Minimizing Interference in Cellular Mobile Communications by Optimal Channel Assignment Using Chaotic Simulated Annealing

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Abstract. In this chapter, we deal with the problem of assigning frequency channels to radio cells in a cellular mobile network so that interference between channels is minimized, while demands for channels are satisfied. We solve the channel assignment problem (CAP) using chaotic simulated annealing (CSA) proposed by Chen and Aihara recently. Simulations show that our results are better than existing results found by other algorithms in several benchmarking CAPs.

Keywords: channel assignment, mobile phone, cellular communication, neural network, chaos

1 Introduction

Over recent years, the demand for cellular mobile communication services has been growing rapidly. But electromagnetic spectrum or frequencies allocated for this purpose are limited. Thus optimal assignments of frequency channels are becoming more and more critical. Careful design for a cellular radio network can greatly improve the traffic capacity of the cellular system to accommodate calls, while minimizing interference between calls and guaranteeing the quality of service.

In 1982, Gamst and Rave defined a general form of channel assignment problems (CAPs) in an arbitrary inhomogeneous cellular radio network [1]: minimizing the span of channels subject to demand and interference-free constraints (denoted as CAP1 in [8]). CAP1 is usually solved by graph coloring algorithms [3] and various techniques have been explored for applying neural networks to CAP1s. Funabiki solved CAP1 by using a parallel algorithm which does not require a rigorous synchronization procedure [9]. Chan et al proposed an approach based on cascaded multilayered feedforward neural networks which showed good performance in dynamic CAP1 [10]. Kim et al proposed a modified Hopfield network to solve CAP1 [11].

In 1991, Kunz used the Hopfield neural network for solving a different CAP [2]: minimizing the severity of interferences, subject to demand constraints (denoted as CAP2 in [8]). Kunz solved CAP2 by minimizing an energy or cost function representing interference and channel demand constraints. Smith and Palaniswami reformulated CAP2 as a generalized quadratic assignment problem [8] and found good solutions to CAP2 using simulated annealing (SA), a modified Hopfield neural network, and a self-organizing neural network.

In recent years, a large body of work has been carried out on chaotic simulated annealing (CSA) proposed by Chen and Aihara [12]-[22]. Aihara et al proposed a chaotic neural network based on a modified Nagumo and Sato neuron model [7]. Nozawa found [15] that Euler approximation of the continuous-time Hopfield neural network [24] with a negative neuronal self-coupling has chaotic dynamics. Chen and Aihara proposed a neural network model with transient chaos for combinatorial optimization problems [12]. Since this model is similar to simulated annealing, not in a stochastic way but in a deterministically chaotic way, it is regarded as chaotic simulated annealing (CSA). CSA can search efficiently because of its reduced search spaces. Chen and Aihara use CSA to solve the traveling salesman problem (TSP) and showed good performance [12].

In this chapter, we use CSA to solve CAP2 and show that CSA can lead to further improvements on solutions for CAP2 in comparison to solutions obtained by SA, a modified Hopfield neural network, and a self-organizing neural network [8]. We are concerned with only CAP2 because in most of cases, the interference-free lower bound is far greater than the number of available channels.

This chapter is organized as follows. Section 2 reviews CAP2 and its mathematical formulation given by Smith and Palaniswami [8]. In section 3, the CSA algorithm is reviewed. In the section 4, we apply CSA to several benchmarking CAP2s. Finally in section 5, we conclude this chapter.

2 Static Channel Assignment Problems

We assume that a mobile radio network has N cells and the total number of available channels is M. The channel requirements for cell i are given by D_i $(i = 1, 2, \cdots, N)$. $D^T = (D_1, D_2, \cdots, D_N)$ is called the demand matrix. In this chapter, A^T stands for the transposed matrix of A. $C = \{C_{ij}\}$ is the compatibility matrix, where C_{ij} is the minimum frequency separation between cell i and cell j to guarantee an acceptably low signal/interference ratio in each region, $i, j = 1, 2, \cdots N$, and N is the number of cells in the mobile network.

The solution to CAP2 is mapped onto a neural network with $N \times M$ neurons [8]. The output of each neuron x_{jk}:

$$x_{jk} = \begin{cases} 1, \text{ if cell j is assigned to channel } k, \\ 0, \text{ otherwise }, \end{cases} \tag{1}$$

for $j = 1, \cdots, N$ and $k = 1, \cdots, M$. Potential interferences considered here come from the co-channel constraint (CCC), the adjacent channel constraint (ACC), and the co-site constraint (SCC) [1]. A cost tensor $P_{ji(m+1)}$ is used to measure the degree of interference between cells j and i caused by such assignments that $x_{jk} = x_{il} = 1$ [8], where $m = |k - l|$ is the distance in the channel domain between channels k and l. The cost tensor P can be calculated recursively as follows:

$$P_{ji(m+1)} = \max\,(0, P_{jim} - 1) \quad , \quad for \quad m = 1, \cdots, M-1 \quad , \tag{2}$$

$$P_{ji1} = C_{ji} \quad , \qquad \forall j, i \neq j \quad , \tag{3}$$

$$P_{jj1} = 0 \quad , \qquad \forall j \quad . \tag{4}$$

CAP2 can then be formulated as:
minimize

$$F(x) \;=\; \sum_{j=1}^{N}\sum_{k=1}^{M} x_{jk} \sum_{i=1}^{N}\sum_{l=1}^{M} P_{ji(|k-l|+1)} x_{il} \quad , \tag{5}$$

subject to

$$\sum_{k=1}^{M} x_{jk} \;= D_j \quad , \qquad \forall j = 1, \cdots, N \quad , \tag{6}$$

where $F(x)$ is the total interference in the mobile network.

