of the lighting and viewing directions are never required in this approach. In these cases, the proposed RBF approach outperforms other conventional models. Fig. 4(c) shows the recovered shape by the Lambertian model. Perceivably, the result is far from satisfactory attributed to the specular components. Fig. 4(d) also shows the result obtained by the Lambertian model without illuminate condition estimation. In this case, the recovered egg lost the major contour because of the abnormal illuminate condition. The reconstructed shapes obtained by a more refined Torrance–Sparrow model under known and unknown reflectivity parameters are shown in Fig. 4(e) and 4(f), respectively. Although its performance is better than that of the Lambertian model, the recovered egg and cylinder are not satisfactory attributed to the incorrect reflectivity parameter estimate. Another promising real image result based on a human hand object is demonstrated as shown in Fig. 5. Clearly, the proposed RBF-based approach is able to deliver much better results than those yielded by the other models under specular effect and unknown reflectivity parameters conditions.

IV. Conclusion

A new learning approach for parametric specular reflectance model is proposed. This new reflectance model is based on learning the parameters of RBF network, which is able to model the physical parameters of the reflectance model under the existence of specular components. Through this RBF-based specular model, the shape from shading algorithm has become much robust and effective for most objects reconstruction with different lighting and specular conditions. The proposed RBF model is very robust under noisy environments. This enables the RBF-based reflectance model to be applicable to most real-world applications where the reflectivity parameters is never given, and the image is suffered from noisy and specular conditions.

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HeteroAssociations of Spatio-Temporal Sequences with the Bidirectional Associative Memory

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***Abstract*—Autoassociations of spatio-temporal sequences have been discussed by a number of authors. We propose a mechanism for storing and retrieving pairs of spatio-temporal sequences with the network architecture of the standard bidirectional associative memory (BAM), thereby achieving hetero-associations of spatio-temporal sequences.**

***Index Terms*—Associative memory, bidirectional associative memory (BAM), image sequence, neural network.**

In the bidirectional associative memory (BAM) proposed by Kosko [5], there are two layers of neurons: layer A with n neurons and layer B with m neurons. The neurons in layer A are connected with those in layer B through a weight matrix M, and the neurons in layer B are connected with those in layer A through the transposed matrix M^T. There are no intralayer connections. Kosko [5] showed that the BAM always settles down to a stable state for any arbitrary real weight matrix M. A stable state for a BAM is in fact a pair of two memory states represented by the final states of the two layers of neurons

$$A_i \underset{M^T}{\overset{M}{\rightleftarrows}} B_i \tag{1}$$

where A_i and B_i are spatial patterns with dimensions n and m, respectively.

In the special case where the two layers contain the same number of neurons and the weight matrices are symmetric, that is, $m = n$, $M = M^T$, and $A_i = B_i$, the BAM is equivalent to the Hopfield network [2]. The BAM is normally referred to as a heteroassociative memory, because it stores memory pairs, whereas the Hopfield network is referred to as an autoassociative memory.

Much research attention has been focused on storage and retrieval of spatio-temporal sequences (e.g., [4], [8], [9], and [15]), which are time-dependent sequences of spatial patterns. Autoassociations of spatio-temporal sequences have been discussed by a number of authors based on static autoassociative networks (e.g., [4] and [8]); however, there does not yet exist any mechanism for heteroassociations of pairs of spatio-temporal sequences. Possible applications of heteroassociations of spatio-temporal sequences may include control for robots or unmanned planes, with one sequence in the pair describing the environment and the other sequence in the pair being the corresponding control signals.

