A Fuzzy Neural Network for System Modeling

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Abstract

In this paper, we describe an incrementally generated fuzzy neural network (FNN) for fuzzy system modelling. This FNN combines the features of initial fuzzy model self-generation, fast input selection, partition validation, parameter optimisation and rule-base simplification. Experimental studies demonstrate that the FNN is able to achieve accuracy comparable to or higher than both a feedforward crisp neural network i.e. NeuroRule, and a decision tree i.e. C4.5, with more compact rule bases for most of the data sets used in our experiment. In addition, the FNN is insensitive to the problem of small disjuncts that affects decision trees. This can be very useful in real-world situations, since the data of interest can often be only a small fraction of the available data.

1. Introduction

System modeling has been an important issue in both engineering and non-engineering areas. To overcome the problems confronted by conventional modeling methods, fuzzy neural modeling have been proposed as viable alternatives and successfully employed in various areas where conventional approaches fail to provide satisfactory solutions.

In the literature, crisp neural networks often have a fixed architecture, i.e., a pre-determined number of layers with pre-determined numbers of neurons. The weights are usually initialized to small random values. Knowledge-based networks [4], [11] use crude domain knowledge to generate the initial network architecture. This helps in reducing the search space and time required for the network to find an optimal solution. There have also been mechanisms to generate crisp neural networks from scratch, i.e., initially there are no neurons or weights which are generated and then refined during training. For example, Mezard and Nadal's tiling algorithm [9], Fahlman and Lebiere's cascade-correlation [3], and Giles et al's constructive learning of recurrent networks [5] are very useful.

For FNNs, it is also desirable to shift from the traditional fixed architecture design methodology [13] to self-generating approaches. Higgins and Goodman [6] proposed an algorithm to create an FNN according to input data. New membership functions are added at the point of maximum error on an as-needed basis, which will be adopted in the present paper. Juang and Lin [14] proposed a self-constructing FNN with on-line learning. New membership functions are added based on input-output space partitioning using a self-organizing clustering algorithm. Chao and Chen [2] applied fuzzy similarity measure to eliminate redundant fuzzy logic rules.

Frayman and Wang [15] proposed a FNN using fuzzy similarity measure to remove irrelative inputs and rule applicability coefficient was used for rule-base simplification. We proposed a new FNN, with much simpler and effective input selection and rule-base simplification abilities.

The paper is organized as follows. Section 2 describes the implementation of our FNN. Some experimental results on data classification using our FNN and comparisons with the pruned feedforward crisp neural network and decision tree approaches are shown in section 3. Section 4 concludes the paper.

2. Implementation of Our FNN

The structure of our FNN is shown in Figure1. The network consists of four layers, i.e., the input layer, the input membership function layer, the rule layer, and the output layer [15].

In databases, data fields are either numerical or categorical. The input membership function layer generates input membership functions for numerical inputs, i.e., numerical values are converted to categorical values.

Each rule node is connected to all input membership function nodes and output nodes for this rule. Each rule node performs a product of its inputs. The input membership functions act as fuzzy weights between the input layer and the rule layer. Links between the rule layer, the output layer and the input membership functions are adaptive during learning. In the output layer each node receives inputs from all rule nodes connected to this output node and produces the actual output of the network.

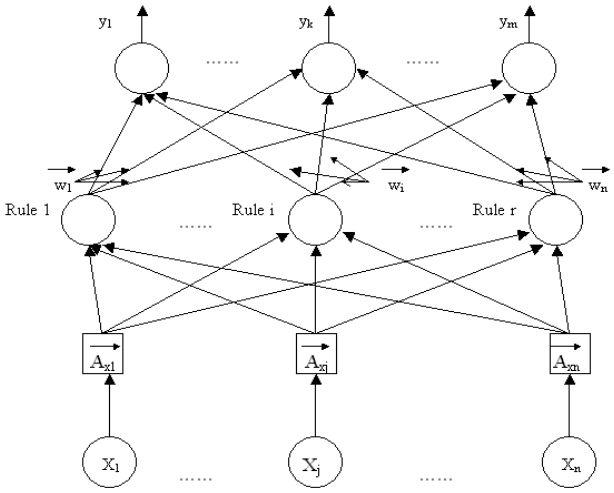

Figure 1: Structure of FNN

The structure generation and learning algorithm of the FNN are as follows in Figure 2.

