APPLICATIONS OF TRANSIENTLY CHAOTIC NEURAL NETWORKS TO IMAGE RESTORATION

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ABSTRACT

Transiently chaotic neural network with continuous neural states is implemented to restore gray level images. The neural network is modeled to represent the image whose gray level function is the simple sum of the neuron state variables. The restoration consists of two phases: parameter estimation and image reconstruction. During the first phase, parameters are estimated by comparing the energy function of the neural network to a constraint error function. The neural network is updated using stochastic chaotic simulated annealing. Hopfield neural network is also implemented to compare the results. Experiments show that transiently chaotic neural network could get good results in much shorter time compared to Hopfield neural network.

1. INTRODUCTION

Image restoration has great application in various fields of signal processing. As most imaging systems are imperfect and they tend to produce noisy blurred images. Image restoration seeks to recover the true image using objective criteria and some prior knowledge about the original image. Restoration techniques are applied to remove 1) system degradations such as blur due to optical system aberrations, atmospheric turbulence, motion and diffraction; and 2) statistical degradations due to noise.

Various methods have been proposed for image restoration, such as inverse filter, Wiener filter [1], Kalman filter [2]. One of the major drawbacks of most of the image restoration algorithms is the computational complexity, so much so that many simplifying assumptions such as wide sense stationary (WSS), availability of second-order image statistics have been made to obtain computationally feasible algorithms.

The inverse filter method works only for extremely high signal-to-noise ratio images. The wiener filter is usually implemented only after the wide sense stationary assumption has bee made for images. Furthermore, knowledge of the power spectrum or correlation matrix of the undegraded image is required. Often times, additional assumptions regarding boundary conditions are made so that fast orthogonal transforms can be used. The Kalman filter approach can be applied to nonstationary image, but is computationally very intensive. It is desirable to develop a restoration algorithm that does not make WSS assumptions and can be implemented in a reasonable time.

An artificial neural network system that can perform extremely rapid computations seems to be very attractive for image restoration in particular and image processing and pattern recognition in general. Zhou et al [3] proposed a method using a neural network model containing redundant neurons with discrete neuron states to restore gray level images degraded by a known shift-invariant blur function and noise. Paik and Katsaggelos [7] modified the model to eliminate the energy check steps by ensuring the energy always decreases during neural updating so that the network eventually converges to a fixed point. However different fix points represent different image quality. Starting from the same initial state the modified neural network may converge to different quality with different neural updating sequences. Sun [8] [9] proposed a generalized updating rule (GUR) and eliminating-highest-error (EHE) criterion to ensure network convergence and improve quality by trying to find the best neural updating sequence. However EHE is very time consuming as it tries to find the neuron that could mostly decrease the energy function before each neural updating step.

Simulated annealing is a general method for obtaining approximate solutions of combinational optimization problems. Since the optimization processing is undertaken in a stochastic manner, it is also called stochastic simulated annealing (SSA). In recent years, a large amount of work has been done in chaotic simulated annealing (CSA). CSA can search efficiently because of its reduced search spaces. Chen and Aihara [4] proposed a transiently chaotic neural networks (TCNN) which adds a large negative self-coupling with slow damping in the Euler approximation of the continuous Hopfield neural network so that neurodynamics eventually converge from strange attractors to an equilibrium point. The TCNN showed good performance in solving traveling salesman problem (TSP). In this paper we try to solve image restoration problem using TCNN.

Comparing the restoration results requires a measurement of image quality. One commonly used measurement is peak signal-to-noise ration. Another image quality measurement m1 is extracted from video quality measurements based on human perception [5]. The equation for m1 is given by

$$m1 = \left| \frac{SI[g'] - SI[g]}{SI[g']} \right| \quad (1)$$

where SI is the spatial information defined by

$$\mathrm{SI[F]=STD_{space}\{Sobel\ [F]\}} \quad (2)$$

STD_{space} is the standard deviation operator over the horizontal and vertical spatial dimensions in a frame, and Sobel is the sobel filter operation. We will use these two measurements to qualify our experimental results.

This paper is organized as follows. Section II reviews neural network representation for an image. In Section III the TCNN is reviewed, and the network parameters are estimated in Section IV. The restoration algorithm is presented in Section IV. Our experimental results appear in Section V, followed by concluding remarks.

