A Mixed Branch-and-bound and Neural Network Approach for the Broadcast Scheduling Problem

Haixiang Shi and Lipo Wang
School of Electrical and Electronic Engineering
Nanyang Technological University
Block S2, Nanyang Avenue, Singapore

Abstract. In this paper we proposed a mixed method to solve the broadcast scheduling problem in packet radio networks. Due to the two objectives of this problem, a two-stage optimization process is adopted. In order to obtain a optimal time slot number, we use an exact method, branch-and-bound algorithm to search the whole solution space in the first stage and obtain the minimal TDMA cycle length. In the second stage, we use stochastic chaotic neural network to find the maximum node transmissions based on the fixed time slots obtained in previous stage. Results show that this mixed method outperforms previous approaches like Mean Filed Annealing, HNN-GA, Sequential Vertex Coloring algorithm (SVC) and Gradually Neural Networks

1 Introduction

A packet radio network is a network that consists of geographically distributed stations or nodes and provides flexible data communication services for them through a shared high-speed radio channel using broadcasting method. Since the communications spectrum is limited, an efficient broadcast scheduling scheme is necessary. Interference may occur when two transmitting nodes are in close proximate. In order to avoid the interference arising on the shared radio channel, a *time-division multiple-access* (TDMA) protocol has been adopted to obtain conflict-free transmissions. A TDMA cycle consist of many time slots which has unit time length require for a single packet to be communicated between adjacent nodes. At each time slot, any node may either transmit or receive a packet from its adjacent nodes. If a node is scheduled to perform both the transmitter and receiver at the same time slot, a *primary conflict* occurs there. If two or more packets reach one node at the same time slot, all of them become annulled by a *secondary conflict* In each TDMA cycle, every node is scheduled to transmit at least once without any conflicts. [5].

The goal of broadcast scheduling problem (BSP) is to find an optimal TDMA frame structure that fulfils the following two objectives. The first objective is to schedule the transmission of all nodes in a minimal TDMA cycle length without packet conflicts. The second objective is to maximize the total node transmission or channel utilization. The optimal frame structure is directly related to the main network performance measures [7]. Firstly, the TDMA length essentially determines the average packet delay. Secondly, for a fixed frame length, the channel utilization is determined by the number of simultaneous transmissions of non-interfering nodes.

The BSP is proven to be an NP-hard combinatorial optimization problem which has been extensively studied [2]- [7]. In [2], Funabiki and Takefuji proposed a parallel algorithm based on an artificial neural network in a TDMA cycle with $n \times m$ neurons. They used hill-climbing to help the system to escape from local minima. In [3], Wang and Ansari proposed a mean field annealing algorithm to find a TDMA cycle with the minimum delay time. They also proved the NP-completeness of the BSP by transforming it to the maximum independent set problem. In [4], Chakraborty and Hirano used genetic algorithm with a modified crossover operator to handle large networks with complex connectivity. In [5], Funabiki and Kitamichi proposed a binary neural network with a gradual expansion scheme to find minimum time slots and maximum transmissions through a two-phase process. They demonstrated the performance of their gradual neural network (GNN) through benchmark instances used in [3] and randomly generated geometric graph instances with up to 1000 vertices. Yeo *et al* [6] proposed a algorithm based on the sequential vertex coloring algorithm. In [7], Salcedo-Sanze *et al* proposed a mixed algorithm which combines a Hopfield neural network for constrain satisfaction and a genetic algorithm for achieving a maximal throughput. They compared their results with meaning field annealing in [3] for three benchmark problems, and showed that the mixed algorithm finds the optimal frame length and outperforms the MFA in the resulting throughput.

In this paper, we adopt a mixed method which combines the branch-and-bound and stochastic chaotic simulated annealing (BnB-SCSA) to obtain an optimal solution. The remainder of the paper is organized as follows. In section 2, we formulate the broadcast scheduling problem. In section 3, we proposed a mixed methods and use the two stage optimization to solve the optimal scheduling problem. Numerical benchmark instances are stated and the performance is evaluated in section 4. Section 5 is the conclusion.