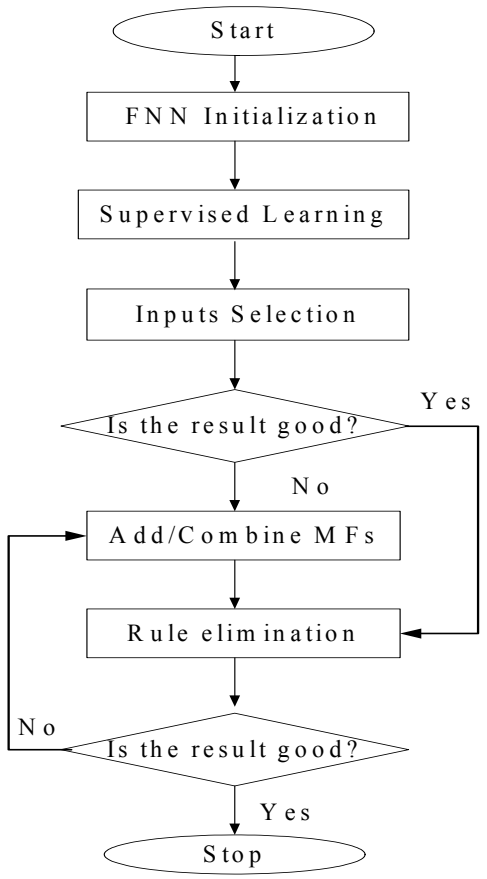

Figure 2: Flow chart of the FNN algorithm

2.1 FNN Initialization

Firstly we create n nodes for the input layer and m nodes for the output layer, where n and m are the number of the input variables (attributes) and the output variables (classes), respectively. The rule layer is empty, i.e., there are initially no rules in the rule base [15].
Two equally spaced triangular membership functions are added along the operating range of each input variable. In such a way these membership functions will satisfy ε-completeness. Piecewise-linear triangular membership function is chosen for computational efficiency [6].

Then we create the initial rule base layer using the following form for rule *i*:

$$\text{Rule } i\text{: IF } x_1 \text{ is } A^i_{x1} \text{ and } \ldots x_n \text{ is } A^i_{xn} \quad \text{Then } y_1=\omega_1^i,\ldots,y_m=\omega_m^i, \quad \text{---- (1)}$$

where x_j (j=1,2,.., n), and y_l (l=1,2,.., m) are the inputs and the outputs, respectively. ω_l^i is a real number. A^i_q (q=x_1, x_2,… x_n) is the membership function of the antecedent part of rule *i* for node *q* in the input layer[16].

The membership value μ_i of the premise of the *i*th rule is calculated as fuzzy AND, using the product operator

$$\mu_i = A_{x1}{}^i(x_1)\times A_{x2}{}^i(x_2)\times\cdots\times A_{xn}{}^i(x_n) \quad \text{---- (2)}$$

The output y_l of the fuzzy inference is obtained using the weighted average [17].

$$y_l=\frac{\sum_i \mu_i \times \omega_l^i}{\sum_i \mu_i} \quad \text{----(3)}$$

2.2 FNN Training

The network is trained using the following general learning rule [18].

$$y_l^i(k+1) = y_l^i(k) - \eta\frac{\partial \varepsilon_l}{\partial y_l^i} \quad \text{----(4)}$$

The learning rules for ω_l^i and A^i_j are:

$$\omega_l^i(k+1) = \omega_l^i(k) - \eta\frac{\partial \varepsilon_l}{\partial \omega_l^i} \quad \text{----(5)}$$

$$A_q^i(k+1) = A_q^i(k) - \eta\frac{\partial \varepsilon_l}{\partial A_q^i} \quad \text{----(6)}$$

where η is the learning rate. The objective is to minimize an error function

$$\varepsilon_l = \frac{1}{2}\times(y_l - y_{dl})^2 \quad \text{----(7)}$$

where y_l is the current output, and y_{dl} is the target output.

