\sum_{q=1,q\neq k}^{M} P_{j,p,|k-q|+1}\, x_{pq}\right\}$.

For comparison, we also plot the three corresponding terms for the 4-city TSP in Figure 5.

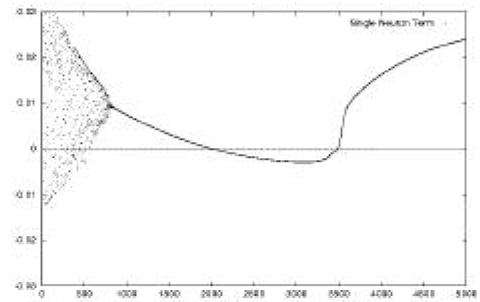
From Figures 2 and 4, for both energy and input, the optimization terms for the CAP are much smaller compared to constraint terms and the single neuron term (for input), whereas Figure 3 and 5 show that the optimization terms in the case of the 4-city TSP are comparable with the constraint terms and the single neuron term (for input). However, our experiments show that the small optimization terms in CAP still make contribution to the global optimal searching: In EX1, while $W_1 = 1$, when $W_2 \in [0.002, 0.027]$ can assure global optima for all simulations. In EX2, while $W_1 = 1$, only $W_2 \in [0.01, 0.027]$ can do so.

It is interesting to notice that the time evolution processes in the CAP are quite different from those in the 4-city TSP [9]. Clear bifurcations are present in the 4-city TSP (Figs. 3 and 5), but absent in the CAP (Figs. 2 and 4).

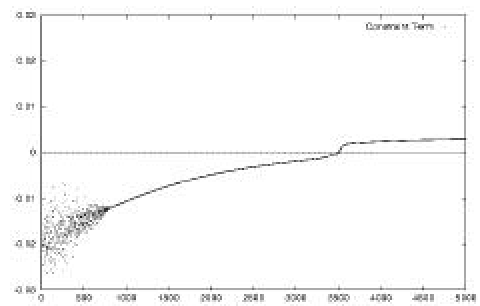
## 5. CONCLUSIONS

The results obtained in this paper indicate that Chen and Aihara's transiently chaotic neural networks (TCNN) can be successfully used in solving channel assignment problems (CAP). Experiments on two CAP's show that the optimal solutions can always be found each case with a set of parameters chosen in the TCNN. This result is better than those of other techniques. Our further analyses show that the dynamic characteristics of the TCNN in the CAP's are quite different from those of the 4-city TSP: (1) the optimization terms in the TSP are comparable in magnitude with the constraint terms and the single neuron term (for input), while in the CAPs the optimization terms are quite smaller than the other two terms; (2) the clear bifurcations cannot be observed in the chaotic searching for optima in the CAPs as those in the 4-city TSP [9,16].

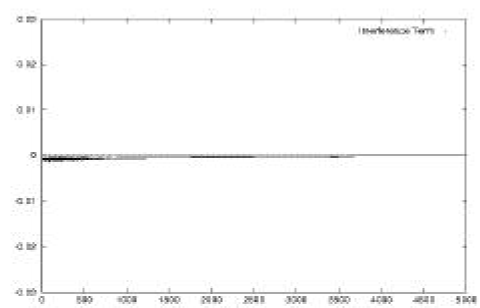
The ability of the TCNN on solving large size CAPs will be a focus in our future work.