2 Broadcast Scheduling Problem

A packet radio network can be described by a graph *G = (I, E)* where *I* is the set of vertices in graph *G* and *E* is the set of edges. Each edge represents a link between two nodes. If $u, v \in I$, then the edge $e = (u, v) \in E$ if and only if u can receive a transmission from v, or vice versa. Here we consider only undirected graphs and the matrix c_{ij} is symmetric, i.e., $c_{ij} = c_{ji}, (i, j = 1, 2, .., n)$. If two nodes are adjacent with $c_{ij} = 1$, then we define two nodes to be one-hop-away, and the two nodes with the same neighboring node to be two-hop-away. An *N*-node undirected graph can be represented by an $N \times N$ connectivity matrix $C = c_{ij}$ defined as:

$$c_{ij} = \begin{cases} 1 & \text{if node } i \text{ and node } j \text{ are adjacent,} \\ \\ 0 & \text{otherwise.} \end{cases}$$

We form a new $N \times N$ matrix called compatibility matrix $D = \{d_{ij}\}$ from connectivity matrix $C = \{c_{ij}\}$ and defined below. Note that $c_{ii} = 0$ and $d_{ii} = 0, \forall i$. Matrix C and D are both symmetric, i.e., c_{ij} = c_{ji} and d_{ij} = d_{ji}.

$$d_{ij} = \begin{cases} 1 & \text{If node } i \text{ and node } j \text{ are one-hop-away or two-hop-away,} \\ 0 & \text{otherwise.} \end{cases}$$

The three constrains can be deduced to ensure conflict-free transmission which list below:

- 1. *No-transmission constraint*. Each node should be scheduled to transmit at least once in a TDMA cycle.
- 2. *Primary conflict*. A node cannot have transmission and reception simultaneously.
- 3. *Secondary conflict*. A node is not allowed to received more than two transmissions simultaneously.

Based on the above constrains, the final optimal solution for a N-node network is a conflict-free transmission schedule consisting of M time slots. Additional transmissions can be arranged provided that the transmission does not violate the constrains. We use an $M \times N$ binary matrix $V = (v_{ij})$ to express such a schedule [3], where

$$v_{ij} = \begin{cases} 1 & \text{if node } i \text{ transmits in slot } j \text{ in a frame,} \\ 0 & \text{otherwise.} \end{cases}$$

One of the indices to evaluate the solution quality is the average time delay η for each node to broadcast packets [5], where the η is defined as:

$$\eta = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{M}{\sum_{j=1}^{M}}v_{ij}\right) = \frac{M}{N}\sum_{i=1}^{N}\left(\frac{1}{\sum_{j=1}^{M} v_{ij}}\right) \tag{1}$$

where M is the time-slot cycle length and v_{ij} is the neuron output.

The goal of BSP is to find a transmission schedule with the shortest TDMA frame length (i.e., M should be as small as possible) which satisfied the above constrains, and the average time delay for each node is minimized (i.e. the total number of node transmissions is maximized). The BSP can be described as bellow:

BSP Minimize M and η

Subject to:
$$\sum_{k=1}^{M} v_{ik} \geq 0 \tag{2}$$
$$\sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{k=1,k\neq i}^{N} d_{ik}v_{ij}v_{kj} = 0 \tag{3}$$

The primary constrain is formulated in eqn. (3), which means each node in network must transmit at least once. The secondary constrains in eqn. (4) indicates that every pair of nodes within two hops away cannot be scheduling in the same time slot.

3 Two Stage Optimization with the Mixed Algorithm

3.1 Stage I: Minimize TDMA Length

Branch and bound is implemented as a *tree search*, where the problem at the root node of the tree is the original problem. Each leaf node represents a feasible solution. The non-leaf node in the search tree represents a subset of feasible solutions. Parent-child relationship represents partition of search space into sub-spaces. The tree is constructed in an iterative fashion with new nodes formed by branching on an existing node. If in some sub-spaces, we can conclude that no feasible solutions can be found by bounding functions, then such sub-spaces will be abandoned and the computation complexity will be reduced. The basic idea of branch and bound is that if we can drastically reduce solution space from one level to next, total number of nodes explored will be reduced.

In order to get a minimal number of frame length which satisfied the constrains, we use a *gradual expansion scheme* in which a initial value of frame length is set with a low bound value. If with current frame length there is no feasible solution which satisfied the constrains, then this value is gradually increased by 1, i.e.,$M = M + 1$. The tree search algorithm restarted and stopped until a feasible frame schedule is obtained, which means every node can transmit at least once in a TDMA cycle. Then the current value of M is the minimal frame length and the current time-slot schedule is the optimal solution for the first stage optimization. The low bound value of initial frame length L_m can be found with many methods. Funabiki *et al* [5] choose the initial value of time slot as the maximum clique of the compatibility matrix D. In this paper, we use the same definition as in [3]:

$$L_m = Max(deg_i) + 1 \tag{4}$$

In above equation, $Max(deg_i)$ means the maximum node degree (deg_i) of all nodes. Where the node degree is defined as follow:

$$deg_i = \sum_{k=1, k \neq i}^{N} c_{ik} \tag{5}$$

The most important part in gradual expansion scheme is the rules to judge whether there is a feasible solution under a certain M value. With each fixed frame length, the branch and bound algorithm build the search tree. The root node of tree is the node with the maximal degree. A Breadth First Search is implemented to obtain a node sequence which will be used as the tree levels. The whole tree is a N-level corresponding to N-node network. Every neighbouring node of root is assigned to different slots. Thus the first M-level is assigned with the M-different time slots. From level M+1, non-conflicting time slots are found for each node. If the number of available time slots found for this node is exactly one, then set the time slot to this node and try the next tree level node. If the number is more than one then set this node with the first available time-slot and add the additional time slots as the brother node of this node. For each brother node, it means a alternative solution for possible combination of solutions. If the number of nonconflicting time slot is zero, this means in current sub-space, there is no feasible time slot schedule, then we stop further search in this sub-space and backtrack to other branches of trees. The backtracking is kept going until all nodes has a feasible schedule. If all alternative branches are tried and for some nodes still have no feasible time slot to assign. Under this condition, we define it is the situation to use gradual expansion scheme. Then, the current time slot number M is added by 1 and the algorithm restarted with the new value of time slot M. Because branch and bound searches among all possible combinations of time slots, furthermore, the searching is started with a gradual expansion scheme, it is guaranteed to get a minimal number of frame length.

3.2 Stage II: Maximize the channel utilization

In the second stage, the objective is to maximize the total node transmissions. As the minimal time slot number is fixed in the previous stage, we only need to find additional conflict free transmissions one by one in all nodes of network. It was shown that it is also a NP-complete problem to maximize the utilization. Thus we attempt to use heuristic method. Since Hopfield neural network has shortcoming of easy to tramped in local minimal, simulated annealing technique is adopted to combined with neural network. Instead of gradient descent dynamics, chaotic dynamics have been studied and shown more complex neurodynamics. Chen and Aihara[8][9] proposed chaotic simulated annealing by starting with a sufficiently large negative self-coupling and then gradually decreasing the self-coupling to stabilize the network, which they called transient chaotic neural network (TCNN). By adding decaying stochastic noise into TCNN, Wang and Tian [10] proposed a new approach to simulated annealing using noisy chaotic neural network, i.e., stochastic chaotic simulated annealing (SCSA), which is described as follows:

$$x_{jk}(t) = \frac{1}{1 + e^{-y_{jk}(t)/\varepsilon}} \tag{6}$$

$$y_{jk}(t+1) = ky_{jk}(t) + \alpha[\sum_{i=1,i\neq j}^{N} \sum_{l=1,l\neq k}^{M} w_{jkil}x_{jk}(t) + I_{ij}] - z(t)(x_{jk}(t) - I_0) + n(t) \tag{7}$$

$$z(t+1) = (1-\beta_1)z(t) \tag{8}$$

$$A[n(t+1)] = (1-\beta_2)A[n(t)] \tag{9}$$

where

x_{jk} : output of neuron jk ;

y_{jk} : input of neuron jk ;

w_{jkil}: connection weight from neuron jk to neuron il, with $w_{jkil} = w_{iljk}$ and $w_{jkjk} = 0$;

$$\sum_{i=1,i\neq j}^{N} \sum_{l=1,l\neq k}^{M} w_{jkil}x_{jk} + I_{ij} = -\partial E/\partial x_{jk} : \quad \text{input to neuron } jk \tag{10}$$

I_{jk} : input bias of neuron jk ;

k : damping factor of nerve membrane $(0 \leq k \leq 1)$;

α : positive scaling parameter for inputs ;

β_1 : damping factor for neuronal self-coupling $(0 \leq \beta_1 \leq 1)$;

β_2 : damping factor for stochastic noise $(0 \leq \beta_2 \leq 1)$;

$z(t)$: self-feedback connection weight or refractory strength $(z(t) \geq 0)$;

I_0 : positive parameter;

ε : steepness parameter of the output function $(\varepsilon > 0)$;

E : energy function;

$n(t)$: random noise injected into the neurons, in $[-A, A]$ with a uniform distribution;

$A[n]$: amplitude of noise n.

The energy function for phase II is defined as follow:

$$E = \frac{W_1}{2} \sum_{i=1}^{N} \sum_{j=1}^{M} \sum_{k=1, k \neq i}^{N} d_{ik} v_{ij} v_{kj} + \frac{W_2}{2} \sum_{i=1}^{N} \sum_{j=1}^{M} (1 - v_{ij})^2 \tag{11}$$

Where W_1 and W_2 are coefficients. W_1 represents the constraint that any pair of nodes which is one-hop away or two-hop away must not transmit simultaneously during each TDMA cycle. W_2 is the optimization which maximized the total number of output firing neurons.