2. NEURAL NETWORK REPRESENTATION FOR IMAGE

The neural network containing redundant neurons, which was proposed by Zhou et al [3], is used. The model consists of $L^2 \times M$ mutually interconnected neurons where L is the size of image and M is the maximum value of the gray level function. Let $V=\{v_{i,k}$ where $1\le i \le L^2$, $1\le k \le M\}$ be a state set of the neural network with $v_{i,k}$ denoting the state of the (i, k)th neuron. $U=\{u_{i,k}$ where $1\le i \le L^2$, $1\le k \le M\}$ denotes the internal state of neuron {i,k}. Let $T_{i,k;j,l}$ denote the strength (possibly negative) of the interconnection between neuron (i,k) and neuron (j,l). We require symmetry:

$$T_{i,k;j,l} = T_{j,l;\,i,k} \quad \text{for } 1\le i,j \le L^2 \text{ and } 1\le l,k \le M. \quad (3)$$

We also allow for neurons to have self-feedback, i.e., $T_{i,k;i,k} \neq 0$. In this model, each neuron (i,k) is randomly and asynchronously updated. The image is described by a finite set of gray level functions {x(i,j) where $1\le i,j \le L$} with x(i,j) denoting the gray level of the pixel (i,j). The image gray level function can be represented by a simple sum of the neural state variables as

$$x(i,j) = \sum_{k=1}^{M} v_{m,k} \quad (4)$$

where m=(i-1)L +j. By using the lexicographic notation, the image degradation model can be written as Y=HX+N, where H is the blur matrix corresponding to a blur function, N is the signal independent white noise, and X and Y are the original and degraded images, respectively.

The shift-invariant blur function can be written as a convolution over a small window. For instance, it takes the form

$$h(k,l) = \begin{cases} \frac{1}{2} & if \quad k=0, l=0 \\ \frac{1}{16} & if\ |k|,|l| \le 1, (k,l) \neq (0,0) \end{cases} \quad (5)$$

The blur matrix H will be a block Toeplitz or block circulant matrix. The block circulant matrix corresponding to above h can be written as:

$$H = \begin{bmatrix} H_0 & H_1 & 0 & \cdots & 0 & H_1 \\ H_1 & H_0 & H_1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ H_1 & 0 & 0 & \cdots & H_1 & H_0 \end{bmatrix} \quad (6)$$

where

$$H_0 = \begin{bmatrix} \frac{1}{2} & \frac{1}{16} & 0 & \cdots & 0 & \frac{1}{16} \\ \frac{1}{16} & \frac{1}{2} & \frac{1}{16} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ \frac{1}{16} & 0 & 0 & \cdots & \frac{1}{16} & \frac{1}{2} \end{bmatrix}$$

$$H_1 = \begin{bmatrix} \frac{1}{16} & \frac{1}{16} & 0 & \cdots & 0 & \frac{1}{16} \\ \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ \frac{1}{16} & 0 & 0 & \cdots & \frac{1}{16} & \frac{1}{16} \end{bmatrix}$$

3. TRANSIENTLY CHAOTIC NEURAL NETWORK

The transiently chaotic neural network is defined as follows [4]:

$$v_{ik}(t) = \frac{1}{1+e^{-u_{ik}(t)/\varepsilon}} \quad (7)$$

$$u_{ik}(t+1) = ku_{ik}(t) + \alpha\left(\sum_{j=1}^{L^2}\sum_{l=1}^{M} T_{ikjl} v_{jl} + I_{ik}\right) - z(t)(v_{ik}(t) - I_o) \quad (8)$$

$$z(t+1) = (1-\beta)\, z(t) \quad (9)$$

where

v_{ik} : output of neuron (i,k);

u_{ik} : internal state of neuron (i,k);

Tikjl= Tjlik; $\sum_{j=1, j\neq i}^{L^2} \sum_{l=1, l\neq k}^{M} T_{ikjl} v_{jl} + I_{ik} = -\partial E / \partial x_{ik}$:

connection weight from neuron (j, l) to neuron (i, k);

I_{ik} : input bias of neuron i, k;
k : damping factor of nerve membrane ($0 \le k \le 1$);
α : positive scaling parameter for inputs;
z(t): self-feedback connection weight or refractory strength ($z(t) \ge 0$);
I_o : positive parameter;
ε : steepness parameter of the output function ($\varepsilon > 0$);

Note when the "temperature" z(t) tends toward zero with time evolution, equations (7)-(9) eventually stabilize.

4. PARAMETER ESTIMATION

The neural model parameters including the interconnection strengths, and bias inputs can be determined using the energy function of the neural network. The energy function of the neural network can be written as [6]

$$E = -\frac{1}{2}\sum_{i=1}^{L^2}\sum_{j=1}^{L^2}\sum_{k=1}^{M}\sum_{l=1}^{M} T_{i,k;j,l} v_{i,k} v_{j,l} - \sum_{i=1}^{L^2}\sum_{k=1}^{M} I_{i,k} v_{i,k} \tag{10}$$

Image restoration seeks to recover the true image from the degraded image. In order to use the spontaneous energy-minimization process of the neural network, Zhou et al [3] reformulated the restoration process as one of the minimizing an error function with constraints defined as

$$E = \frac{1}{2}\left\| Y - H\hat{X} \right\|^2 + \frac{1}{2}\lambda \left\| D\hat{X} \right\|^2 \tag{11}$$

where $\|Z\|$ is the L_2 norm of Z and λ is a constant. Such a constrained error function is widely used in the image restoration problems. In general D is a high-pass filter. A common choice of D is a second-order differential operator, which can be approximated as a local window operator in the 2-D discrete case. For instance, D may be a Laplacian operator.

