

# A Noisy Chaotic Neural Network Approach for Broadcast Scheduling Problem in Packet Radio Networks

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## Abstract

*Broadcast scheduling problem (BSP) in packet radio(PR) networks is proved to be NP-complete. The goal of this problem is to find a conflict free transmission schedule in a fixed-length time-slots. This paper presents a novel algorithm called "Noisy Chaotic Neural Network (NCNN)" to solve this problem. A two-phase optimization is adopted with two different energy functions with which NCNN not only find the minimal TDMA cycle length but also maximize the node transmissions. The performance is evaluated through several benchmark examples. The results show that NCNN always finds better TDMA cycles than existing algorithms, such as mean filed annealing, mixed algorithm of Hopfield neural network and genetic algorithm, sequential vertex coloring algorithm and the gradually neural network.*

## 1 Introduction

Packet Radio (PR) network [1] provide a good option for high-speed wireless data communications, especially over a broad geographic region. It uses shared radio channels as the broadcast medium to interconnect stations or nodes. In order to avoid any collision and interference, a time-division multiple-access (TDMA) protocol has been used to schedule conflict free transmissions. A TDMA cycle is divided into distinct frames consisting of a number of time slots. A time slot has a unit time to transmit one data packet between adjacent nodes. At each time slot, each node can either transmit or receive a packet, but no more than two packets can be received from neighbor nodes. If a node is scheduled to both transmit and receive at the same time slot, a *primary* conflict occurs. If two or more packets reach one node at the same time slot, a *second* conflict occurs.

The BSP has been extensively studied [2]- [5]. In [2], Funabiki and Takefuji proposed a parallel algorithm based on an artificial neural network in a TDMA cycle with  $n \times m$  neurons. In [3], Wang and Ansari proposed a mean field annealing algorithm to find a TDMA cycle with the minimum delay time. In [4], Chakraborty and Hirano used genetic algorithm with a modified crossover operator to handle large networks with complex connectivity. In [5], Funabiki and Kitamichi proposed a binary neural network with a gradual expansion scheme to find minimum time slots and maximum transmissions through a two-phase process. In [6], Yeo *et al* proposed a algorithm based on the sequential vertex coloring algorithm. In [7], Salcedo-Sanchez *et al* proposed a mixed algorithm which combines a Hopfield neural network for constrain satisfaction and a genetic algorithm for achieving a maximal throughput. In this paper, we present a novel neural network model with complex neurodynamics, i.e., noisy chaotic neural network (NCNN). Numerical results show that this NCNN method outperforms existing algorithms in both the average delay time and the minimal TDMA length. The organization of this paper is as follows. In section 2, we formulate the broadcast scheduling problem. The Noisy Chaotic Neural network (NCNN) model is proposed in section 3. In section 4, NCNN is applied to solving the optimal scheduling problem. Numerical benchmark instances are stated and the performance is evaluated in section 5. Section 6 is the conclusion.

## 2 Problem Formulation

A packet radio network can be described by a graph  $G = (I, E)$  where  $I$  is the set of vertices in graph  $G$  and  $E$  is the set of edges. Each edge represents a link between two nodes. If  $u, v \in I$ , then the edge  $e = (u, v) \in E$  if and only if  $u$  can receive a transmission from  $v$ , or vice versa. Here

we consider only undirected graphs and the matrix  $c_{ij}$  is symmetric, i.e.,  $c_{ij} = c_{ji}$ , ( $i, j = 1, 2, \dots, n$ ). If two nodes are adjacent with  $c_{ij} = 1$ , then we define two nodes to be one-hop-away, and the two nodes with the same neighboring node to be two-hop-away. An  $N$ -node undirected graph can be represented by an  $N \times N$  connectivity matrix  $C = c_{ij}$  defined as:

$$c_{ij} = \begin{cases} 1 & \text{if node } i \text{ and node } j \text{ are adjacent,} \\ 0 & \text{otherwise.} \end{cases}$$

We form a new  $N \times N$  matrix called compatibility matrix  $D = \{d_{ij}\}$  from connectivity matrix  $C = \{c_{ij}\}$  and defined below. Note that  $c_{ii} = 0$  and  $d_{ii} = 0, \forall i$ . Matrix  $C$  and  $D$  are both symmetric, i.e.,  $c_{ij} = c_{ji}$  and  $d_{ij} = d_{ji}$ .

$$d_{ij} = \begin{cases} 1 & \text{If node } i \text{ and node } j \text{ are one-hop-away} \\ & \text{or two-hop-away,} \\ 0 & \text{otherwise.} \end{cases}$$

The final optimal solution for a  $N$ -node network is a conflict-free transmission schedule consisting of  $M$  time slots. Additional transmissions can be arranged provided that the transmission does not violate the constraints. We use an  $M \times N$  binary matrix  $V = (v_{ij})$  to express such a schedule [3], where

$$v_{ij} = \begin{cases} 1 & \text{if node } i \text{ transmits in slot } j \text{ in a frame,} \\ 0 & \text{otherwise.} \end{cases}$$