We let the learning rate η vary to improve the speed of convergence, as well as the learning performance (accuracy). We update η according to the following two heuristic rules:

1) If the error measure undergoes five consecutive reductions, increase η by 5%.
2) If the error measure undergoes three consecutive combinations of one increase and one reduction, decrease η by 5%.

Furthermore, due to this dynamical update strategy, the initial value of η is usually not critical as long as it is not too large.

The learning error ε_l is reduced towards zero or a pre-specified small value $\varepsilon_{def} > 0$ as the iteration number k increases.

2.3 Input Selection

Based on the initial fuzzy model that incorporates all possible input variables, we can evaluate the importance of each input variable. The objective of this work is to reduce the input dimensionality of the model without significant loss in accuracy. Elimination of redundant input features may even improve accuracy.

It is known that the change of system output is contributed to by all input variables. The larger the output change caused by a specified input variable, the more important this input may be. The fuzzy inference system provides an easy mechanism to test the importance of each input variable without having to generate new models [7]. The basic idea is to let the antecedent clauses associated with a particular input variable i in the rules be assigned a truth-value of 1 and then compute the fuzzy output, which is due to the absence of input i. Then we rank those fuzzy outputs from worst to best, and the worse result indicates the associated input is most important.

Further more, we have to decide how many inputs should be selceted. We start from using the most important input as the only input to the FNN, setting antecedents associated with all other inputs to 1. In the following steps, each time we add one more input according to the ranking order. The subset with best output result is selected as the inputs group; other inputs and associated membership functions are deleted. This approach is proved to be very simple, fast and effective.

2.4 Partition Validation

If the degree of overlapping of membership functions is greater than a threshold, we need to combine them [2]. We use the following *fuzzy similarity measure* [19]

$$E(A_1, A2) = \frac{M(A_1 \cap A_2)}{M(A_1 \cup A_2)} \qquad \text{----(8)}$$

where $\cap$ and $\cup$ denote the intersection and union of two fuzzy sets A_1 and A_2, respectively. M (•) is the size of a fuzzy set, and $0 \le E(A_1, A2) \le 1$.

From the formula above, we find that the computation of the similarity of two fuzzy sets requires calculating the size of intersection and union of two triangular membership functions. As triangular membership functions are used in our FNN, it makes such calculations simple and straightforward.

If an input variable ends up with only one membership function, which means that this input is irrelevant, we delete the input. We can thus eliminate irrelevant inputs and reduce the size of the rule base. If the classification accuracy of the FNN is below the requirement, and the number of rules is less than the specified maximum, we modify the rule base following the method introduced in next section.

2.5 Rule Base Modification

An additional membership function is added for each input at its value at the point of the maximum output error, following Higgins and Goodman [6]. One vertex of the additional membership function is placed at the value at the point of the maximum output error and has the membership value unity; the other two vertices lie at the centers of the two neighboring regions, respectively, and have membership values zero. As the output of the network is not a binary 0 or 1, but a continuous function in the range from 0 to 1, by firstly eliminating the error whose deviation from the target value is the greatest, we can speed up the convergence of the network substantially.

The rules generated above are then evaluated for accuracy and simplicity. We use a weighting parameter between accuracy and simplicity, which is the compatibility grade (*CG*) of each fuzzy rule. *CG* of rule j is calculated by the product operator as:

$$\mu_j(x) = \mu_{j1}(x_1) \times \mu_{j2}(x_{21}) \times \cdots \times \mu_{jn}(x_n) \quad \text{---- (9)}$$

when the system provides correct classification result.

All rules whose *CG* falls below a pre-defined threshold are deleted. Elimination of rule nodes is rule by rule, i.e., when a rule node is deleted, its associated input membership nodes and links are deleted as well. By varying the *CG* threshold the user is able to specify the degree of rule base compactness. The size of the rule base can thus be kept minimal. If the classification accuracy of the FNN after the elimination of rule nodes is below the prescribed

requirement we will add another rule as described above, otherwise we stop the process.