In addition to the static BAM, Kosko [5] further proposed a method for storing and retrieving sequences of k spatial patterns:

$$A_1 \underset{M^T}{\overset{M}{\rightleftarrows}} B_1 \quad \ldots \quad \underset{M^T}{\overset{M}{\rightleftarrows}} B_i \quad \ldots \quad \underset{M^T}{\overset{M}{\rightleftarrows}} B_k. \tag{2}$$

But the network structure that is capable of storing and retrieving sequences as in (2) is no longer that of the standard BAM. Rather, the network needs to have multiple slabs in parallel or other hierarchical structures. There are two drawbacks in the above system. First, the network structure is complex and depends on the lengths of the sequences to be stored. Second, a sequence represented by (2) may be stored and retrieved with an autoassociative network such as the Hopfield network

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TABLE I
RETRIEVAL OF THE SPATIO-TEMPORAL SEQUENCE PAIR SHOWN IN (3). INITIALLY ($t = 0$) THE STATE OF LAYER B IS B_1' (CLOSE TO PATTERN B_1)

t	0	$2\tau-1$	$4\tau-1$	...	$2i\tau-1$	...	$2p\tau-1$
Layer A		A_1	A_2	...	A_i	...	A_p
Layer B	B_1'	B_1	B_2	...	B_i	...	B_p

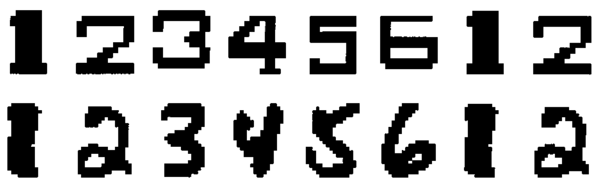

Fig. 1. Memory image pair. The images for layer A (top row) and layer B (bottom row) are 12×11 and 22×16 pixels in size, respectively. Each sequence consists of six images and repeats itself.

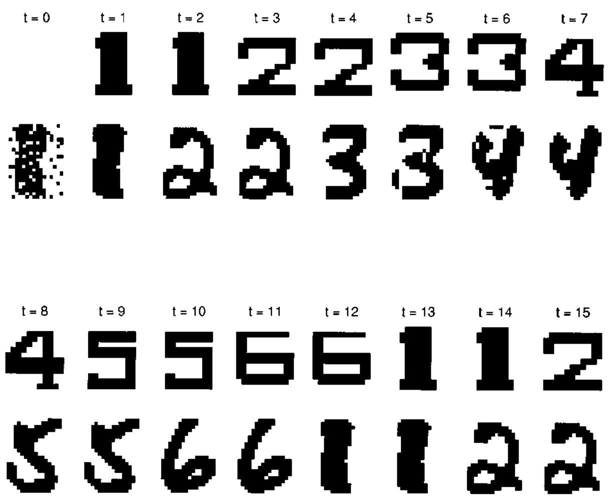

Fig. 2. Retrieval of the image pair shown in Fig. 1 with a noisy cue image presented to layer B at $t = 0$.

[8], [4]. A heteroassociative network, such as the BAM, should be able to handle more complicated problems.

In this Letter, we propose the following mechanism for storing and retrieving pairs of spatiotemporal sequences:

$$\begin{aligned} \tilde{S}_1 &: A_1 \longrightarrow A_2 \longrightarrow \ldots \longrightarrow A_p \\ \tilde{S}_2 &: B_1 \longrightarrow B_2 \longrightarrow \ldots \longrightarrow B_p \end{aligned} \tag{3}$$

with the standard two-layer BAM structure

$$\tilde{S}_1 \underset{M_{BA}}{\overset{M_{AB}}{\rightleftarrows}} \tilde{S}_2 \tag{4}$$

thereby achieving heteroassociations of spatio-temporal sequences. Here p is the number of pairs of spatial patterns stored in the system and is also the length of the spatio-temporal sequences.

