Noisy Chaotic Neural Networks for Solving Combinatorial Optimization Problems

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***Abstract* – Chaotic simulated annealing (CSA) recently proposed by Chen and Aihara has been shown to have higher searching ability for solving combinatorial optimization problems compared to both the Hopfield-Tank approach and stochastic simulated annealing (SSA). However, CSA is not guaranteed to relax to a globally optimal solution no matter how slowly annealing takes place. In contrast, SSA is guaranteed to settle down to a global minimum with probability 1 if the temperature is reduced sufficiently slowly. In this paper, we attempt to combine the best of both worlds by proposing a new approach to simulated annealing using a noisy chaotic neural network, i.e., stochastic chaotic simulated annealing (SCSA). We demonstrate this approach with the 48-city traveling salesman problem.**

1. Introduction

In recent years, a large body of work has been created on theory and applications of chaotic neural networks due to extensive research interests and efforts in this area (e.g., [1]-[16]). In particular, Freeman and co-workers [1] have demonstrated strong evidence, through both biological experiments and theoretical investigations, that chaos play an important role in information processing in real and artificial neural systems.

Aihara et al [3] proposed a chaotic neural network based on a modified Nagumo and Sato neuron model. Nozawa [4] showed that the Euler approximation of the continuous-time Hopfield neural network [17] with a negative neuronal self-coupling has chaotic dynamics and that this model is equivalent to Aihara et al's chaotic neural network after a simple variable transformation. Nozawa further showed [4][6] that this chaotic neural network have much higher searching ability for solving the traveling salesman problem (TSP), in comparison with the Hopfield neural network [17][18][19], the Boltzmann machine, and the Gaussian machine.

Chen and Aihara [7][8] proposed chaotic simulated annealing (CSA) by starting with a sufficiently large negative self-coupling in the above-mentioned chaotic neural network and gradually decreasing the self-coupling so that the network eventually stabilizes, thereby obtaining a transiently chaotic neural network. Their computer simulations showed that CSA obtains good solutions for TSP much more easily compared to the Hopfield-Tank approach [17]-[19] and stochastic simulated annealing (SSA) [20]. Chen and Aihara [15] provided the following theoretical explanation for the global searching ability of the chaotic neural network: its attracting set contains all global and local minima of the optimization problem, and since the chaotic attracting set has a fractal structure and covers only a very small fraction of the entire state space, the chaotic neural network is more efficient in finding good solutions for optimization problems compared to other global search algorithms such as SSA.

SSA is known to relax to a global minimum if the annealing takes place sufficiently slowly, i.e., no faster than logarithmatically. In practical terms, this means that SSA is capable of producing good (optimal or near-optimal) solutions for many applications, if the annealing parameter (temperature) is reduced exponentially but with a reasonably small exponent. However, unlike SSA, CSA has completely deterministic dynamics and is not guaranteed to settle down at a global minimum no matter how slowly the annealing parameter (the self-coupling) is reduced. Practically speaking, this implies that CSA sometimes may not be able to provide a good solution at the conclusion of annealing.

In this paper, we attempt to combine the best features of both SSA and CSA, i.e., noisy nature and efficient chaotic searching, by adding a decreasing noise in the transiently chaotic neural network of Chen and Aihara [7][8][15]. We therefore obtain a novel method for solving optimization problems: stochastic chaotic simulated annealing (SCSA).

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We then use the noisy chaotic neural network or SCSA to solve the 48-city TSP and show marked improvement over CSA.