3 Chaotic Simulated Annealing

Chen and Aihara's chaotic simulated annealing (CSA) is described by the following equations [12]:

$$x_{jk}(t) = \frac{1}{1 + e^{-y_{jk}(t)/\varepsilon}} \quad , \tag{7}$$

$$y_{jk}(t+1) = k y_{jk}(t) + \alpha\Big(\sum_{i=1, i\neq j}^{N} \sum_{l=1, l\neq k}^{M} w_{jkil} x_{jk}(t) + I_{ij}\Big) - z(t)(x_{jk}(t) - I_0) \quad , \tag{8}$$

$$z(t+1) = (1 - \beta) z(t) \quad , \tag{9}$$

where
x_{jk} : output of neuron jk ;
y_{jk} : input of neuron jk ;

w_{jkil}: connection weight from neuron jk to neuron il, with $w_{jkil} = w_{iljk}$ and $w_{jkjk} = 0$;

$$\sum_{i=1,i\neq j}^{N} \sum_{l=1,l\neq k}^{M} w_{jkil} x_{jk} + I_{ij} = -\partial E / \partial x_{jk} : \text{ input to neuron } jk \tag{10}$$

I_{jk} : input bias of neuron jk ;
k : damping factor of nerve membrane ($0 \leq k \leq 1$);
α : positive scaling parameter for inputs ;
β : damping factor ($0 \leq \beta \leq 1$);
$z(t)$: self-feedback connection weight or refractory strength ($z(t) \geq 0$) ;
I_0 : positive parameter;
ε : steepness parameter of the output function ($\varepsilon > 0$) ;
E : energy function.

For any given optimization problem, once we know the energy function E to be minimized, a chaotic neural network can be designed using eqs. (7) - (10) to effectively minimize the energy function E [16][12].

The corresponding energy function E for CAP2 can be obtained by combining a constraint term and an interference term suggested by eqs. (5) and (6), respectively :

$$E = \frac{W_1}{2} \sum_{j=1}^{N} (\sum_{k=1}^{M} x_{jk} - D_j)^2 + \frac{W_2}{2} \sum_{j=1}^{N} \sum_{k=1}^{M} x_{jk} \sum_{i=1}^{N} \sum_{l=1}^{M} P_{ji(|k-l|+1)} x_{il}, \tag{11}$$

where W_1 and W_2 represent the relative strength (or importance) of the constraint and the interference, respectively. With eq. (11), the input to neuron jk given in eq. (10) becomes :

$$\begin{aligned} y_{jk}(t+1) &= k y_{jk}(t) - z(t)(x_{jk}(t) - I_0) \\ &+ \alpha \{ -W_1 \sum_{q\neq k}^{M} x_{jq} + W_1 D_j - W_2 \sum_{p=1,p\neq j}^{N} \sum_{q=1,q\neq k}^{M} P_{jp(|k-q|+1)} x_{pq} \} \quad . \end{aligned} \tag{12}$$

In eq. (8), the term $z(t)(x_{jk}(t) - I_0)$ is related to inhibitory self-feedback with a bias I_0. This term gives the neural network the transiently chaotic dynamics which eventually converges to a stable equilibrium point. Eq. (9) represents an exponential cooling schedule for annealing. $z(t)$ corresponds to the temperature in usual stochastic annealing processes.

4 Benchmarking Problem Description

The first benchmarking CAP2 was suggested by Sivarajan [3], denoted as EX1. The number of cells is $N = 4$. The number of channels available is

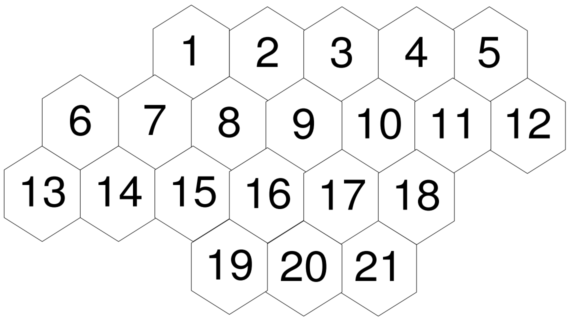

Fig. 1. A 21-cell hexagonal network used in our simulations.

$M = 11$. The demand of channels is given by $D^T = (1, 1, 1, 3)$. We also use a slightly larger extension of EX1, denoted as EX2 [8]:

$N = 5, M = 17, D^T = (2, 2, 2, 4, 3)$.

The Second benchmarking CAP2 used in our simulations is the 21-cell cellular system (HEX1-HEX4) found in [4] (Fig.1). Two different demands are used for HEX as follows.

