A NOISY CHAOTIC NEURAL NETWORK APPROACH TO IMAGE DENOISING

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ABSTRACT

This paper presents a new approach to address image denoising based on a new neural network, called noisy chaotic neural network (NCNN). The original Bayesian framework of image denoising is reformulated into a constrained optimization problem using continuous relaxation labeling. The NCNN, which combines the simulated annealing technique with the Hopfield neural network (HNN), is employed to solve the optimization problem. It effectively overcomes the local minima problem which may be incurred by the HNN. The experimental results show that the NCNN could offer good quality solutions.

1. INTRODUCTION

The objective of image denoising is to estimate the original image from the noisy image with some knowledge of the degradation process. There are many existing models and algorithms for solving this problem [1] [2] [3] [4]. Here we adopt a Bayesian framework because it is highly parallel and it can decomposite a complex computation into a network of simple local computation [3], which is important in hardware implementation of neural networks. In this study the approach computes the maximum *a posteriori* (MAP) estimate of the original image given the noisy image. The MAP estimation involves the prior distribution of the original image and the conditional distribution of the data. The prior distribution of the original images imposes the contextual constraints, and can be modeled by Markov random field (MRF) or equivalently Gibbs distribution. Maximizing the *a posteriori* problem is equivalent to maximizing the energy function in the Gibbs distribution. The MAP-MRF principle centers on applying MAP estimation on the MRF modeling of the images.

Li proposed the augmented Lagrange Hopfield method to solve the optimization problem [5]. He transformed the combinatorial optimization problem into real constrained optimization using the notion of continuous relaxation labeling. The HNN was then used to solve the real constrained optimization.

Previous studies have shown that neural networks are powerful for solving optimization problems [6] [7]. The HNN is an artificial neural network that is capable of solving quadratic optimization problems. However, it suffers from convergence to local minima [8]. To overcome this shortcoming, different simulated annealing techniques have been combined with the HNN to solve optimization problems [9] [8] [10] [11] [12]. Kajiura et al [11] proposed the gaussian machine which combines stochastic simulated annealing (SSA) with neural network for solving assignment problems. Convergence to globally optimal solutions is guaranteed if the cooling schedule is sufficiently slow, i.e., no faster than logarithmic progress [3]. SSA searches the entire solution spaces, which is time consuming. Chen and Aihara [9] proposed a transiently chaotic neural network (TCNN) which adds a large negative self-coupling with slow damping in the Euler approximation of the continuous HNN so that neurodynamics eventually converge from strange attractors to an equilibrium point. This Chaotic simulated annealing (CSA) can search efficiently because of its reduced search spaces. The TCNN showed good performance in solving traveling salesman problem. However CSA is deterministic and is not guaranteed to settle down at a global minimum. In view of this, Wang and Tian [12] proposed a novel algorithm called stochastic chaotic simulated annealing (SCSA) which combines both stochastic manner of SSA and chaotic manner of CSA. In this paper the NCNN, which performs SCSA algorithm, is applied to solve the constrained optimization in the MAP-MRF fomulated image denoising. Experimental results show that the NCNN outperforms the HNN and the TCNN.

The rest of the paper is organized as follows: Section 2 introduces the MAP-MRF framework in image restoration and the transformation of the combinatorial optimization to a real unconstrained optimization. Section 3 presents the NCNN and the derivation of the neural network dynamics. The experimental results are shown in Section 4. Section 5 concludes the paper.

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2. MAP-MRF IMAGE RESTORATION

Let $\mathcal{S} = \{1, \dots, N\}$ indexes the set of sites corresponding to the image pixels. $x = \{x_i \mid i \in \mathcal{S}\}$, $\hat{x} = \{\hat{x}_i \mid i \in \mathcal{S}\}$ and $y = \{y_i \mid i \in \mathcal{S}\}$ are the random variables denoting the original image, the restored image and the degraded image respectively.

When the original image is degraded by identical independently distributed (i.i.d.) Gaussian noise, the degraded image is modeled by:

$$y_i = x_i + e_i \tag{1}$$

where $e_i \sim N(0, \sigma^2)$ is the zero mean Gaussian distribution with standard deviation σ. The objective of image denoising is to find an $\hat{x}$ that approximates x.