**(a) Single neuron term**

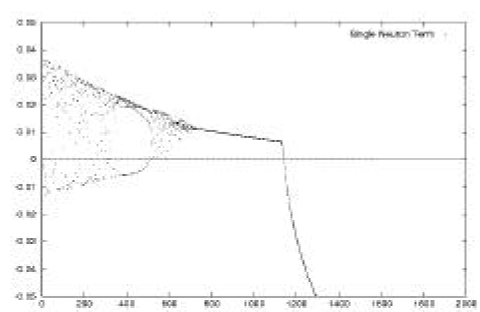


**(b) Constraint term**

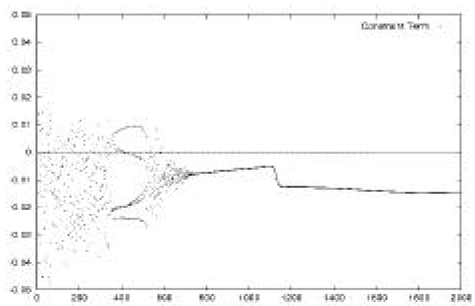


**(c) Optimization (interference) term**

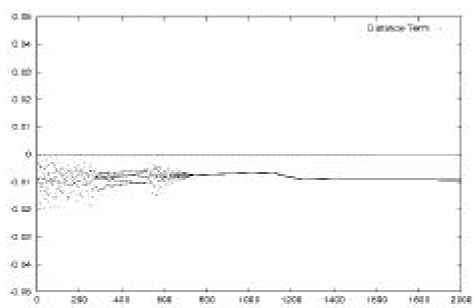
**Figure 4. The three input terms in the CAP**



**(a) Single neuron term**



**(b) Constraint term**



**(c) Optimization (distance) term**

**Figure 5. The three input terms in the 4-city TSP**

## REFERENCES


[1] Hale, W. K., "Frequency assignment: theory and applications", *Pro. IEEE*, vol. 68, pp.1497-1514, Dec. 1980.

[2] Box, F., "A heuristics technique for assigning frequencies to mobile radio net," *IEEE Trans. Veh. Technol.*, vol. VT-27, no. 2, pp. 57-64, 1978.

[3] Kunz, D., " Channel assignment for cellular radio using neural networks," *IEEE Trans. Veh. Technol.*, vol. 40, no. 1, pp. 188-193, 1991.

[4] Funabiki, N. and Takefuji, Y., " A neural network parallel algorithm for channel assignment problems in cellular radio networks," *IEEE Trans. Veh. Technol.*, vol. 41, pp. 430-436, Nov. 1992.

[5] Chan, P. T., Palaniswami, M. and Everitt, D., "Neural network-based dynamic channel assignment for cellular mobile communication systems," *IEEE Trans. Veh. Technol.*, vol.43, pp. 279-288, 1994.

[6] Kim, J. -S., et al, "Cellular radio channel assignment using a modified Hopfield network," *IEEE Trans. Veh. Technol.*, vol. 46, pp. 957-967, Nov. 1997.

[7] Smith, K. and Palaniswami, M., "Static and dynamic channel assignment using neural networks," *IEEE Journal on Selected Areas in Communications*, vol. 15, No. 2, Feb. 1997.

[8] Hopfield, J. J. and Tank, D. W., "Neural computation of decisions in optimization problems," *Biological Cybernetics*, vol. 52, pp. 141-152, 1985.

[9] Chen, L. and Aihara, K., "Chaotic simulated annealing by a neural network model with transient chaos," *Neural Networks*, vol. 8, no. 6, pp. 915-930, 1995.

[10] Wang, L. and Ross, J., "Osillations and chaos in neural networks: an exactly solvable model." *Proc. National Academy of Sciences* (USA) vol. 87, pp. 9467-9471, 1990.

[11] Wang, L. and Alkon, D.L., (eds.), *Artificial neural networks: Oscillations, chaos, and sequence processing*. (IEEE Computer Society Press), Los Alamitos, California, 1993.

[12] Wang, L., "Oscillatory and chaotic dynamics in neural networks under varying operating conditions," *IEEE Transactions on Neural Networks*, vol.7, no. 6, pp. 1382-1388, 1996.

[13] Kwok, T., Smith, K. and Wang, L., "Incorporating chaos into the Hopfield neural network for combinatorial optimization," in *Proc. 1998 World Multiconference on Systemics, Cybernetics and Informatics*, N. Callaos, O. Omolayole, and L. Wang, (eds.) vol.1, pp. 646-651, 1998.

[14] Wang, L. and Smith, K., "Chaos in the discretized analog Hopfield neural network and potential applications to optimization," *Proc. International Joint Conference on Neural Networks*, vol.2, pp. 1679-1684, 1998.

[15] Wang, L. and Smith, K., "On chaotic simulated annealing," *IEEE Transactions on Neural Networks*, vol. 9, no. 4, pp. 716-718, 1998.

[16] Tian, F. and Wang, L., "Solving optimization problems using transiently chaotic neural networks: choices of network parameters," *Proc. Second International Conference on Information, Communications & Signal Processing*, Singapore, Dec 7-10, 1999.