$D_1^T = (2, 6, 2, 2, 2, 4, 4, 13, 19, 7, 4, 4, 7, 4, 9, 14, 7, 2, 2, 4, 2)$;

$D_2^T = (1, 1, 1, 2, 3, 6, 7, 6, 10, 10, 11, 5, 7, 6, 4, 4, 7, 5, 5, 5, 6)$.

Two compatability matrices for HEX problems are generated by considering the first two rings of cells around a particular cell as interfering cells. Because HEX1 and HEX3 contain only CCC and CSC, they have the same matrix $C^{(1)}$. HEX2 and HEX4 include CCC, CSC and ACC, so they have the same matrix $C^{(2)}$. We produce the matrices in term of the details of HEX1-HEX4 listed in table 1 [8] and show the two matrices $C^{(1)}$ and $C^{(2)}$ in Appendix A (not given in [8]).

Table 1. The descriptions for HEX problems.

Problem	N	M	D	co-channel	adjacent	C_{ii}	C
HEX1	21	37	D_1	yes	no	2	$C^{(1)}$
HEX2	21	91	D_1	yes	yes	3	$C^{(2)}$
HEX3	21	21	D_2	yes	no	2	$C^{(1)}$
HEX4	21	56	D_2	yes	yes	3	$C^{(2)}$

We chose the last CAP2 generated from the topographical and morphostructure data from the area of 24×21 km around Helsinki, Finland [23]. Twenty five base station locations were distributed unequally over the area. Kunz used these data to calculate the traffic demand and interference relationships between the 25 base stations [2]. The compatability matrix $C^{(3)}$

is obtained from Kunz data [8][9], which we reproduce in Appendix A. The demand vector is:

$D_3^T = (10, 11, 9, 5, 9, 4, 5, 7, 4, 8, 8, 9, 10, 7, 7, 6, 4, 5, 5, 7, 6, 4, 5, 7, 5)$.

Smith and Palaniswami divided this benchmarking CAP2 into four classes by considering only the first 10 regions (KUNZ1), 15 regions (KUNZ2), 20 regions (KUNZ3), and the entire area (KUNZ4)[8]. The detail is listed in Table 2.

Table 2. The descriptions for KUNZ problems.

Problem	N	M	C	D
KUNZ1	10	30	$[C^{(3)}]_{10}$	$[D_3]_{10}$
KUNZ2	15	44	$[C^{(3)}]_{15}$	$[D_3]_{15}$
KUNZ3	20	60	$[C^{(3)}]_{20}$	$[D_3]_{20}$
KUNZ4	25	73	$C^{(3)}$	D_3

5 Simulation Results

Our simulation results are shown in Table 3. For comparison, Table 3 also includes the simulation results given in [8], i.e., the performances of GAMS/MINOS-5 (labeled GAMS), the traditional heuristics of steepest descent (SD), stochastic simulated annealing (SSA), the original Hopfield network (HN) (with no hill-climbing), the hill-climbing Hopfield network (HCHN), and the self-organizing neural network (SONN). Each of the techniques (except GAMS/MINOS-5) is run from ten different random initial conditions. In Table 3, “Min” means the minimum total interference (eq. (5)) found during these ten times, and “Ave” is the average total interference for the ten runs [8]. The results in Table 3 show that CSA is able to further improve on results obtained by other approaches. In addition, the actual channel assignment results of all CAP2s with minimum interference are shown in Appendix B, in case the reader wishes to verify and compare with our results.

To show the dynamics of the system, the total energy function E (eq. (11)) in HEX2 is plotted as a function of time in Figure 2. We also plot the constraint energy term (eq. (13)) in Figure 3 and the optimization (interference) energy term (eq. (14)) in Figure 4. Figures 5-7 show the three neuronal input terms (eqs. (15), (16) and (17)) for HEX2, respectively.

The constraint energy term in eq. (11) enforces the demand constraint:

$$\frac{W_1}{2}\sum_{j=1}^{N}\left(\sum_{k=1}^{M} x_{jk} - D_j\right)^2 \quad . \tag{13}$$

Table 3. The simulation results of CSA and other heuristics.

	GAMS	SD		SSA		HN		HCHN		SONN		**CSA**	
problem	Min	Ave	Min	Ave	Min	Ave	Min	Ave	Min	Ave	Min	Ave	Min
EX1	2	0.6	0	0.0	0	0.2	0	0.0	0	0.4	0	**0.0**	**0**
EX2	3	1.1	0	0.1	0	1.8	0	0.8	0	2.4	0	**0.0**	**0**
HEX1	54	56.8	55	50.7	49	49.0	48	48.7	48	53.0	52	**48.1**	**47**
HEX2	27	28.9	25	20.4	19	21.2	19	19.8	19	28.5	24	**18.9**	**18**
HEX3	89	88.6	84	82.9	79	81.6	79	80.3	78	87.2	84	**77.1**	**76**
HEX4	31	28.2	26	21.0	17	21.6	20	18.9	17	29.1	22	**17.7**	**17**
KUNZ1	28	24.4	22	21.6	21	22.1	21	21.1	20	22.0	21	**21.0**	**21**
KUNZ2	39	38.1	36	33.2	32	32.8	32	31.5	30	33.4	33	**31.3**	**31**
KUNZ3	13	17.9	15	13.9	13	13.2	13	13.0	13	14.4	14	**13.0**	**13**
KUNZ4	7	5.5	3	1.8	1	0.4	0	0.1	0	2.2	1	**0.0**	**0**

Table 4. The parameters used in CSA for various CAP2s.