Each site in $\mathcal{S}$ takes on a discrete value in the label set $\mathcal{L} = 1, \dots, M$. The spatial relationship of the sites is determined by a neighborhood system $\mathcal{N} = \{\mathcal{N}_i \mid i \in \mathcal{S}\}$ where $\mathcal{N}_i$ is the set of sites neighboring i. A single site or a set of neighboring sites form a clique denoted by c. $\mathcal{C}$ is the set of all cliques. In this paper only the pair-site cliques are considered.

There are many different ways the pixels can influence each other through contextual interactions. Such contextual interactions can be modeled as MRFs. According to Markov-Gibbs equivalence, an MRF is equivalent to a Gibbs distribution. Therefore, the prior distribution of the original image which imposes the contextual constraints can be expressed in terms of the MRF clique potentials.

In (1) the noise is independent Gaussian noise. The conditional distribution can be expressed in terms of y, x and σ. Knowing the prior distribution and the conditional distribution, the energy in the posterior distribution is given by [4]

$$E(x) = \sum_{i \in \mathcal{S}} \frac{(x_i - y_i)^2}{2\sigma^2} + \sum_{\{i,i'\} \in C} V_2(x_i, x_{i'}) \tag{2}$$

where $\sum_{\{i,i'\} \in C} V_2(x_i, x_{i'})$ is the pair-site clique potential of the MRF model.

Among various MRFs, the multi-level logistic (MLL) model is a simple mechanism for encoding a large class of spatial patterns [13]. In MLL, the pair-site clique potentials take the form: $V_2(x_i, x_{i'}) = -{}_c$ if sites in the clique $\{i, i'\}$ have the same label; otherwise, $V_2(x_i, x_{i'}) = {}_c$ where ${}_c$ is a parameter associated with the type of the clique c. However as the MLL pair-site potentials are strong constraints, we adopt the modified potential as:

$$V_2(x_i, x_{i'}) = -{}_c * g(x_i, x_{i'}) \tag{3}$$

where $g(x_i, x_{i'}) = 2e^{-|x_i - x_{i'}|/\tau} - 1$, τ is a positive constant. $g(x_i, x_{i'})$ in the modified potential function is an exponential function in $(-1, 1]$. Compared to the potential function in the MLL model, the modified potential function allows the pixel to be slightly different from the neighboring pixels. This is logical as most real images have smooth non-uniform regions.

As image pixels can only take discrete values, the minimization in (2) is a combinatorial optimization problem. It can be transformed into a constrained optimization in real space using continuous relaxation labeling. Let $p_i(I) \in [0, 1]$ represent the strength with which label I is assigned to i, the energy with the p variables is given by

$$\begin{aligned} E(p) &= \sum_{i \in \mathcal{S}} \sum_{I \in \mathcal{L}} r_i(I) p_i(I) + \\ & \sum_{i \in \mathcal{S}} \sum_{I \in \mathcal{L}} \sum_{i' \in \mathcal{S}, i' \neq i} \sum_{I' \in \mathcal{L}} r_{i,i'}(I, I') p_i(I) p_{i'}(I') \end{aligned} \tag{4}$$

where $I = x_i$, $I' = x_{i'}$, $r_i(I) = V_1(I|y) = (I - y_i)^2 / 2\sigma^2$ is the single-site clique potential function and $r_{i,i'}(I, I') = V_2(I, I'|y) = V_2(I, I')$ is the pair-site clique potential function in the posterior distribution $P(x|y)$.

With such a representation, the combinatorial minimization is reformulated as the following constrained minimization

$$\min_p \quad E(p) \tag{5}$$

$$\text{subject to} \quad C_i(p) = 0 \qquad i \in \mathcal{S} \tag{6}$$

$$p_i(I) \geq 0 \qquad \forall i \in \mathcal{S},\ \forall I \in \mathcal{L} \tag{7}$$

where $C_i(p) = \sum_I p_i(I) - 1 = 0$.

The final solution p^* is subject to additional constraints: $p_i^*(I) \in \{0, 1\}$. The HNN is prone to trappings at local minima. In view of this, we propose a new network NCNN to perform the optimization.