One of the indices to evaluate the solution quality is the average time delay  $\eta$  for each node to broadcast packets [5], where the  $\eta$  is defined as:

$$\eta = \frac{1}{N} \sum_{i=1}^N \left( \frac{M}{\sum_{j=1}^M v_{ij}} \right) = \frac{M}{N} \sum_{i=1}^N \left( \frac{1}{\sum_{j=1}^M v_{ij}} \right) \quad (1)$$

where  $M$  is the time-slot cycle length and  $v_{ij}$  is the neuron output.

The goal of BSP is to find a transmission schedule with the shortest TDMA frame length (i.e.,  $M$  should be as small as possible) which satisfied the above constraints, and the total number of node transmissions is maximized.

### 3 Noisy Chaotic Neural Network Model

Since Hopfield neural network (HNN) can be easily trapped in local minima, stochastic simulated annealing (SSA) technique has been combined with the HNN [9]. Chen and Aihara [8][9] proposed chaotic simulated annealing (CSA) by starting with a sufficiently large negative self-coupling in the neurons and then gradually decreasing the self-coupling to stabilize the network. They called this model the transiently chaotic neural network (TCNN). By adding decaying stochastic noise into the TCNN, Wang

and Tian [10] proposed a new approach to simulated annealing using a noisy chaotic neural network (NCNN), i.e., stochastic chaotic simulated annealing (SCSA). This neural network model has been applied successfully in solving several optimization problems including the travelling salesman problem (TSP) and the channel assignment problem (CAP) [10] [11]. The NCNN model is described as follows [10]:

$$x_{jk}(t) = \frac{1}{1 + e^{-y_{jk}(t)/\varepsilon}} \quad (2)$$

$$y_{jk}(t+1) = ky_{jk}(t) + \alpha \left( \sum_{i=1, i \neq j}^N \sum_{l=1, l \neq k}^M w_{jkil} x_{jk}(t) + I_{ij} \right) - z(t)(x_{jk}(t) - I_0) + n(t) \quad (3)$$

$$z(t+1) = (1 - \beta_1)z(t) \quad (4)$$

$$A[n(t+1)] = (1 - \beta_2)A[n(t)] \quad (5)$$

where

$x_{jk}$  : output of neuron  $jk$  ;

$y_{jk}$  : input of neuron  $jk$  ;

$w_{jkil}$ : connection weight from neuron  $jk$  to neuron  $il$ ,

with  $w_{jkil} = w_{iljk}$  and  $w_{jjkk} = 0$ ;

$$\sum_{i=1, i \neq j}^N \sum_{l=1, l \neq k}^M w_{jkil} x_{jk} + I_{ij} = -\partial E / \partial x_{jk} \quad \text{input to neuron } jk \quad (6)$$

$I_{jk}$  : input bias of neuron  $jk$  ;

$k$  : damping factor of nerve membrane ( $0 \leq k \leq 1$ );

$\alpha$  : positive scaling parameter for inputs ;

$\beta_1$  : damping factor for neuronal self-coupling ( $0 \leq \beta_1 \leq 1$ );

$\beta_2$  : damping factor for stochastic noise ( $0 \leq \beta_2 \leq 1$ );

$z(t)$  : self-feedback connection weight or refractory strength ( $z(t) \geq 0$ ) ;

$I_0$  : positive parameter;

$\varepsilon$  : steepness parameter of the output function ( $\varepsilon > 0$ ) ;

$E$  : energy function;

$n(t)$ : random noise injected into the neurons, in  $[-A, A]$  with a uniform distribution;

$A[n]$ : amplitude of noise  $n$ .

## 4 Solving BSP Using Noisy Chaotic Neural Network

### 4.1 Energy Function in Phase I

The energy function  $E_1$  for phase I is given as following:

$$E_1 = \frac{W_1}{2} \sum_{i=1}^N \left( \sum_{k=1}^M v_{ik} - 1 \right)^2 + \frac{W_2}{2} \sum_{i=1}^N \sum_{j=1}^M \sum_{k=1, k \neq i}^N d_{ik} v_{ij} v_{kj} \quad (7)$$

where  $W_1$  and  $W_2$  are weighting coefficients. The  $W_1$  term represents the constraints that each of  $N$  nodes must transmit exactly once during each TDMA cycle. The  $W_2$  term indicates the constraint that any pair of nodes which is one-hop away or two-hop away must not transmit simultaneously during each TDMA cycle.

