# Channel Assignment for Mobile Communications Using Stochastic Chaotic Simulated Annealing

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**Abstract.** The channel assignment problem (CAP), the task to assign the required number of channels to each radio cell in such a way that interference is precluded and the frequency spectrum is used efficiently, is known to be an NP-complete optimization problem. In this paper, we solve CAP using a stochastic chaotic neural network that we proposed recently. The performance of stochastic chaotic simulated annealing (SCSA) is compared with other algorithms in several benchmark CAPs. Simulation results showed that this approach is able to further improve on results obtained by other algorithms.

## 1 Introduction

In recent years, there is a steady increasing demand for cellular mobile telephone communication systems. But the usable frequency spectrum is limited. Thus optimal frequency channel assignment is becoming more and more important and it can greatly enhance the traffic capacity of a cellular system.

Gamst and Rave defined a general form of channel assignment problems in an arbitrary inhomogeneous cellular radio network [1]: minimize the span of channels subject to demand and interference-free constraints. The CAP is usually solved by graph coloring algorithms [3]. Kunz used the Hopfield neural network for solving the CAP [2]. The algorithm minimizes an energy or cost function representing interference constraints and channel demand. Since then, various techniques have been explored for the application of neural networks to CAP. Funabiki used a parallel algorithm which does not require a rigorous synchronization procedure [10]. Chan et al proposed an approach based on cascaded multilayered feedforward neural networks which showed good performance in dynamic CAP [11]. Kim et al proposed a modified Hopfield network without fixed frequencies [12]. Smith and Palaniswami reformulated the CAP as a generalized quadratic assignment problem [8]. They then found remarkably good

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solutions to CAPs using simulated annealing, a modified Hopfield neural network, and a self-organizing neural network. Potential interferences considered in their paper come from the cochannel constraint (CCC), the adjacent channel constraint (ACC), and the co-site constraint (CSC) [1].

Simulated annealing is a general method for obtaining approximate solutions of combinatorial optimization problems [6]. Since the optimization processing is undertaken in a stochastic manner, it is also called stochastic simulated annealing (SSA). Convergence to globally optimal solutions is guaranteed if the cooling schedule is sufficiently slow, i.e., no faster than logarithmically [7]. In recent years, a large amount of work has been done in chaotic simulated annealing (CSA) ([12]-[21]). CSA can search efficiently because of its reduced search spaces. Chen and Aihara proposed a transiently chaotic neural networks (TCNN) which adds a large negative self-coupling with slow damping in the Euler approximation of the continuous Hopfield neural network so that neurodynamics eventually converge from strange attractors to an equilibrium point [13]. The TCNN showed good performance in solving traveling salesman problem (TSP). But CSA is deterministic and is not guaranteed to settle down at a global minimum no matter how slowly the annealing takes place. In [15], we generalized CSA by adding a decreasing random noise into the neuron inputs in the TCNN to combine the best feasures of both SSA and CSA, thereby abtaining stochastic chaotic simulated annealing (SCSA). In this paper, we show that SCSA can lead to further improvements on solutions for CAP.

This paper is organized as followings. Section 2 reviews the static channel assignment problem and its mathemmatical formulation as given in [8]. In section 3, the SCSA is reviewed. In the section 4, we apply SCSA to several benchmarking CAPs. Finally in section 5, we conclude this paper.

# 2 Static Channel Assignment Problem

In this section, we review the formulation of CAP as given by Smith and Palaniswami [8]. Suppose the number of available channels in an N-cell static mobile communication system is M and the number of channels requiremed in cell iis  $D_i$ . The minimum distances in the frequency domain by which two channels must be separated in order to guarantee an acceptably low signal/interference ratio in each region are stored in the compatibility matrix  $C = \{C_{ij}\}$ , an  $n \times n$ symmetric matrix, where n is the number of cells in the mobile networks and  $C_{ij}$  is the minimum frequency separation between cell i and one in cell j. In most of cases, the number of available channels is less than the lower bound, i.e., the minimum number of channels required for an interference-free assignment. A useful version of the CAP is defined as follows [8]:

 $X_{j,k} = \begin{cases} 1, \text{ if cell j is assigned to channel } k \\ 0, \text{ otherwise }. \end{cases}$ 

A cost tensor  $P_{j,i,m+1}$  is used to measure the degree of interference between cells j and i caused by such assignments that  $X_{j,k} = X_{i,l} = 1$  [8], where m = |k - l| is the distance in the channel domain between channels k and l. The cost tensor P can be calculated recursively as follows:

$$P_{j,i,m+1} = \max(0, P_{j,i,m} - 1), \quad for \ m = 1, \cdots, M - 1 , \quad (1)$$

$$P_{j,i,1} = C_{ji}, \qquad \forall j, i \neq j \quad , \tag{2}$$

$$P_{j,j,1} = 0, \qquad \forall j \quad . \tag{3}$$

Thus the CAP can be formulated to minimize the total interference of all assignments in the network:

minimize

$$F(X) = \sum_{j=1}^{N} \sum_{k=1}^{M} X_{j,k} \sum_{i=1}^{N} \sum_{l=1}^{M} P_{j,i,(|k-l|+1)} X_{i,l} , \qquad (4)$$

subject to

$$\sum_{k=1}^{M} X_{j,k} = D_j, \qquad \forall j = 1, \cdots, N \quad .$$
(5)

where F(X) is the total interference.