3. NOISY CHAOTIC NEURAL NETWORK

Let $u_i(I)$ denote the internal state of the neuron (i, I) and $p_i(I)$ denote the output of the neuron (i, I). $p_i(I) \in [0, 1]$ represents the strength that the pixel at location i takes the value I. The NCNN is formulated as follows [12]:

$$p_i^{(t)}(I) = \frac{1}{1 + e^{-u_i^{(t)}(I)/\epsilon}} \tag{8}$$

$$\begin{aligned} u_i^{(t+1)}(I) &= k u_i^{(t)}(I) - z^{(t)}(p_i^{(t)}(I) - I_o) + n^{(t)} + \\ & \Big(\sum_{i'=1, i' \neq i}^{N} \sum_{I'=1}^{M} T_{iI;i'I'} p_{i'}^{(t)}(I') + \mathcal{I}_i(I) \Big) \end{aligned} \tag{9}$$

$$z^{(t+1)} = (1 - {}_z) z^{(t)} \tag{10}$$

$$A[n^{(t+1)}] = (1 - {}_n) A[n^{(t)}] \tag{11}$$

where

$T_{iI;i'I'}$: connection weight from neuron (i', I') to neuron (i, I);

$\mathcal{I}_i(I)$: input bias of neuron (i, I);

k : damping factor of nerve membrane (0 $\quad k \quad$ 1)

: positive scaling parameter for inputs;

ϵ : steepness parameter of the output function ($\epsilon \geq 0$);

z : self-feedback connection weight or refractory strength ($z \geq 0$);

I_o : positive parameter;

n : random noise injected into the neurons;

$_z$: positive parameter ($0 < \quad_z < 1$);

$_n$: positive parameter ($0 < \quad_n < 1$);

$A[n]$: the noise amplitude.

When we set $n^{(t)} = 0$ in (9), the NCNN becomes TCNN. When we further set $z^{(t)} = 0$, the TCNN becomes similar to the HNN with stable fixed point dynamics. The basic difference between the HNN and the TCNN is that a nonlinear term $z^{(t)}(p_i^{(t)}(I) - I_o)$ is added to the HNN. Since the "temperature" $z^{(t)}$ tends toward zero with time evolution, the updating equations of the TCNN eventually reduce to those of the HNN. In (9) the variable $z^{(t)}$ can be interpreted as the strength of negative self-feedback connection of each neuron, the damping of $z^{(t)}$ produces successive bifurcations so that the neurodynamics eventually converge from strange attractors to a stable equilibrium point [8].

CSA is deterministic and is not guaranteed to settle down to a global minimum. In view of this, Wang and Tian [12] added a noise term $n^{(t)}$ in (9). The noise term continues to search for the optimal solution after the chaos of the TCNN disappears.

From (5)-(7) and (9), we obtain the dynamics of the NCNN:

$$u_i^{(t+1)}(I) = ku_i^{(t)}(I) - z^{(t)}(p_i^{(t)}(I) - I_o) + n^{(t)} + \quad q_i^{(t)}(I) \tag{12}$$

where

$$q_i(I) = -\frac{\partial L \ (p, \gamma)}{\partial p_i(I)}$$

$$L \ (p, \gamma) = E(p) + \sum_{k \in \mathcal{S}} \gamma_k C_k(p) + \frac{}{2} \sum_{k \in \mathcal{S}} [C_k(p)]^2$$

γ_k are the Lagrange multipliers and $\quad > 0$ is the weight for the penalty term. Note that the Lagrange multipliers are updated with neural outputs according to $\gamma_k^{(t+1)} = \gamma_k^{(t)} + \quad C_i(p^{(t)})$.

4. EXPERIMENTAL RESULTS

The experimental results presented here demonstrate the performance of the NCNN on image restoration.

We chose the Lena image of size 128 $\quad$ 128 with $M = 256$ gray levels. The label set $\mathcal{L} = \{0, 1, 2, \ldots, 255\}$. The degraded image was generated by adding zero-mean identical independently distributed Gaussian noise with standard deviation $\sigma = 16$.