The first term in (11) is to seek an $\hat{X}$ such that H $\hat{X}$ approximates Y in a least squares sense. Meanwhile, the second term is a smoothness constraint on the solution $\hat{X}$. The constant λ determines their relative importance.

$\lambda=\infty$: $\hat{X}$ is smooth, but we ignore the data;

$\lambda=0$: $\hat{X}$ approximate X, but $\hat{X}$ may not be smooth.

Expanding (11) and then replacing x_i by (4), we have

$$E = \frac{1}{2}\sum_{p=1}^{L^2}\left(y_p - \sum_{i=1}^{L^2} h_{p,i} x_i \right)^2 + \frac{1}{2}\lambda \sum_{p=1}^{L^2}\left(\sum_{i=1}^{L^2} d_{p,i} x_i \right)^2$$

$$= \frac{1}{2}\sum_{i=1}^{L^2}\sum_{j=1}^{L^2}\sum_{k=1}^{M}\sum_{l=1}^{M}\sum_{p=1}^{L^2} h_{p,i} h_{p,j} v_{i,k} v_{j,l}$$

$$+ \frac{1}{2}\lambda \sum_{i=1}^{L^2}\sum_{j=1}^{L^2}\sum_{k=1}^{M}\sum_{l=1}^{M}\sum_{p=1}^{L^2} d_{p,i} d_{p,j} v_{i,k} v_{j,l}$$

$$- \sum_{i=1}^{L^2}\sum_{k=1}^{M}\sum_{p=1}^{L^2} y_p h_{p,i} v_{i,k} + \frac{1}{2}\sum_{p=1}^{L^2} y_p^2$$

By comparing the terms above to the corresponding terms in (10) and ignoring the constant term $\frac{1}{2}\sum_{p=1}^{L^2} y_p^2$, we can determine the interconnection strengths and bias inputs as

$$T_{i,k;j,l} = -\sum_{p=1}^{L^2} h_{p,i} h_{p,j} - \lambda \sum_{p=1}^{L^2} d_{p,i} d_{p,j} \tag{12}$$

$$I_{i,k} = \sum_{p=1}^{L^2} y_p h_{p,i} \tag{13}$$

where $h_{i,j}$ and $d_{i,j}$ are the elements of the matrices H and D respectively. Two interesting aspects of (12) and (13) should be pointed out: 1) the interconnection strengths are independent of subscripts k and l and the bias inputs are independent of subscript k, and 2) the self-connection $T_{i,k;j,l}$ is not equal to zero which requires self-feedback for neurons.

For an L $\times$ L image with M gray levels, L^2M neurons and $1/2L^4M^2$ interconnections are required to represent the image. The space complexity is $O(L^4M^2)$, which is difficult to represent on a conventional computer. However simplification is possible if the neurons are sequentially updated. As noted earlier, the interconnection strengths given in (12) are independent of subscripts k and l and the bias inputs given in (13) are independent of subscript k; the M neurons used to represent the same image gray level function have the same interconnection strengths and bias inputs. Hence, the dimensions of the interconnection matrix T and bias input matrix I can be reduced [3].

5. RESTORATIONS

Restoration consists of two steps: neural evaluation and image construction. Once the parameters $T_{i,k;j,l}$ and $I_{i,k}$ are obtained using (12) and (13), each neuron can randomly and asynchronously evaluate and update its state using (8) and (7) accordingly. When one quasi-minimum energy point is reached, the image can be constructed using (4).

The neural network energy does not decrease monotonically. To successfully restore the image the neural network has to find an optimal solution, which corresponding to the minimum energy of the network.

