A Noisy Chaotic Neural Network Approach to Topological Optimization of a Communication Network with Reliability Constraints

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Abstract. Network topological optimization in communication network is to find the topological layout of network links with the minimal cost under the constraint that all-terminal reliability of network is not less than a given level of system reliability. The all-terminal reliability is defined as the probability that every pair of nodes in the network can communicate with each other. The topological optimization problem is an NP-hard combinatorial problem. In this paper, a noisy chaotic neural network model is adopted to solve the all-terminal network design problem when considering cost and reliability. Two sets of problems are tested and the results show better performance compared to previous methods, especially when the network size is large.

1 Introduction

An important stage of network design is to find the best way to layout all the components to optimize costs while meeting a performance criterion, such as transmission delay, throughput, or reliability [1]. This paper focuses on network design of large backbone communication networks with an all-terminal reliability (ATR) constraint, i.e., an acceptable probability that every pair of nodes can communicate with each other [1]. The network topology problem can be formulated as a combinatorial optimization problem which is NP-hard [2]. Previous work on this problem can be categorized to enumerative-based and heuristic methods. Jan *et al* [1] developed an algorithm using decomposition based on branch-and-bound to find the exact, optimal solution. They divided the problem into several subproblems by the number of links of subnetworks. The maximum network size of fully connected graphs for which a solution has been found is 12 nodes (60 links) [4]. Due to the NP-hard nature of the problem, heuristic approaches are often adopted to solve this problem. Berna *et al* [3] used genetic algorithms to solve the all-terminal network design problem. They formulate the network design as an integer vector which is the chromosome. Each gene of the

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chromosome represents a possible link of the network. Because the network reliability measure is also an NP-hard problem, instead of the exact calculation of reliability, the network reliability is estimated through three steps. First step is the connectivity check for a spanning tree. The next step is the evaluation of a 2-connectivity measure and the last step is to compute Jan's upper bound [1]. The precise estimation of the reliability of the network can be obtained through Monte Carlo simulations. Hosam *et al* [4] adopted a Hopfield neural network called optimized neural network (OPTI-nets) to solve this problem. The links in the backbone network are represented by neurons, where each neuron represents a link. A neuron is set if its output is a nonzero value and the corresponding link is hence selected [4]. They tested their OPTI-nets on a large network size with 50 nodes and 1225 edges.

In this paper, we use a noisy chaotic neural network (NCNN) [7]-[9] to solve this NP-hard problem. In section II, we briefly introduce the NCNN model [7]-[9]. Section III presents the NCNN solution to the topological optimization problem. Section IV includes the results. We draw conclusions in Section V.

2 Noisy Chaotic Neural Network

Chen and Aihara [5][6] proposed chaotic simulated annealing (CSA) by starting with a sufficiently large negative self-coupling in the neurons and then gradually decreasing the self-coupling to stabilize the network. They called this model the transiently chaotic neural network (TCNN). By adding decaying stochastic noise into the TCNN, Wang and Tian [7] proposed a new approach to simulated annealing using a noisy chaotic neural network (NCNN). This novel model has been applied successfully in solving several challenging optimization problems including the traveling salesman problem (TSP) and the channel assignment problem (CAP) [7] [8] [9]. The NCNN model is described as follows [7]:

$$x_{jk}(t) = \frac{1}{1 + e^{-y_{jk}(t)/\varepsilon}} \quad , \tag{1}$$

$$y_{jk}(t+1) = ky_{jk}(t) + \alpha \left[\sum_{i=1, i\neq j}^{N} \sum_{l=1, l\neq k}^{N} w_{jkil} x_{jk}(t) + I_{ij} \right] - z(t) \left[x_{jk}(t) - I_0 \right] + n(t) \, , \tag{2}$$

$$z(t+1) = (1 - \beta_1) z(t) \quad , \tag{3}$$

$$A[n(t+1)] = (1 - \beta_2) A[n(t)] \quad , \tag{4}$$

where

x_{jk} : output of neuron jk ;

y_{jk} : input of neuron jk ;

w_{jkil}: connection weight from neuron jk to neuron il, with $w_{jkil} = w_{iljk}$ and $w_{jkjk} = 0$;

$$\sum_{i=1,i\neq j}^{N} \sum_{l=1,l\neq k}^{N} w_{jkil} x_{jk} + I_{ij} = -\partial E / \partial x_{jk} : \quad \text{input to neuron } jk \tag{5}$$