2. Noisy Chaotic Neural Networks

Our noisy chaotic neural network is defined as follows:

$$x_{ij}(t) = \frac{1}{1+e^{-y_{ij}(t)/\varepsilon}} \tag{1}$$

$$y_{ij}(t+1) = ky_{ij}(t) + \alpha\Big(\sum_{k,l=1,k,l\neq i,j}^{n} w_{ijkl}x_{kl}(t) + I_{ij}\Big) - z(t)(x_{ij}(t) - I_0) + n(t) \tag{2}$$

$$z(t+1) = (1-\beta)z(t) \qquad (i,j,k,l = 1,\ldots,n) \tag{3}$$

$$A[\,n(t+1)\,] = (1-\beta)\,A[\,n(t)\,] \tag{4}$$

where

x_{ij} =output of neuron i, j ; $y_{i,j}$ =internal state of neuron i, j ; $I_{i,j}$ =input bias of neuron i, j ;

k =damping factor of nerve membrane ($0 \le k \le 1$); α =positive scaling parameter for inputs;

$z(t)$ =self-feedback connection weight or refractory strength $(z(t) \ge 0)$;

β =damping factor of the time dependent ($0 \le \beta \le 1$); I_0 =positive parameter;

ε =steepness parameter of the output function (ε >0);

$w_{ijkl} = w_{klij}$; $w_{ijij} = 0$; $\sum_{k,l=1,k,l\neq i,j}^{n} w_{ijkl}x_{kl} + I_{ij} = -\partial E / \partial x_{ij}$: connection weight from neuron k, l to neuron i, j .

n(t) : random noise injected into the neurons, i.e., in [*-A, A*] with a uniform distribution, where *A[n]* is the noise amplitude. If *n(t)* = 0, for all *t*, then the noisy chaotic neural network reduces to the transiently chaotic neural network.

3. Solving the Traveling Salesman Problem Using the Noisy Chaotic Neural Network

A classical combinatorial optimization problem is the travelling salesman problem (TSP), which is NP-hard. It is to seek the shortest route through n cities, visiting each once and only once, and returning to the starting point. Since Hopfield and Tank applied their neural networks to TSP, TSP has been intensively studied in the field of artificial neurocomputing.

Here the solution of TSP with n cities is represented by a network with $n \times n$ neurons. The neuron output $x_{ij} = 1$ represents visiting city i in visiting order j . The computational energy function to be minimized consists of two parts:

$$E = \frac{W_1}{2}\left\{\sum_{i=1}^{n}\left(\sum_{j=1}^{n} x_{ij} - 1\right)^2 + \sum_{j=1}^{n}\left(\sum_{i=1}^{n} x_{ij} - 1\right)^2\right\} + \frac{W_2}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\left(x_{kj+1} + x_{kj-1}\right)x_{ij}d_{ik} \tag{5}$$

where $x_{i0} = x_{in}$, $x_{in+1} = x_{i1}$, d_{ij} is the distance between city i and city j . Coefficient W_1 and W_2 reflect the relative strength of the constraints and the tour length term. Hence a global minimum of E represents a shortest valid tour. We experiment on the 48-city TSP [21]. To compare the performance with the CSA, we use a set of $k,\alpha,\beta,\varepsilon$ similar to Chen and Aihara's [7][8]:

$$k,\alpha,\beta,\varepsilon = 0.9, 0.015, 0.00005, 0.004;\ I_0 = 0.5; z(0) = 0.10\ ,\ W_1 = 1\ ,\ A[n(0)] = 0.002\ .$$

For each choice of W_2, we run 100 simulations with different randomly generated initial neuron states. Overall results obtained by the noisy chaotic neural network is much better compared to CSA (an example is show in Figs. 1 and 2). As shown in Fig. 3, the noisy chaotic neural network continues to search after chaos disappears.

4. Summary

In this paper, we proposed stochastic chaotic simulated annealing (SCSA) by adding noise to Chen-Aihara's transiently chaotic neural network. Application of this noisy chaotic neural network to the 48-city TSP showed marked improvement over chaotic simulated annealing (CSA). In contrast to the conventional stochastic simulated annealing (SSA), SCSA restricts the random search to a sub-space of chaotic attracting set which is much smaller than the entire state space searched by SSA. In contrast of CSA, SCSA continues to search after the disappearance of chaos. Future work will include applications of SCSA to other practical optimization problems.