From eqn. (8), eqn. (11), and eqn. (12), we obtain the dynamics of SCSA for the BSP phase II as follow:

$$y_{jk}(t+1) = ky_{jk}(t) + \alpha[-W_1(\sum_{k=1, k \neq i}^{N} d_{ik} v_{kj} - 1) + W_2(1 - v_{ij})] - z(t)(v_{jk}(t) - I_0) + n(t) \tag{12}$$

Where y_{jk} is the neuron input and v_{ij} is the neuron output.

In the above models of BSP, the network with $N \times M$ neurons is updated cyclically and asynchronously. All neurons are cyclically updated in a fixed order. After all neurons updated once, one iteration finished. And the new state information is immediately available for the other neurons in next iteration.

The choice of parameters in eqn. (12) and eqn. (13) for this noisy chaotic neural network is obtained through try and error. The best values of these parameters for solving this combinatorial optimization problem are listed as follow:

$$\begin{aligned} &k, \alpha, \beta_1, \beta_2, \varepsilon = 0.9, 0.015, 0.001, 0.0002, 0.004 \\ &I_0 = 0.65, z_0 = 0.08, A[n(0)] = 0.009; W_1, W_2 = 1, 1 \end{aligned} \tag{13}$$

4 Experimental Results

Three benchmark problems from [3] have been also solved by [3], [5], [6], and [7]. The three examples are instances with 15-node-29-edge, 30-node-70-edge, and 40-node-66-edge respectively. As we can see from Table 1, the low bound for each instance is 8, 9, and 8 respectively. The resulting schedules of our mixed algorithm are shown in Fig. 1, Fig. 2 and Fig. 3 respectively, where a black box represents an assigned time slot.

The computational results are summarized in Table 2 in comparison with mixed HNN-GA algorithm from [7], Sequential Vertex Coloring (SVC) from [6], Gradual Neural Network (GNN) from [5] and Mean Field Annealing (MFA) from [3]. In respect of the TDMA frame length, our mixed algorithm can find optimal schedule for all three examples. Our mixed algorithm also outperforms all the other algorithms in obtaining the minimal value of η, i.e., our algorithm is more efficient in searching the solutions with maximal channel utilization.

5 Conclusion

In this paper, we proposed a mixed algorithm which combined the branch and bound algorithm and stochastic chaotic simulated annealing (SCSA) to solve broadcast scheduling problem in packet radio network with two stages of optimization. We use branch and bound

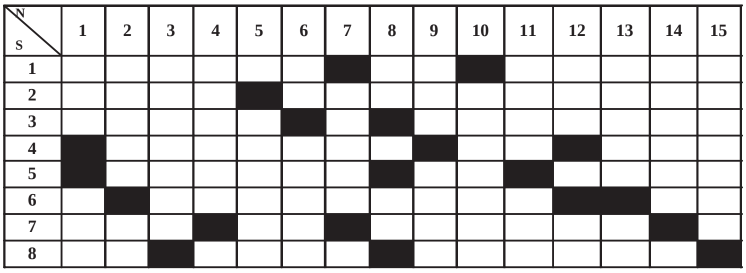

Figure 1: The broadcast schedule for the 15-node network. N and S stand for the number of nodes and the number of time slots, respectively. The black square stands for the transmission of node i in slot j.

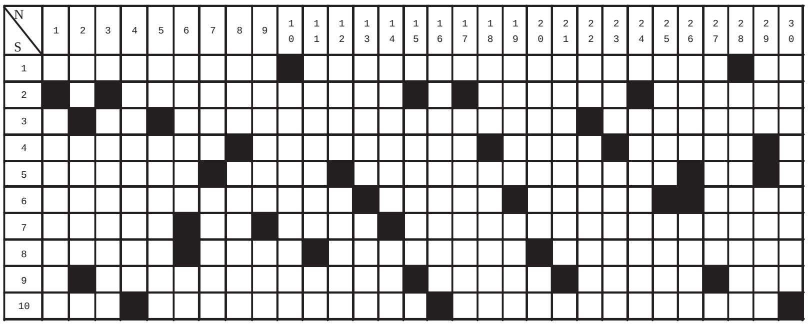

Figure 2: Same as Fig. 3 for the 30-node network.