#### 3 Stochastic Chaotic Simulated Annealing

The stochastic chaotic simulated annealing (SCSA) is formulated as follows [15]:

$$x_{jk}(t) = \frac{1}{1 + e^{-y_{jk}(t)/\varepsilon}}$$
, (6)

$$y_{jk}(t+1) = ky_{jk}(t) + \alpha \left(\sum_{i=1, i \neq j}^{N} \sum_{l=1, l \neq k}^{M} w_{jkil} + I_{ij}\right) - z_{jk}(t) (x_j k(t) - I_0) + n(t) \quad , \quad (7)$$

$$z_{jk}(t+1) = (1-\beta)z_{jk}(t)$$
  $(i = 1, \dots, n)$  , (8)

$$A[n(t+1)] = (1-\beta)A[n(t)] , \qquad (9)$$

where

$$\begin{array}{l} x_{jk}: \text{output of neuron j,k };\\ y_{jk}: \text{input of neuron j,k };\\ w_{jkil} = w_{iljk}; \; w_{jkjk} = 0; \sum_{l=1, l \neq k}^{M} w_{jkil} + I_{ij} = -\partial E / \partial x_{jk}:\\ \text{ connection weight from neuron j,k to neuron i,l }; \end{array}$$

 $I_{jk}$ : input bias of neuron j, k;

k: damping factor of nerve membrane  $(0 \le k \le 1)$ ;

 $\alpha$  : positive scaling parameter for inputs ;

 $\beta$ : damping factor of the time dependent  $(0 \le \beta \le 1)$ ;

 $z_{jk}(t)$  : self-feedback connection weight or refractory strength  $(z(t) \ge 0)$ ;

 $I_0$ : positive parameter;

 $\varepsilon$  : steepness parameter of the output function  $(\varepsilon>0)$  ;

E : energy function;

n(t): random noise injected into the neurons, in [-A, A] with a uniform distribution;

A[n]: the noise amplitude.

If n(t)=0 for all t, then the above noisy chaotic neural network reduces to the transiently chaotic neural network of Chen and Aihara [13].

The corresponding energy function E for CAP is given by (1) and (2):

$$E = \frac{W1}{2} \sum_{j=1}^{N} (\sum_{k=1}^{M} X_{jk} - D_j)^2 + \frac{W2}{2} \sum_{j=1}^{N} \sum_{k=1}^{M} X_{jk} \sum_{i=1}^{N} \sum_{l=1}^{M} P_{j,i,(|k-l|+1)} X_{il} \quad (10)$$



Fig. 1. Layout of a 21-cell hexagonal network

# 4 Application of SCSA to Benchmark CAPs

#### 4.1 Descriptions of Various Problems

First we use the data set suggested by Sivarajan [3], denoted as EX1. The number of cells is N = 4, the number of channels available is M = 11, the demand of channels is given by  $D^T = (1, 1, 1, 3)$ . We also use a slightly larger extension of EX1, denoted as EX2 [8]:

 $N = 5, M = 17, D^T = (2, 2, 2, 4, 3).$ 

The Second example considered is the 21-cell cellular system (HEX1-HEX4) found in [4] (Fig.1). We used two sets of demands for the 21-cell as follows.

 $D_1^T = (2, 6, 2, 2, 2, 4, 4, 13, 19, 7, 4, 4, 7, 4, 9, 14, 7, 2, 2, 4, 2);$ 

3

yes

 $D_2^T = (1, 1, 1, 2, 3, 6, 7, 6, 10, 10, 11, 5, 7, 6, 4, 4, 7, 5, 5, 5, 6).$ 

The details of HEX1-HEX4 are shown in Table 1 [8]. There are two different compatability matrices for HEX problems by considering the first two rings of cells around a particular cell as interferences [8]. The first one does not include ACC, so the relative off-diagonal terms  $C_{ij}$  are 1. The second one includes ACC, the off-diagonal terms of  $C_{ij}$  in second matrix are 1 or 2 respectively corresponding to CCC and ACC.

Problem adjacent Ν Μ D co-channel  $C_{ii}$ 2137  $\overline{2}$ HEX1  $D_1$ yes no HEX2 2191 $D_1$ 3 yes yes HEX3 2121 $D_2$ 2yes no

yes

Table 1. Descriptions for hexagonal CAPs

The final set of CAP is generated from the topographical data of an actual  $24 \times 21$  km area around Helsinki, Finland [22]. Kunz calculated the traffic demand and interference relationships between the 25 regions around the base stations of this area. The compatability matrix is abtained from Kunz data as  $C_3$  [8]. The demand vector is:

 $D_3^T = (10, 11, 9, 5, 9, 4, 5, 7, 4, 8, 8, 9, 10, 7, 7, 6, 4, 5, 5, 7, 6, 4, 5, 7, 5).$ 

This benchmarking CAP is divided into four classes by considering only the first 10 regions (KUNZ1), 15 regions (KUNZ2), 20 regions (KUNZ3), and the entire area (KUNZ4) (Table 2) [8].