From eqn. (3), eqn. (6), and eqn. (7), we obtain the dynamics of NCNN for the BSP as below:

$$y_{jk}(t+1) = ky_{jk}(t) + \alpha \left\{ -W_1 \left( \sum_{k=1}^M v_{ik} - 1 \right) - W_2 \left( \sum_{k=1, k \neq i}^N d_{ik} v_{kj} \right) \right\} - z(t)(x_{jk}(t) - I_0) + n(t) \quad (8)$$

In order to get a minimal number of frame length which satisfied the constrains, we use a *gradual expansion scheme* in which a initial value of frame length is set with a lower bound value of  $M$ . If with current frame length there is no feasible solution which satisfied the constrains, then this value is gradually increased by 1, i.e.,  $M = M + 1$ . The algorithm compute iteratively until every node can transmit at least once in the cycle without conflicts, then the algorithm stopped and the current value of  $M$  is the minimal frame length. In this way, the scheduled frame length would be minimized.

The lower bound  $L_m$  of the initial frame length can be found with many methods. In [5], the initial value is set as the maximum clique of the compatibility matrix. We use the same definition as in [3] and find the lower bound using the following steps:

1) Find the node with the maximum degree  $deg_i$  in all nodes. Here the node degree is defined as follows:

$$deg_i = \sum_{k=1, k \neq i}^N c_{ik} \quad (9)$$

2) Lower bound  $L_m$  is defined as:

$$L_m = \text{Max}(deg_i) + 1 \quad (10)$$

### 4.2 Energy Function in Phase II

In phase II, the objective is to maximize the total number of transmissions based on the minimal TDMA length  $M$  obtained in the previous phase.  $M$  and the scheduled transmissions in phase I are fixed. In order to take advantage of the results in phase I and . We use the energy function for phase II is defined as follow:

$$E_2 = \frac{W_3}{2} \sum_{i=1}^N \sum_{j=1}^M \sum_{k=1, k \neq i}^N d_{ik} v_{ij} v_{kj} + \frac{W_4}{2} \sum_{i=1}^N \sum_{j=1}^M (1 - v_{ij})^2 \quad (11)$$

Where  $W_3$  and  $W_4$  are coefficients.  $W_3$  represents the constraint term that any pair of nodes which is one-hop away or two-hop away must not transmit simultaneously during each TDMA cycle.  $W_4$  is the optimization term which maximized the total number of output firing neurons.

From eqn. (3), eqn. (6), and eqn. (11), we obtain the dynamics of NCNN for phase II of the BSP as follow:

$$y_{jk}(t+1) = ky_{jk}(t) + \alpha \left\{ -W_3 \left( \sum_{k=1, k \neq i}^N d_{ik} v_{kj} - 1 \right) + W_4 (1 - v_{ij}) \right\} - z(t)(x_{jk}(t) - I_0) + n(t) \quad (12)$$

In the above models of BSP, the network with  $N \times M$  neurons is updated cyclically and asynchronously. The new state information is immediately available for the other neurons in the next iteration. The iteration is terminated once a feasible transmission schedule is obtained, i.e., the transmission of all nodes are conflict free.

## 5 Simulation Results

We use two evaluation indices to compare with different algorithms. One is the TDMA cycle length  $M$ . The second is the average time delay  $\eta$  defined in eqn. (1). The another definition of average time delay can be found in [3] and [6] which is calculated with the Pollaczek-Khinchin formula [12], which models the network as  $N M/D/1$  queues. We will use both definitions in order to compare with other methods.

We choose the set of parameters in eqn. (7), eqn. (8), eqn. (11) and eqn. (12) as follow:

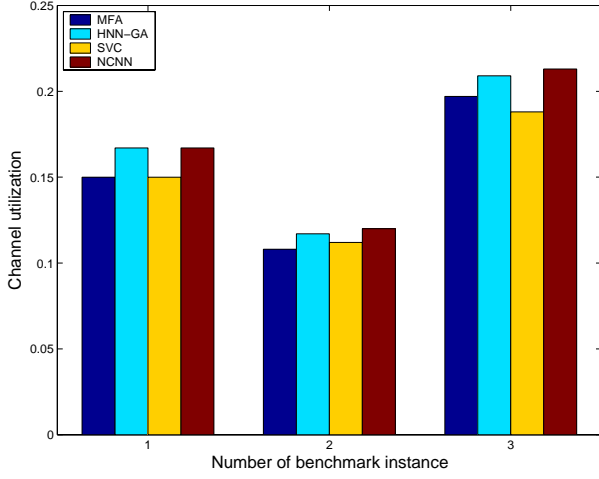


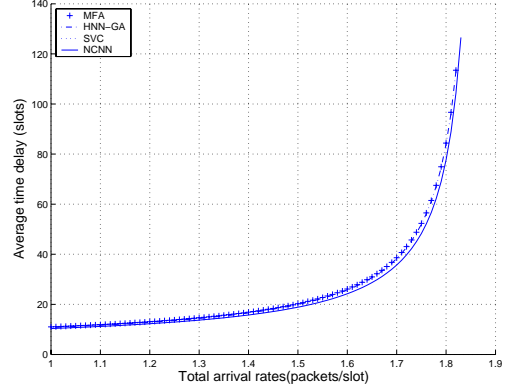
Figure 1: Comparisons of channel utilization for three benchmark problems. 1, 2, and 3 in the horizontal axis stand for instance with 15, 30, and 40 nodes, respectively.