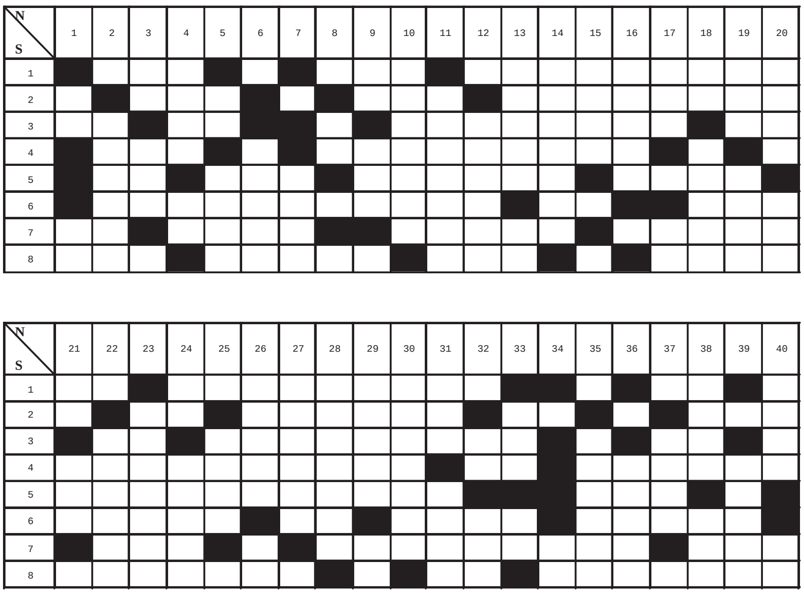

Figure 3: Same as Fig. 3 for the 40-node network.

Table 1: Specifications of three benchmark instances.

Case	# of Nodes N	# of edges E	Maximum degree	Minimum degree	Low bound on M
BM #1	15	29	7	2	8
BM #2	30	70	8	2	9
BM #3	40	66	7	1	8

Table 2: Comparisons of BnB-SCSA with other algorithms in three benchmark problems.

Case	BnB-SCSA		HNN-GA		SVC		GNN		MFA	
	η	M	η	M	η	M	η	M	η	M
BM #1	6.8	8	7.0	8	7.2	8	7.1	8	7.2	8
BM #2	9.2	10	9.3	10	10.0	10	9.5	10	10.5	12
BM #3	5.8	8	6.3	8	6.76	8	6.2	8	6.9	9

to find the minimal frame length in the first stage and use SCSA to find optimal schedule with maximal channel utilization in the second stage. We evaluate our mixed algorithm in three benchmark examples and compared with all existing methods on this problem. The results show that our BnB-SCSA always finds better solutions than all other existing algorithms.

References

[1] A.S. Tanenbaum, Computer Networks, Prentice Hall (1981).

[2] N. Funabiki, Y. Takefuji, A Parallel Algorithm for Broadcast Scheduling Problems in Packet Radio Networks, IEEE Trans. on Communications, **6**, (1993) 828–831.

[3] G. Wang and N. Ansari, Optimal Broadcast Scheduling in Packet Radio Networks Using Mean Field Annealing, IEEE Journal on selected areas in Communications, bf 15 (1997) 250–260.

[4] G. Chakraborty and Y. Hirano, Genetic Algorithm for Broadcast Scheduling in Packet Radio Networks, IEEE World Congress on Computational Intelligence, **E82-A** (1998) 183–188.

[5] N. Funabiki and J. Kitamichi, A Gradual Neural Network Algorithm for Broadcast Scheduling Problems in Packet Radio Networks, IEICE Trans. Fundamentals, (1999) 815–824.

[6] J. Yeo, H. Lee and S. Kim, An efficient broadcast scheduling algorithm for TDMA ad-hoc networks, Computer & Operations Research, **29**, (2002) 1793–1806.

[7] S. Salcedo-Sanz, C. Bousoño-Calzón and A. R. Figueiras-Vidal, A Mixed Neural-Genetic Algorithm for the Broadcast Scheduling Problem, IEEE Trans. on wireless communications, **2**, (2003) 277–283.

[8] L. Chen and K. Aihara, Transient chaotic neural networks and chaotic simulated annealing, M.Yamguti(ed.), Towards the Harnessing of chaos. Amsterdam, Elsevier Science Publishers B.V., (1994) 347–352.

[9] L. Chen and K. Aihara, Chaotic simulated annealing by a neural network model with transient chaos, *Neural Networks*,**8**, (1995) 915–930.

[10] L. Wang and F. Tian, Noisy chaotic neural networks for solving combinatorial optimization problems, Proc. International Joint Conference on Neural Networks, IJCNN 2000, (2000).