Define the state change $\Delta v_{i,k}$ of the neuron (i,k) and energy change ΔE as

$$\Delta v_{i,k} = v_{i,k}^{new} - v_{i,k}^{old} \quad \text{and} \quad \Delta E = E^{new} - E^{old}$$

Consider the energy function in (10), the change ΔE due to a change $\Delta v_{i,k}$ is given by

$$\Delta E = -\left(\sum_{j=1}^{L^2} \sum_{l=1}^{M} T_{i,k;j,l} v_{j,l} + I_{i,k} \right) \Delta v_{i,k} - \frac{1}{2} T_{i,k;i,k} \left(\Delta v_{i,k} \right)^2 \tag{14}$$

At any one time only one neuron is updating. The new state $v_{i,k}^{new}$ of neuron (i,k) is calculated according to (7)-(9), and the energy change ΔE is calculated according to (14). If $\Delta E \leq 0$, then new state of neuron (i,k) $v_{i,k}^{new}$ is accepted, and the network with new neural state is used as the starting point of the next step. Then case $\Delta E \geq 0$ is treated probabilistically: then probability that the new neural state $v_{i,k}^{new}$ is accepted is $P(\Delta E) = \exp(-\Delta E / k_B T)$. Random numbers uniformly distributed in the interval (0,1) are a convenient means of implementing then random part of the algorithm. One such number is selected and compared with $P(\Delta E)$. If it is less than $P(\Delta E)$, then new state is retained; if not, the original state is used.

The algorithm is summarized as below:

1) Set the initial state of the neurons.
2) Sequentially visit all numbers (image pixels). For each number, use (7)-(9) to update it repeatedly until there is no further change.
3) Check the energy function; if the energy variation is less than a limit, go to step 4); otherwise go back to step 2).
4) Construct the image using (4).

6. EXPERIMENTAL RESULTS

Experimental results presented here demonstrate the performance of noisy chaotic neural network on image restoration. Performances of Hopfield neural network and transiently chaotic neural network are also shown.

We used Lena image of size 128 × 128 with 256 gray levels. The image was degraded by a 5 × 5 blur function:

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 8 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} / 32$$

Then we further degraded the image by adding a small amount of quantization noise.

The high pass filter we use to impose the smoothing constraint is

$$D = \begin{bmatrix} 1 & 4 & 1 \\ 4 & -20 & 4 \\ 1 & 4 & 1 \end{bmatrix} / 6$$

The neural network starts from three different initial states. The three initial states are randomly generated.

The simulation results are shown in Figure 1 and Table 1. The PSNR and m1 of the degraded image are 24.7036 and 0.2882, respectively.

Both neural networks improve the degraded image. The restoration results starting from the three random initial states are consistent. HNN takes more than 100 iterations to converge to a stable point, TCNN takes around 20 iterations to stabilize. HNN can improve the PSNR from 25 dB to 41 dB, TCNN to 37 dB. The other measurement, m1, shows consistent result with PSNR.

(a) (b) (c) (d)

Fig. 1. Restoration of noisy blurred real image. (a) Original image. (b) Blurred image. (c)-(d) Restored image using Hopfield neural network and transiently chaotic neural network, respectively.

Table 1. Restoration results in terms of PSNR and m1

		Iterations	PSNR	m1
Random initial state1	HNN	107	41.4912	0.0132
	TCNN	24	37.1758	0.0309
Random initial state2	HNN	106	41.483	0.0132
	TCNN	21	37.1433	0.0313
Random initial state3	HNN	106	41.4678	0.0133
	TCNN	26	37.2336	0.0309

7. CONCLUSION

In this report we have implemented Hopfield neural network and transiently chaotic neural network to restore an image. The restoration procedure consists of two phases: parameter estimation and image reconstruction. The restored image is generated iteratively by updating the neurons representing the image gray levels via a simple sum scheme. The networks were updated in stochastic manners. From the simulation results we see that transiently chaotic neural network is capable of generating good restoration results in much shorter time compared to Hopfield neural.

8. REFERENCES

[1] H. C. Andrews and B. R. Hunt, *Digital Image Restoration.* Englewood Cliffs, NJ: Prentice-Hall, 1977

[2] J. W. Woods and V. K. Ingle, "*Kalman filtering in two dimensions: Further results,*" IEEE Trans. Acoust., Speech, Signal Processing, vol. ASSP-29, pp. 188-197, Apr. 1981

[3] Y.T. Zhou, R. Chellappa, and B.K. Jenkins, "*Image Restoration Using a Neural Network*", IEEE Trans. Acoust,. Speech, Sign. Proc., vol. 36, pp. 38-54, 1988

[4] L. Chen and K. Aihara, "*Chaotic simulated annealing by a neuralnetwork model with transient chaos,*" Neural Networks, vol. 8, no. 6, pp. 915--930, 1995

[5] L. Wang and F. Tian, "*Noisy chaotic neural networks for solving combinational optimization problems*", Proc. International Joint Conference on Neural Networks, IJCNN2000

[6] A. A. Webster, Coleen T. Jones, Margaret H. Pinson, Stephen D. Voran and Stephen Wolf, "*An objective video quality assessment system based on human perception*", SPIE 1913, Human Vision, Visual Processing and Digital Display, pp. 15-26, 1993

[7] J. J. Hopfield and D. W. Tank, "*Neural computation of decisions in optimization problems,*" Biol. Cybern., vol. 52, pp. 141-152, 1985