Our FNN contains the powerful features of initial fuzzy model self-generation [16], fast input selection, partition validation [2], [6], parameter optimisation [18] and rule-base simplification, and combines them together to achieve better performance. Our fast inputs selection is added as an effective supplementation, which is much simpler than others' method, such as [14], [15].

3. Experimental Evaluation

To test the FNN we used ten classification problems of different complexity defined by R.Agrawal in [1] on synthetic database with nine attributes. Attributes elevel, car and zipcode are categorical, and all others are non-categorical.

The first classification problem has predicates on the values of only one attribute. The 2nd to 3rd problems have predicates with two attributes, and the 4th to 6th problems have predicates with three attributes. Problems 7 to 9 are linear functions and problem 10 is a non-linear function of attribute values.

For comparisons with the FNN, we used decision tree construction algorithms C4.5 and C4.5rules [10] on the same data sets. C4.5 and C4.5rules Release 8 from the pruned trees with default parameters were used. We also compared our results with those of a pruned feedforward crisp neural network (NeuroRule) [8] for the classification problems.

Table 1 shows the accuracy on each test data set and the number of rules for all three approaches for the problems.

Func	Accuracy (%)			Number of Rules		
	NR	C4.5	FNN	NR	C4.5	FNN
1	99.9	100	100	2	3	2
2	98.1	95.0	93.6	7	10	5
3	98.2	100	97.6	7	10	7
4	95.5	97.3	94.3	13	18	8
5	97.2	97.6	97.1	24	15	7
6	90.8	94.3	95.0	13	17	8
7	90.5	93.5	95.7	7	15	13
8	N/A	98.6	99.3	N/A	9	2
9	91.0	92.6	94.1	9	16	15
10	N/A	92.6	96.2	N/A	10	8

Table 1 Experimental Results 1

Compared to NeuroRule, the FNN produces rule bases of less complexity for all problems, except problem 7 and 9. The FNN gives better accuracy for problems 6, 7 and 9, and comparable but lower accuracy for the rest. Compared to C4.5rules, the FNN gives less complex rules for all problems. The FNN gives higher accuracy than C4.5rules on problems 6 to 10, and comparable but lower accuracy for the rest. The results for problems 8 and 10 were not reported in [8] (indicated by "N/A" in Table 1).

If very high accuracy is required, without consideration of compactness of rule base, we can set a lower threshold for rule elimination stage. From Table 2, it is found that we got higher accuracy compared with both methods in most of problems, except in problem 3 and 5. However, rule base becomes much larger than results in Table 1, which may make the analysis of database more difficult.

Func	Accuracy (%)			Number of Rules (%)		
	NR	C4.5	FNN	NR	C4.5	FNN
1	99.9	100	100	2	3	11
2	98.1	95.0	98.5	7	10	25
3	98.2	100	99.3	7	10	36
4	95.5	97.3	98.1	13	18	120
5	97.2	97.6	97.2	24	15	81
6	90.8	94.3	96.5	13	17	50
7	90.5	93.5	97.7	7	15	52
8	N/A	98.6	99.1	N/A	9	15
9	91.0	92.6	95.2	9	16	65
10	N/A	92.6	97.7	N/A	10	50

Table 2 Experimental Results 2

4. Discussion and Conclusions

In Table 1, we attempted to obtain the most compact rule base for the FNN to allow for easy analysis of the rules for very large databases, and subsequently easy decision making in real-world situations. If accuracy is more important than compactness of the rule base, it is possible to use the FNN with more strict accuracy requirements, i.e., a lower threshold for pruning the rule base, thereby producing more accurate results at the expense of a higher rule base complexity. The final decision regarding complexity versus accuracy of the rules is application specific.

Moreover, the performance of the FNN is significantly better for classes with relatively small amounts of available data, in comparison with C4.5rules, i.e., the FNN approach to data mining is insensitive to *the problem of small disjuncts*, in contrast to decision tree approaches such as C4.5rules [12]. This is due to the fact that the IG-FNN treats all classes with equal importance, while decision trees give preference to more commonly occurred classes and treat classes with less data as less important. This

property of the FNN can be very useful for real-world situations, e.g., medical diagnosis, where important information may be contained in a small fraction of the available data.

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