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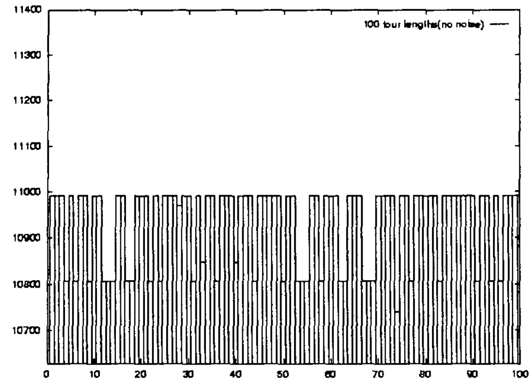


Fig.1 The tour lengths obtained using Chen-Aihara's chaotic simulated annealing (in the absence of noise) from 100 runs starting from different randomly generated initial states. W_2=0.80. Shortest tour length found (10739): 1%; 2nd shortest (10805): 28%; 3rd shortest (10847): 2%; Longer than 10900: 69%.

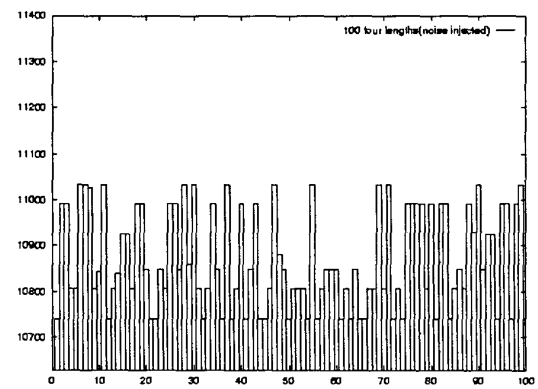
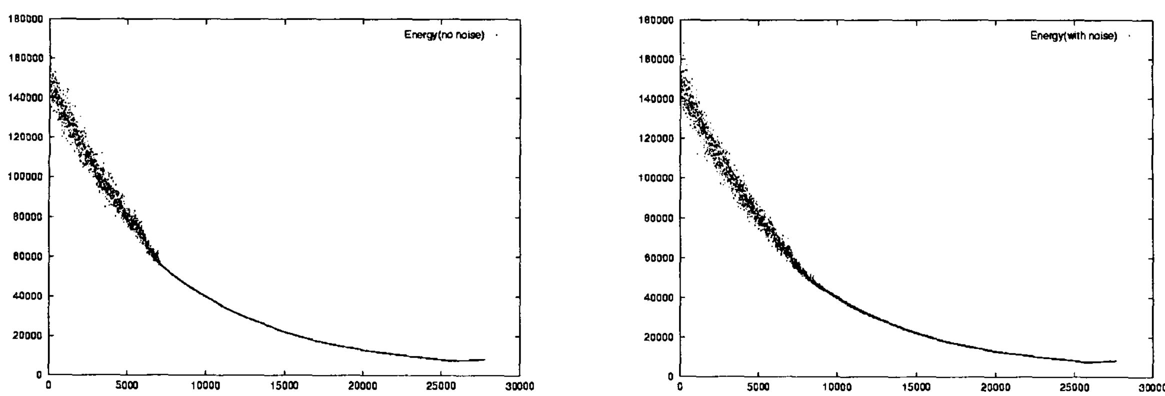


Fig.2 The tour lengths obtained using our stochastic chaotic simulated annealing (noise) from 100 runs starting from different randomly generated initial states. W_2=0.80. Shortest tour length found (10739): 24%; 2nd shortest (10805): 22%; 3rd shortest (10840 - 10882): 16%; Longer than 10900:38 %.



(a) **(b)**

Fig. 3 Energy for 48-city TSP as a function of time obtained from Chen-Aihara's transiently chaotic neural network (a) without and (b) with noise injected.