Problem	K	ε	I_0	α	β	$z(0)$	W_1	W_2
EX1	0.9	1/250	0.65	0.0045	0.0005	0.1	1.0	0.02
EX2	0.9	1/250	0.65	0.0045	0.0005	0.1	1.0	0.02
HEX1	0.9	1/150	0.05	0.05	0.0005	0.08	1.0	0.25
HEX2	0.9	1/150	0.05	0.05	0.0005	0.08	1.0	0.25
HEX3	0.9	1/250	0.05	0.05	0.0005	0.08	1.0	0.2
HEX4	0.9	1/250	0.05	0.05	0.0005	0.08	1.0	0.3
KUNZ1	0.9	1/150	0.05	0.05	0.0004	0.08	1.0	0.45
KUNZ2	0.9	1/150	0.05	0.05	0.0005	0.08	1.0	0.45
KUNZ3	0.9	1/150	0.05	0.05	0.0005	0.08	1.0	0.45
KUNZ4	0.9	1/150	0.05	0.05	0.0005	0.08	1.0	0.45

The optimization (interference) energy term in eq. (11) minimizes the interference:

$$\frac{W_2}{2}\sum_{j=1}^{N}\sum_{k=1}^{M}x_{jk}\sum_{i=1}^{N}\sum_{l=1}^{M}P_{ji(|k-l|+1)}x_{il} \quad . \tag{14}$$

The single-neuron input term in eq. (12) is responsible for generating chaotic dynamics:

$$ky_{jk}(t) - z(t)(x_{jk}(t) - I_0) \quad . \tag{15}$$

The constraint input term in eq. (12) enforces the demand contraint:

$$\alpha\{-W_1\sum_{q\neq k}^{M}x_{jq} + W_1 D_j\} \quad . \tag{16}$$

The interference input term in eq. (12) minimizes interference:

$$\alpha\{-W_2 \sum_{p=1,p\neq j}^{N} \sum_{q=1,q\neq k}^{M} P_{jp(|k-q|+1)} x_{pq}\} \quad . \tag{17}$$

Parameters (Table 4) are chosen so that the constraint energy term in eq. (13) is comparable in magnitude to the interference energy term in eq. (14). Similarly, the three neuronal input terms in eqs. (15) - (17) need also to be comparable in magnitude, so that each term can efficiently play its role.

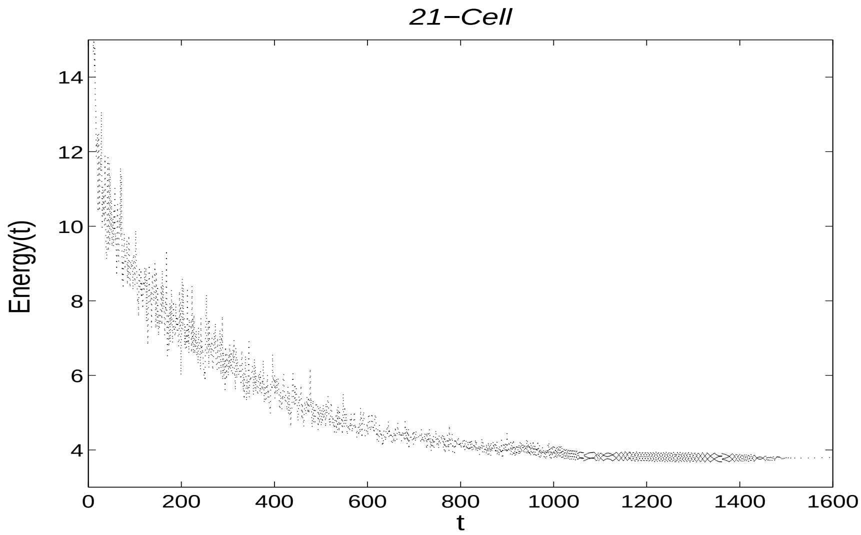

Fig. 2. The Energy (eq. (11)) as a function of time in HEX2.

6 Conclusions

In this chapter, we have considered the CAP2 and demonstrated that chaotic simulated annealing (CSA) is able to further improve on results obtained by other algorithms in several benchmarking CAP2s.

Our work shows the potential of CSA in CAP2s, but it is concerned with only static CAPs. In a dynamic CAP, the demand becomes a function of time. Furthermore, CSA is deterministic and is not guaranteed to settle down at a global minimum. Therefore overcoming those deficiencies and implementation of CSA to solve other practical optimization problems, such as the dynamic CAP, will be studied in future work.