In the forward direction, i.e., from layer A to layer B, the weight matrix consists of two parts

$$\begin{aligned} M_{AB} &= \lambda_1 \sum_{i=1}^{p} A_i^T B_i + \lambda_2 \sum_{i=1}^{p-1} A_i^T B_{i+1} \\ &\equiv \lambda_1 M_1 + \lambda_2 M_2. \end{aligned} \tag{5}$$

The weight matrix in the backward direction, i.e., from layer B to layer A, is

$$\begin{aligned} M_{BA} &= \lambda_1 \sum_{i=1}^{p} B_i^T A_i + \lambda_2 \sum_{i=1}^{p-1} B_{i+1}^T A_i \\ &\equiv \lambda_1 M_1^T + \lambda_2 M_2^T. \end{aligned} \tag{6}$$

The memory patterns in (5) and (6) are bipolar, i.e., on-pixels and off-pixels are represented by $+1$ and -1, respectively, rather than one and zero. The coefficients in (5) and (6) are chosen such that only one part of each matrix is active at any instance of time, i.e.,

$$\begin{aligned} &\lambda_1 = 1 \text{ and } \lambda_2 = 0, \quad \text{if } 2j\tau \le t < (2j+1)\tau \\ &\lambda_1 = 0 \text{ and } \lambda_2 = 1, \quad \text{if } (2j+1)\tau \le t < (2j+2)\tau \end{aligned} \tag{7}$$

where $j = 0, 1, \ldots$. Suppose all neurons in a layer (layer A or B) complete a round of sequential updating within one time step. τ in (7) is the number of time steps that is sufficiently large for the BAM to settle down to a memory pair starting from any initial state (the convergence or relaxation time). For the reasons that will be clear below, we also require that τ is an odd number.

Matrix M_1 in (5) and (6) stores the standard memory pairs present in the static BAM [5], whereas matrix M_2 helps to store heteroassociative sequence pairs by associating A_1 with B_2, A_2 with B_3, and so on. The operation of the present spatio-temporal BAM, which we shall call TBAM, is shown schematically in the following equations and is described below:

$$t = 0, 1, \ldots, \tau - 1 \quad A_1 \underset{M_1^T}{\overset{M_1}{\rightleftarrows}} B_1 \tag{8}$$

$$t = \tau, \tau + 1, \ldots, 2\tau - 1 \quad A_1 \underset{M_2^T}{\overset{M_2}{\rightleftarrows}} B_2 \tag{9}$$

$$t = 2\tau, 2\tau + 1, \ldots, 3\tau - 1 \quad A_2 \underset{M_1^T}{\overset{M_1}{\rightleftarrows}} B_2 \tag{10}$$

$$t = 3\tau, 3\tau + 1, \ldots, 4\tau - 1 \quad A_2 \underset{M_2^T}{\overset{M_2}{\rightleftarrows}} B_3 \tag{11}$$

$$\cdots \qquad \cdots. \tag{12}$$

Suppose all neurons in layer A complete a round of sequential updating within one time step starting at *even* time steps $t = 0, 2, \ldots,$ whereas all neurons in layer B complete a round of sequential updating within one time step starting at *odd* time steps $t = 1, 3, \ldots$.

During the first interval of τ time steps [see (8)], the forward and the backward weight matrices are M_1 and M_1^T, respectively. Layer A is the first of the two layers to update (at $t = 0$): the double arrow ($\Longrightarrow$) in (8) indicates that signals first travel from layer B to layer A. Suppose the neural state of layer B at $t = 0$ is near pattern B_1. The state of layer A is thus thrown near pattern A_1 by connection matrix M_1 after one time step. At the end of this interval $t = \tau - 1$, the TBAM settles down to memory pair (A_1, B_1).

During the second interval of τ time steps [see (9)], the forward and the backward weight matrices are M_2 and M_2^T, respectively, according to (5)–(7). Since τ is an odd number, layer B is the first of the two layers to update (at $t = \tau$) and is thrown near pattern B_2 by connection matrix

TABLE II
OVERLAPS (NORMALIZED DOT-PRODUCT) BETWEEN THE SIX IMAGES FOR LAYER A SHOWN IN FIG. 1. ORTHOGONALITY IS INDICATED BY ZERO-OVERLAP: IMAGE "2" IS ORTHOGONAL WITH IMAGE "6", IMAGE "4" IS ORTHOGONAL WITH IMAGE "5"