$k, \alpha, \beta_1, \beta_2, \epsilon = 0.9, 0.015, 0.001, 0.0002, 0.004$ ;  $I_0 = 0.65$ ;  $z_0 = 0.08$ ,  $A[n(0)] = 0.009$ ;  $W_1, W_2, W_3, W_4 = 1, 1, 1, 1$

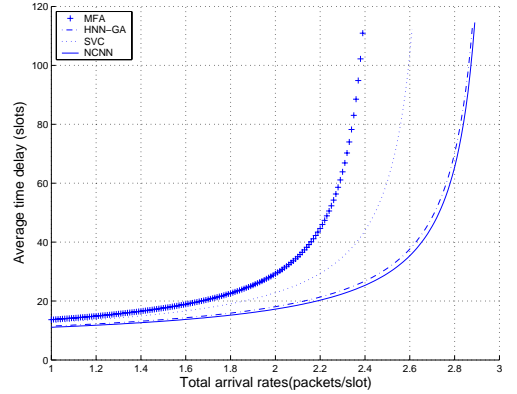
Three benchmark problems from [3] have been chosen to compared with other algorithms in [5],[6], and [7]. The three examples are instances with 15-node-29-edge, 30-node-70-edge, and 40-node-66-edge respectively.

The comparison of channel utilization for three benchmark problems is plotted in Fig. 1, which shows that the NCNN can find solutions with the highest channel utilization among all algorithms. The average time delay is plotted in Fig. 2. From this figure, it can be seen that the time delay experienced by the NCNN is much less than that of the MFA algorithm in all three instances. In the 15-node instance, the MFA, SVC and HNN-GA have the same delay, but the delay obtained by our NCNN is less than them, as we can see from Fig. 2(a). Also in other two instances, time delay obtained by the NCNN is less than the MFA, SVC and HNN-GA as we can see from 2(b), and 2(c).

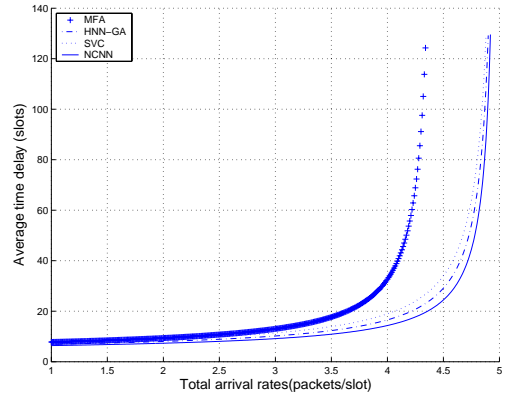
The computational results are summarized in Table 1 in comparison with the mixed HNN-GA algorithm from [7], sequential vertex coloring (SVC) from [6], gradual neural network (GNN) from [5] and mean field annealing (MFA) from [3]. In respect of the frame length, our algorithm can find optimal schedules for all three examples. In respect of the average time delay, our algorithm outperforms the all other algorithms in obtaining the minimal value of  $\eta$ .



(a)



(b)



(c)

Figure 2: Comparison of average time delay and channel utilization among different approaches for three benchmark problems: (a) 15-node, (b) 30-node, (c) 40-node.

	NCNN $\eta / M$	HNN-GA $\eta / M$	SVC $\eta / M$	GNN $\eta / M$	MFA $\eta / M$
#1	6.8 / 8	7.0 / 8	7.2 / 8	7.1 / 8	7.2 / 8
#2	9.0 / 10	9.3 / 10	10.0 / 10	9.5 / 10	10.5 / 12
#3	5.8 / 8	6.3 / 8	6.76 / 8	6.2 / 8	6.9 / 9

Table 1: Comparisons of average delay time  $\eta$  and time slot  $M$  obtained by the NCNN with other algorithms for the three benchmark problems given by [3].

## 6 Conclusion

In this paper, we present a noisy chaotic neural network model for solving broadcast scheduling problem in packet radio networks. This noisy chaotic neural network (NCNN) consists of  $N \times M$  neurons for the  $N$ -node- $M$ -slot problem. We evaluate the NCNN in three benchmark examples. We compare our results with existing methods including mean field annealing, HNN-GA, sequential vertex coloring algorithm, and the gradually neural network. The results of three benchmark instances show that NCNN always finds better solutions with minimal average time delay and maximal channel utilization.

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