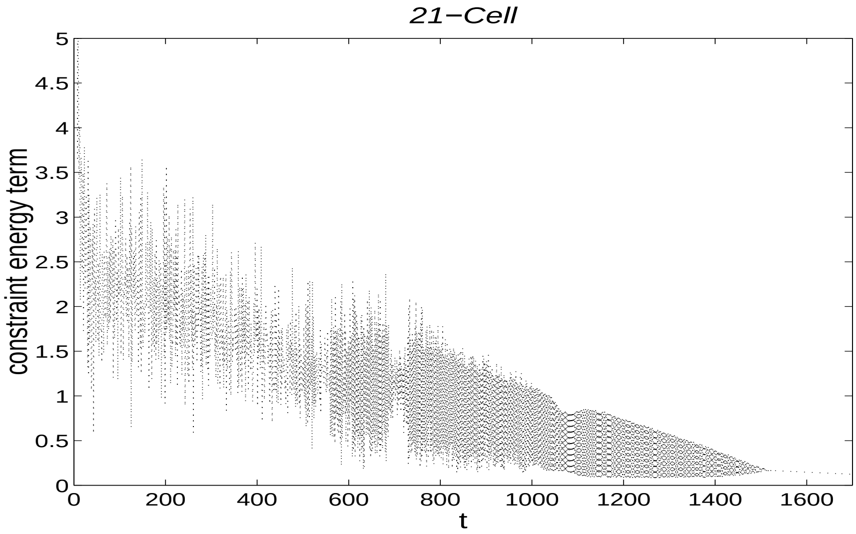

Fig. 3. The constraint energy term (eq. (13)) as a function of time in HEX2.

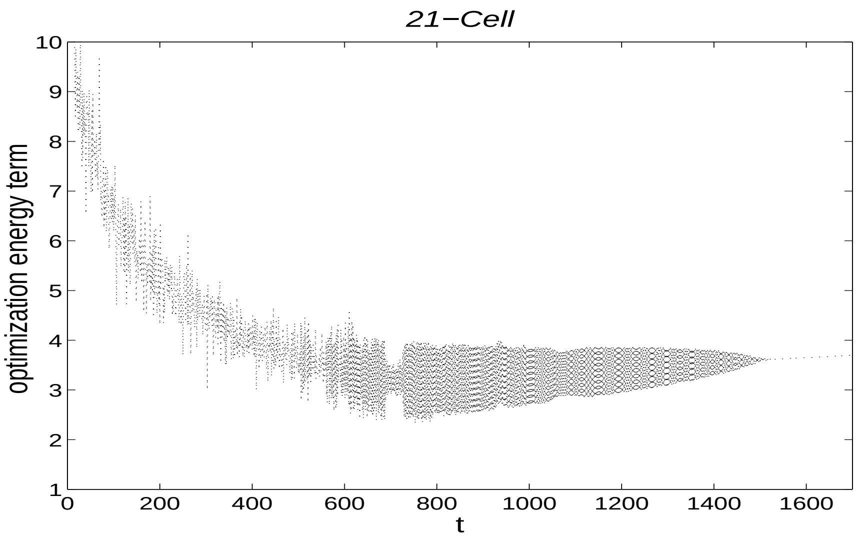

Fig. 4. The optimization (interference) energy term (eq. (14)) as a function of time in HEX2.

APPENDIX A. Compatability Matrices of HEX and KUNZ Problems

There are two different compatability matrices used for the four HEX problems. The matrices are calculated as follows by considering the first two rings

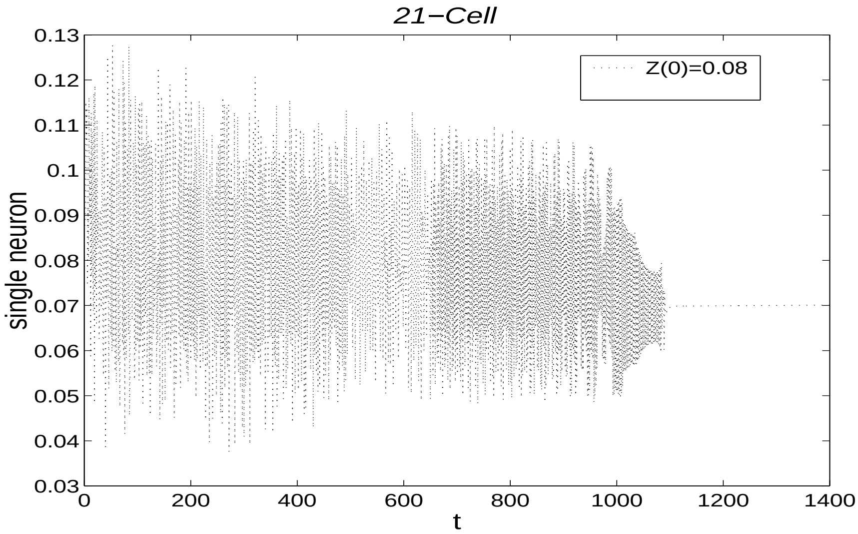

Fig. 5. The single-neuron input term (eq. (15)) as a function of time in HEX2.