The degraded image was set to be the input of the neural networks. After the neural networks were initialized, each neuron was updated using (12) and (8). After all neurons in the neural networks were updated once, γ_k, z and n are updated. The updating scheme is cyclic and asynchronous. When all the neurons are updated once, we call it one iteration. Once the state of a neuron is updated, the new state information is immediately available to other neurons in the network (asynchronous).

The parameters that we used for the NCNN are: $k = 1$, $\epsilon = 0.01$, $\quad = 0.0001$, $I_0 = 0.65$, $z^{(0)} = 0.05$, $n^{(0)} = 0.01$. $\quad$ is increased from 1 to 50 according to $\quad \leftarrow 1.01 \quad$. The decreasing rate of z and n, $\quad_z$ and $\quad_n$ are 0.005. For the TCNN and the HNN we use the same parameters as the NCNN except that $n = 0$ for the TCNN, $n = 0$ and $z = 0$ for the HNN. The MRF pair-site clique potential parameter $_c$=8.

Table 1 shows the required iteration numbers and the peak signal-to-noise ratio (PSNR) of the restored images. The higher the PSNR, the better the image quality. It can be seen from the table that the NCNN offers the best performance. The PSNR of the restored image using the NCNN is higher than those of the restored images using the HNN and the TCNN. In addition, the NCNN use less iterations to converge than the HNN and the TCNN. The restored images are shown in Fig. 1.

Table 1. Numerical restoration results of the Lena image (PSNR_r: PSNR of the restored image, PSNR_d: PSNR of the degraded image)

	Iterations	PSNR_r	PSNR_d
HNN	1489	25.6227	24.0549
TCNN	1173	25.9077	24.0549
NCNN	1114	26.7554	24.0549

5. CONCLUSION

A new neural network, called noisy chaotic neural network (NCNN), is used to address the MAP-MRF formulated image denoising problem. SCSA effectively overcomes the local minima problem. We have shown that the NCNN gives better quality solutions compared to the HNN and the TCNN.

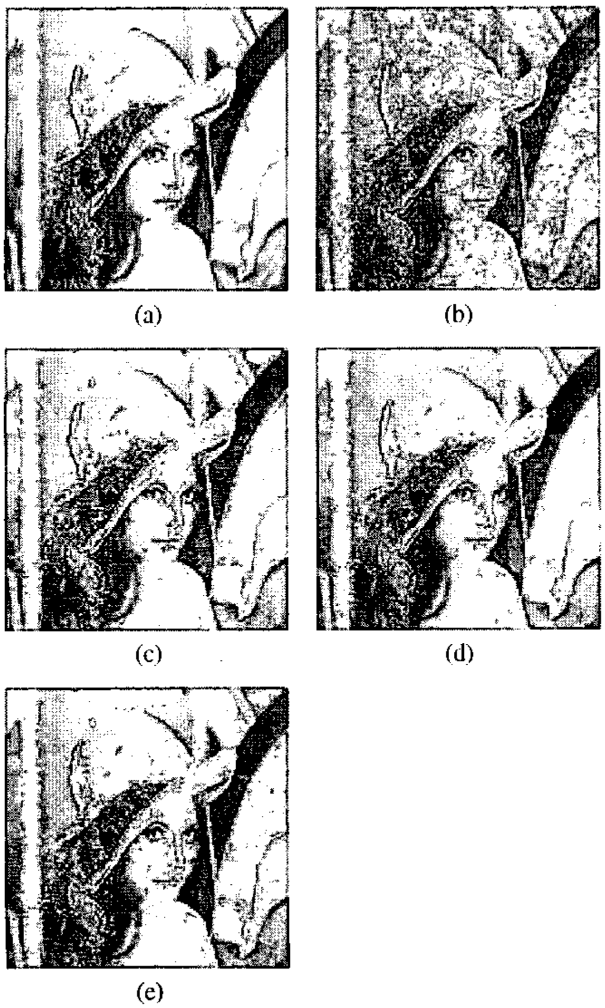

Fig. 1. Restoration of Lena Image: (a) Original image. (b) Degraded image. (c)-(e) Restored images using the HNN, the TCNN and the NCNN, respectively

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