E : energy function;
t : time steps;
N : number of network nodes;
I_{jk} : input bias of neuron jk ;
k : damping factor of nerve membrane ($0 \leq k \leq 1$);
α : positive scaling parameter for inputs ;
β_1 : damping factor for neuronal self-coupling ($0 \leq \beta_1 \leq 1$);
β_2 : damping factor for stochastic noise ($0 \leq \beta_2 \leq 1$);
$z(t)$: self-feedback connection weight or refractory strength ($z(t) \geq 0$), $z(0)$ is a constant;
I_0 : positive parameter;
ε : steepness parameter of the output function ($\varepsilon > 0$) ;
$n(t)$: random noise injected into the neurons, in $[-A, A]$ with a uniform distribution;
$A[n]$: amplitude of noise n.

3 Applying NCNN to Topological Optimization Problem

3.1 Problem Formulation

A communication network can be modeled by a graph $G = (N, L, P, C)$. Here N and L are network sites and communication links, respectively. P is the set of reliability for all links and C is the set of costs for all links. The optimization problem is [3]:

$$\text{Minimize:} \qquad Z = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} c_{ij} v_{ij} \tag{6}$$

$$\text{Subject to:} \qquad R \geq R_0$$

where the $C = \{c_{ij}\}$ is the cost of link (i, j) and $V = \{v_{ij}\}$ is the neuron binary output which is defined as:

$$v_{ij} = \begin{cases} 1, & \text{if link } (i,j) \text{ is selected for the optimized network design ;} \\ 0, & \text{otherwise.} \end{cases}$$

R is the all-terminal reliability of the network and R_0 is the network reliability requirement.

We formulate the energy function of the NCNN as follows:

$$E = -W_1 R + W_2 \sum_{i=1}^{N} \sum_{j=i+1}^{N} v_{ij} c_{ij} + W_3 \eta^{\varsigma} \times \left| 1 - \frac{R}{R_0} \right| + W_4 \sum_{i=1}^{N} \sum_{j=1,j\neq i}^{N} v_{ij} (1 - v_{ij}) \,, \tag{7}$$

where $\varsigma = u(1 - \frac{R}{R_0})$ and $u(x)$ is a unit step function, i.e., $u(x) = 0$ for $x < 0$ and $u(x) = 1$ for $x \geq 0$.

W_1, W_2, W_3, and W_4 are weight factors. The W_1 term encourages the network to increase the network reliability. The W_2 term is the costs of network links to be minimized. The W_3 term is the constraint term which discourages the network from adding more links to increase the network reliability unnecessarily over R_0 and η is the penalty factor. The W_4 term is used to help the convergence of neurons. When the energy goes toa minimum, the W_4 term forces the output of neurons to a value of 0 or 1.

The reliability term in energy function R is replaced with a upper bound instead of exact calculation due to the NP-hard complexity for an exact calculation [4]. The upper bound on the ATR is [4]:

$$R(G) \leq \prod_{i=1, i\neq s}^{N} \left[1 - \prod_{j \in A(i)} (1 - p_{ij}) \right] , \tag{8}$$

where $A(x)$ is defined as the set of vertexes connected to vertex x, p_{ij} is the reliability of link (i, j). Node s is selected as the node with maximum node degree [4].

From dynamic equations (2), (5) and (7), the motion equation of the NCNN is:

$$y_{jk}(t+1) = k y_{jk}(t) + \alpha \left\{ W_1 \frac{\partial R}{\partial v_{ij}} - W_2 c_{ij} - \frac{W_3}{R_0} (-1)^{1-\varsigma} \right.$$
$$\left. \eta^{\varsigma} \frac{\partial R}{\partial v_{ij}} - W_4 (1 - 2 v_{ij}) \right\} - z(t) \left[x_{jk}(t) - I_0 \right] + n(t) , \tag{9}$$

where from equation (8) [4],

$$\frac{\partial R}{\partial v_{ij}} = \prod_{k=1, k\neq s,i,j}^{N} \left[1 - \prod_{l=1, l\neq k}^{N} \psi_{kl} \right] p_{ij}$$
$$\left[\prod_{l=1, l\neq i}^{N} \psi_{jl} + \prod_{l=1, l\neq j}^{N} \psi_{il} - 2 \prod_{l=1, l\neq j}^{N} \psi_{lj} \prod_{l=1, l\neq i,j}^{N} \psi_{li} \right] \tag{10}$$

and

$$\psi_{ab} \equiv 1 - p_{ab} \times v_{ab} . \tag{11}$$

4 Results

The parameters for the energy function is determined empirically as below: $W_1 = 1.5, W_2 = 0.0001, w_3 = 1.2, w_4 = 1, \eta = 10$.