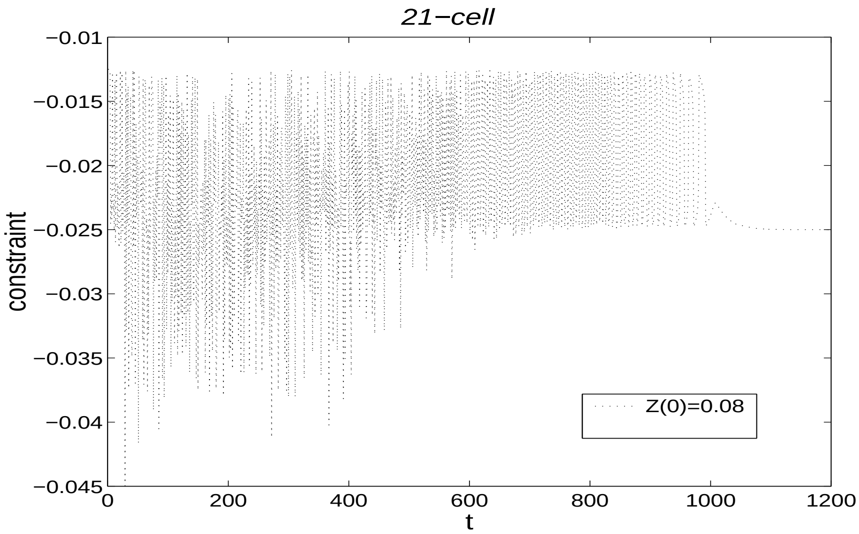

Fig. 6. The constraint input term (eq. (16)) as a function of time in HEX2.

of cells around a particular cell as the sources of interference (Figure 1). The two matrices are both 21×21 in dimension, and they are symmetric matrices according to Gamst and Rave's definition [1]. Each diagonal term C_{ii} represents the minimum separation distance between any two frequencies assigned to cell i, which corresponds to CSC.

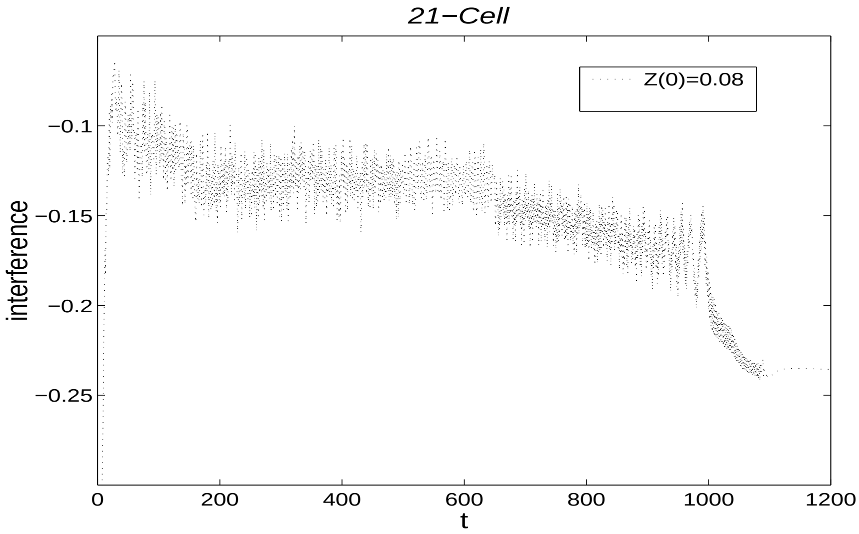

Fig. 7. The interference input term (eq. (17)) as a function of time in HEX2.

Table 1 denotes that diagonal terms $C_{ii}^{(1)}$ in $C^{(1)}$ are 2. The CCC is represented by off-diagonal element C_{ij}=1, and ACC is represented by C_{ij}=2. C_{ij}=0 means that cells i and j are allowed to use the same frequency. From Table 1, $C^{(1)}$ used in HEX1 and HEX3 includes only CCC and CSC but ACC, thus the off-diagonal terms $C_{ij}^{(1)}$ are 1 and 0 corresponding to CCC and no interference, respectively. For Example, in Figure 1, cell 2 and cell 3 are in the first two rings of cells around cell 1, so CCC exists between cell 1 and cell 2 as well as cell 1 and cell 3. Thus $C_{12}^{(1)}$=$C_{21}^{(1)}$=1 and $C_{13}^{(1)}$=$C_{31}^{(1)}$=1. Cell 4 and cell 5 are not among the first two rings of cells around cell 1, so CCC does not exist between cell 1 and cell 4 or between cell 1 and cell 5. Thus $C_{14}^{(1)}$=$C_{41}^{(1)}$=0 and $C_{15}^{(1)}$=$C_{51}^{(1)}$=0.

$$
C^{(1)} = \begin{pmatrix}
2 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 2 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\
1 & 1 & 2 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\
0 & 1 & 1 & 2 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 2 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 2 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 1 & 2 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 & 1 & 1 & 2 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\
1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 2 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 2 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\
0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 2 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 2 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 2 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 2 & 1 & 1 & 0 & 1 & 1 & 0 \\
1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 \\
0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 2 & 1 & 1 & 1 & 1 \\
0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 2 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 2 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 2 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 2
\end{pmatrix}
$$

The matrix $C^{(2)}$ used in HEX2 and HEX4 includes CCC, CSC and ACC. The diagonal terms $C_{ii}^{(2)}$ in $C^{(2)}$ are all 3 as shown in Table 1, and the off-diagonal terms of $C_{ij}^{(2)}$ are 1 or 2 corresponding to CCC and ACC, respectively.