Image	1	2	3	4	5	6
1	1.00	0.09	-0.18	0.06	0.09	-0.15
2	0.09	1.00	0.09	-0.21	0.30	0.00
3	-0.18	0.09	1.00	-0.27	0.09	0.48
4	0.06	-0.21	-0.27	1.00	0.00	0.03
5	0.09	0.30	0.09	0.00	1.00	0.15
6	-0.15	0.00	0.48	0.03	0.15	1.00

TABLE III
OVERLAPS (NORMALIZED DOT-PRODUCT) BETWEEN THE SIX IMAGES FOR LAYER B SHOW IN FIG. 1. EACH IMAGE IS NOT ORTHOGONAL WITH ANY OTHER IMAGES

Image	1	2	3	4	5	6
1	1.00	0.03	0.09	0.03	0.23	0.25
2	0.03	1.00	0.09	0.09	0.16	0.24
3	0.09	0.09	1.00	0.18	0.28	0.03
4	0.03	0.09	0.18	1.00	0.07	-0.02
5	0.23	0.16	0.28	0.07	1.00	0.19
6	0.25	0.24	0.03	-0.02	0.19	1.00

M_2. At the end of this interval, i.e., $t = 2\tau - 1$, the TBAM settles down to the memory pair $(A_1, B_2) \ldots$ [see (10) and (11),. . .].

The system dynamics of the TBAM is summarized in Table I. Pairs of spatio-temporal sequences can thus stored and retrieved.

We illustrate the proposed algorithm with the example shown in Figs. 1 and 2. The image sequence pair shown in Fig. 1 is stored in the TBAM with (5) and (6). The images for layer A (top row) and layer B (bottom row) are 12×11 and 22×16 pixels in size, respectively. Each sequence consists of six images and repeats itself. In Fig. 2, a test image, i.e., a digit image "1" corrupted by noise, is presented to layer B, which retrieves the image sequence pair. In this example, the network stabilizes after one iteration, i.e., $\tau = 1$.

Several important issues need to be addressed:

1) *Memory Capacity*—The number of sequence pairs and the length of each sequence pair that can be stored in and retrieved from the TBAM depend on the number of *static* image pairs that can be stored in the corresponding BAM. That is, the memory capacity of the TBAM needs to be determined by the memory capacity of the BAM. Although there have been studies on the memory capacity of a *second-order* BAM [6], [3], the memory capacity of the original BAM used in the present system has not yet been derived in the literature. Since the original BAM bears certain resemblance with the Hopfield network [2], the methods used to study the memory capacity of the Hopfield network may be used to study the memory capacity of the BAM; however, such a study is expected to be as lengthy as the Hopfield case (e.g., [1] and [7]).
2) *Orthogonality and Noise*—Similarly, detailed studies on the issues of orthogonality and noise for the original BAM are still lacking and need to be carried out before a study can be carried out for the TBAM, which is based on the BAM. The learning algorithm used in the TBAM, i.e., (5) and (6), are Hebb-type dot-product rules, similar to that used in the Hopfield network [2]. It is thus expected that orthogonality and noise in the stored patterns will have significant effects on the performance of the TBAM, as in the case of the Hopfield network [10]. However, the TBAM is able to tolerate some nonorthogonality and noise (see Fig. 2, Tables II and III).
3) *Degree of Sequence Complexity*—During retrieval with the proposed TBAM, the image pair at a given time step is uniquely determined by the image pair at the previous time step, as facilitated by (6). Hence the network is capable of dealing with only one-to-one mappings, i.e., *simple sequences*. Generalization to incorporate multiple time-delays will enable the TBAM to process certain *complex sequences* [9], [15]. Similarly, a generalized TBAM with multiple time-delays will be required to deal with *temporal noise*, e.g., gaps or incorrect images in a sequence, as opposed to noise within each spatial image (referred to as *spatial noise*).

In-depth studies on these issues, as well as applications of the proposed TBAM and its generalizations, e.g., incorporations of complex-valued multistate neurons []–[]are out of the scope for the present Letter and will be the subject of future work.

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