#### 4.2 The Simulation Results Of SCSA

HEX4

21 56  $D_2$ 

We chose a set of parameters for each CAP as in Table 3.

Problem	Ν	Μ	С	D
KUNZ1	10	30	$[C_3]_{10}$	$[D_3]_{10}$
KUNZ2	15	44	$[C_3]_{15}$	$[D_3]_{15}$
KUNZ3	20	60	$[C_3]_{20}$	$[D_3]_{20}$
KUNZ4	25	73	$C_3$	$D_3$

Table 2. Descriptions for KUNZ problems

Our results are presented in Table 4. For comparison, Table 4 also includes results given in [8], i.e., the performances of GAMS/MINOS-5 (labeled GAMS), the traditional heuristics of steepest descent (SD), simulated annealing (SA), the original Hopfield network (HN) (with no hill-climbing), hill-climbing Hopfield

Problem	K	ε	$I_0$	$\alpha$	$\beta$	z(0)	n(0)	W1	W2
EX1	0.9	1/250	0.65	0.0045	0.0005	0.1	0.5	1.0	0.02
$\mathbf{EX2}$	0.9	1/250	0.65	0.0045	0.0005	0.1	0.5	1.0	0.02
HEX1	0.9	1/250	0.05	0.05	0.0005	0.08	0.08	1.0	0.25
HEX2	0.9	1/250	0.08	0.05	0.0005	0.08	0.08	1.0	0.25
HEX3	0.9	1/250	0.05	0.05	0.0005	0.08	0.08	1.0	0.2
HEX4	0.9	1/250	0.07	0.05	0.0005	0.08	0.08	1.0	0.3
KUNZ1	0.9	1/250	0.06	0.05	0.0004	0.08	0.08	1.0	0.45
KUNZ2	0.9	1/250	0.01	0.05	0.0005	0.08	0.08	1.0	0.45
KUNZ3	0.9	1/250	0.05	0.05	0.0005	0.08	0.08	1.0	0.45
KUNZ4	0.9	1/250	0.05	0.05	0.0005	0.08	0.08	1.0	0.45

**Table 3.** The parameters for various CAPs

network (HCHN), and the self-organizing neural network (SONN). Each of the heuristics is run from ten different random initial conditions. In Table 4, "Min" means the minimum total interference (eq. 4) found during these ten times, and "Av" is the average total interference [8]. We also computed the standard deviations (STDD) during these ten runs, as shown in Table 5.

	GAMS	S	D	SA		HN		HCHN		SONN		SCSA	
problem	Min	Av.	Min	Av.	Min	Av.	Min	Av.	Min	Av.	Min	Av.	Min
EX1	2	0.6	0	0.0	0	0.2	0	0.0	0	0.4	0	0.0	0
EX2	3	1.1	0	0.1	0	1.8	0	0.8	0	2.4	0	0.0	0
HEX1	54	56.8	55	50.7	49	49.0	48	48.7	48	53.0	52	47.7	47
HEX2	27	28.9	25	20.4	19	21.2	19	19.8	19	28.5	24	18.5	18
HEX3	89	88.6	84	82.9	79	81.6	79	80.3	78	87.2	84	77.3	76
HEX4	31	28.2	26	21.0	17	21.6	20	18.9	17	29.1	22	17.2	16
KUNZ1	28	24.4	22	21.6	21	22.1	21	21.1	20	22.0	21	20.0	19
KUNZ2	39	38.1	36	33.2	32	32.8	32	31.5	30	33.4	33	30.3	30
KUNZ3	13	17.9	15	13.9	13	13.2	13	13.0	13	14.4	14	13.0	13
KUNZ4	7	5.5	3	1.8	1	0.4	0	0.1	0	2.2	1	0.0	0

 Table 4. The results of SCSA and other techniques

The results in Table 4 show that the SCSA is able to further improve on results obtained by other approaches.

## 5 Conclusion

In this paper, we showed that stochastic chaotic simulated annealing (SCSA) [15] is very effective in solving combinatorial optimization problems, such as channel assignment problems in radio network planning. This approach has both noisy nature and chaotic

Problem	HEX1	HEX2	HEX3	HEX4	KUNZ1	KUNZ2	KUNZ3	KUNZ4
STDD	0.265	0.173	0.267	0.354	2.095	0.173	0.0	0.0

 Table 5. The standard deviations for various CAPs

searching characteristics, so it can search in a smaller space and continue to search after the disappearance of chaos. Implementation of SCSA to solve other practical optimization problems will be studied in future work.

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