$$
C^{(2)} = \begin{pmatrix}
3 & 2 & 1 & 0 & 0 & 1 & 2 & 2 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
2 & 3 & 2 & 1 & 0 & 0 & 1 & 2 & 2 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\
1 & 2 & 3 & 2 & 1 & 0 & 0 & 1 & 2 & 2 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\
0 & 1 & 2 & 3 & 2 & 0 & 0 & 0 & 1 & 2 & 2 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 2 & 3 & 0 & 0 & 0 & 0 & 1 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 3 & 2 & 1 & 0 & 0 & 0 & 0 & 2 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 1 & 0 & 0 & 0 & 2 & 3 & 2 & 1 & 0 & 0 & 0 & 1 & 2 & 2 & 1 & 0 & 0 & 1 & 0 & 0 \\
2 & 2 & 1 & 0 & 0 & 1 & 2 & 3 & 2 & 1 & 0 & 0 & 0 & 1 & 2 & 2 & 1 & 0 & 1 & 1 & 0 \\
1 & 2 & 2 & 1 & 0 & 0 & 1 & 2 & 3 & 2 & 1 & 0 & 0 & 0 & 1 & 2 & 2 & 1 & 1 & 1 & 1 \\
0 & 1 & 2 & 2 & 1 & 0 & 0 & 1 & 2 & 3 & 2 & 1 & 0 & 0 & 0 & 1 & 2 & 2 & 0 & 1 & 1 \\
0 & 0 & 1 & 2 & 2 & 0 & 0 & 0 & 1 & 2 & 3 & 2 & 0 & 0 & 0 & 0 & 1 & 2 & 0 & 0 & 1 \\
0 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 1 & 2 & 3 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 3 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 2 & 2 & 1 & 0 & 0 & 0 & 0 & 2 & 3 & 2 & 1 & 0 & 0 & 1 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 1 & 2 & 2 & 1 & 0 & 0 & 0 & 1 & 2 & 3 & 2 & 1 & 0 & 2 & 1 & 0 \\
1 & 1 & 1 & 0 & 0 & 0 & 1 & 2 & 2 & 1 & 0 & 0 & 0 & 1 & 2 & 3 & 2 & 1 & 2 & 2 & 1 \\
0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 2 & 2 & 1 & 0 & 0 & 0 & 1 & 2 & 3 & 2 & 1 & 2 & 2 \\
0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 2 & 2 & 1 & 0 & 0 & 0 & 1 & 2 & 3 & 0 & 1 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 2 & 2 & 1 & 0 & 3 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 2 & 2 & 1 & 2 & 3 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 2 & 2 & 1 & 2 & 3
\end{pmatrix}
$$

The compatability matrix $C^{(3)}$ is given by [8][9]:

$$C^{(3)} = \begin{pmatrix}
2&1&1&0&1&0&1&1&1&1&0&1&1&1&1&0&0&0&0&0&0&0&0&0&0\\
1&2&1&0&1&0&1&1&0&1&0&1&1&1&1&0&0&0&0&0&0&0&0&0&0\\
1&1&2&1&1&1&1&1&1&1&1&1&1&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&2&0&0&1&1&1&1&1&1&1&0&0&0&0&0&0&0&0&0&1&1&1\\
1&1&1&0&2&0&0&0&0&1&1&1&1&1&1&1&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&2&1&1&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
1&1&1&1&0&1&2&1&1&1&1&1&1&0&0&0&0&0&0&0&0&0&0&0&0\\
1&1&1&1&0&1&1&2&1&1&1&1&1&0&0&0&0&0&0&0&0&0&0&1&0\\
1&0&1&1&0&1&1&1&2&1&1&1&0&0&0&0&0&0&0&0&0&0&0&1&1\\
1&1&1&1&1&1&1&1&1&2&1&1&1&1&1&1&0&0&0&0&0&1&0&1&0\\
0&0&1&1&1&0&1&1&1&1&2&0&1&1&1&1&0&1&1&1&1&1&1&1&1\\
1&1&1&1&1&0&1&1&1&1&0&2&1&1&0&0&0&0&0&0&0&0&0&0&0\\
1&1&1&1&1&0&1&1&0&1&1&1&2&1&1&1&1&1&1&1&0&0&0&0&0\\
1&1&1&0&1&0&0&0&0&1&1&1&1&2&1&1&1&1&1&1&0&0&0&0&0\\
1&1&0&0&1&0&0&0&0&1&1&0&1&1&2&1&1&1&1&1&1&1&0&0&0\\
0&0&0&0&1&0&0&0&0&1&1&0&1&1&1&2&1&1&1&1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&1&2&1&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0&1&1&1&1&1&2&1&1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0&1&1&1&1&1&1&2&1&1&1&1&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0&1&1&1&1&0&1&1&2&1&1&1&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0&0&0&1&0&0&0&1&1&2&1&1&0&0\\
0&0&0&0&0&0&0&0&0&1&1&0&0&0&1&0&0&0&1&1&1&2&1&1&1\\
0&0&0&1&0&0&0&0&0&0&1&0&0&0&0&0&0&0&1&1&1&1&2&1&1\\
0&0&0&1&0&0&0&1&1&1&1&0&0&0&0&0&0&0&0&0&0&1&1&2&1\\
0&0&0&1&0&0&0&0&1&0&1&0&0&0&0&0&0&0&0&0&0&1&1&1&2
\end{pmatrix}$$

APPENDIX B. Channel Assignments Obtained

The actual channel assignments solutions obtained by CSA with minimum interference in various CAP2s are listed as follows, in case the reader wishes to verify and compare with our results.