The benchmark problems are adopted from [3] in which the results for $n < 10$ are verified by using the exact branch-and-bound method [1]. The problems contains 20 different cases. Case 1-17 are problems concerning fully connected networks and case 18-20 are for non-fully connected networks. The comparison of results is listed in Table 1, where p is the link probability for each link and R_0 is the objective network reliability. The simulation of each case was run 10 times and only the best results are listed as in the references to which we shall compare our results [1][3][4]. We compare the results with three other methods as showed in Table 1. From the results, it can be seen that branch-and-bound cannot be applied to problems with larger node sizes but it can obtain optimal solution. Genetic algorithms are effective but failed to find good solutions when the node size becomes large as showed in case 15-17. And it also can be seen that our NCNN can find better solutions than OPTI-net does on these cases.

Table 1. Comparison of results of test cases from [3]

Case	# of Nodes	# of edges	p	R_0	BnB[1][3]	GA[3]	OPTI-net[4]	NCNN
1	5	10	0.8	0.9	255	255	255	255
2	5	10	0.9	0.95	201	201	201	201
3	7	21	0.9	0.9	720	720	720	720
4	7	21	0.9	0.95	845	845	845	845
5	7	21	0.95	0.95	630	630	630	630
6	8	28	0.9	0.9	208	208	208	208
7	8	28	0.9	0.95	247	247	247	247
8	8	28	0.95	0.95	179	179	179	179
9	9	36	0.9	0.9	239	239	239	239
10	9	36	0.9	0.95	286	286	308	286
11	9	36	0.95	0.95	209	209	209	209
12	10	45	0.9	0.9	154	156	154	154
13	10	45	0.9	0.95	197	205	197	197
14	10	45	0.95	0.95	136	136	136	136
15	15	105	0.9	0.95		317	304	304
16	20	190	0.95	0.95		926	270	202
17	25	300	0.95	0.9		1606	402	377
18	14	21	0.9	0.9	1063	1063	1063	1063
19	16	24	0.9	0.95	1022	1022	1077	1022
20	20	30	0.95	0.9	596	596	596	596

5 Conclusion

In this paper we apply the noisy chaotic neural network to solve the topological optimization problem in backbone networks with all-terminal reliability constraint. The results on 20 benchmark problems show that our NCNN outperforms other methods, especially in large problems.

References

1. Jan, R.H., Hwang, F.J., Chen, S.T.: Topological Optimization of a Communication Network Subject to a Reliability Constraint. IEEE Trans. Reliability, Vol. 42. (1993) 63−70
2. Garey, M.R., Johnson, D.S.: Computers and Intractability: A Guide to the Theory of NP-Completeness. W. H. Freeman & Co. (1979)
3. Dengiz, B., Altiparmak, F., Smith, A.E.: Efficient Optimization of All-terminal Reliability Networks, Using an Evolutionary Approach. IEEE Trans. Reliability, Vol. 46. (1997) 18−26
4. AboElFotoh, Hosam M.F., Al-Sumait, Loulwa S.: A Neural Approach to Topological Optimization of Communication Networks, with Reliability Constraints. IEEE Trans. Reliability, Vol. 50. (2001) 397−408
5. Chen, L., Aihara, K.: Transient Chaotic Neural Networks and Chaotic Simulated Annealing, M. Yamguti(ed.). Towards the Harnessing of Chaos. Amsterdam, Elsevier Science Publishers B.V. (1994) 347−352
6. Chen, L., Aihara, K.: Chaotic Simulated Annealing by a Neural Network Model with Transient Chaos. Neural Networks, Vol. 8. (1995) 915−930
7. Wang, L., Tian, F.: Noisy Chaotic Neural Networks for Solving Combinatorial Optimization Problems. Proc. International Joint Conference on Neural Networks. IJCNN (2000)
8. Li, S., Wang, L.: Channel Assignment for Mobile Communications Using Stochastic Chaotic Simulated Annealing. The 6th International Workshop-Conference on Artificial and Natural Neural Networks. IWANN (2001)
9. Wang, L., Li, S., Tian, F., Fu, X.: A Noisy Chaotic Neural Network for Solving Combinatorial Optimization Problems: Stochastic Chaotic Simulated Annealing. IEEE Trans. System, Man, Cybern, Part B - Cybernetics, August, (2004)