Table 5. Channel assignment for EX1 problem with interference 0.

Base Station No.	# Channels	Assigned channels
1	1	11
2	1	2
3	1	3
4	3	1,6,11

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Table 6. Channel assignment for EX2 problem with interference 0.

Base Station No.	# Channels	Assigned channels
1	2	1,17
2	2	5,13
3	2	4,14
4	4	1,6,12,17
5	3	3,8,15

Table 7. Channel assignment for HEX1 problem with interference 47.

Base Station No.	# Channels	Assigned channels
1	2	15,21
2	6	6,13,17,22,26,29
3	2	24,26
4	2	11,32
5	2	20,29
6	4	6,20,22,35
7	4	4,18,21,28
8	13	3,5,7,9,11,14,19,23,25,27,30,32,37
9	19	1,2,4,6,8,10,12,14,16,18,20,22,25,27,29,31,33,35,37
10	7	7,15,17,21,28,34,36
11	4	5,13,23,30
12	4	2,4,14,24
13	7	8,10,12,15,17,23,27
14	4	2,29,31,33
15	9	3,7,9,11,19,24,28,34,36
16	14	1,3,5,8,10,12,16,20,23,25,27,30,32,35
17	7	2,4,8,18,21,31,33
18	2	9,19
19	2	15,17
20	4	6,13,22,26
21	2	11,24

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Table 8. Channel assignment for HEX2 problem with interference 18.

Base Station No.	# Channels	Assigned channels
1	2	9,72
2	6	18,49,57,62,70,81
3	2	33,55
4	2	76,86
5	2	5,66
6	4	17,41,74,86
7	4	13,39,63,76
8	13	5,20,23,27,34,37,42,47,54,59,67,83,87
9	19	2,3,8,11,14,22,26,29,31,35,40,43,51,65,68,74,77,85,89
10	7	16,19,46,60,63,72,80
11	4	1,12,23,54
12	4	10,41,48,68
13	7	12,23,32,45,62,72,80
14	4	49,57,60,78
15	9	7,10,15,25,30,52,66,69,73
16	14	1,12,17,28,32,45,50,56,61,64,71,75,79,91
17	7	6,24,38,48,53,58,82
18	2	78,88
19	2	21,88
20	4	4,9,41,86
21	2	13,34

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Table 9. Channel assignment for HEX3 problem with interference 76.

Base Station No.	# Channels	Assigned channels
1	1	3
2	1	14
3	1	20
4	2	16,18
5	3	2,6,17
6	6	4,9,12,15,17,21
7	7	1,5,7,11,13,16,20
8	6	6,8,10,14,16,18
9	10	2,4,6,8,10,12,15,17,19,21
10	10	1,3,5,7,9,11,13,15,19,21
11	11	1,3,5,7,9,11,13,14,16,18,21
12	5	4,8,10,12,20
13	7	3,6,8,10,12,14,19
14	6	2,4,7,13,15,19
15	4	5,11,18,20
16	4	6,9,12,17
17	7	2,4,8,12,15,17,19
18	5	1,3,5,11,14
19	5	1,3,5,9,21
20	5	4,7,13,16,20
21	6	2,8,10,14,18,20

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Table 10. Channel assignment for HEX4 problem with interference 17.

Base Station No.	# Channels	Assigned channels
1	1	4
2	1	53
3	1	9
4	2	21,33
5	3	4,23,40
6	6	15,22,26,32,36,41
7	7	13,17,29,44,48,51,55
8	6	6,10,25,33,37,40
9	10	3,14,18,22,28,31,43,47,50,56
10	10	5,11,16,24,31,36,41,46,51,54
11	11	2,7,15,19,27,30,35,38,44,48,52
12	5	10,17,25,42,55
13	7	1,5,9,18,30,43,50
14	6	3,7,11,39,46,53
15	4	19,23,27,42
16	4	12,15,45,52
17	7	1,8,20,26,34,39,45
18	5	13,29,32,49,56
19	5	2,9,21,32,35
20	5	4,17,30,48,55
21	6	6,10,23,37,42,53

Table 11. Channel assignment for KUNZ1 problem with interference 21.

Base Station No.	# Channels	Assigned channels
1	10	2,4,6,10,12,18,20,22,24,26
2	11	2,4,8,10,12,14,16,20,24,26,29
3	9	1,3,6,11,15,22,25,28,30
4	5	2,4,12,20,24
5	9	5,7,9,13,17,19,21,23,27
6	4	2,10,12,26
7	5	7,9,19,21,23
8	7	5,9,13,17,21,23,27
9	4	8,14,16,29
10	8	1,3,11,15,18,25,28,30

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Table 12. Channel assignment for KUNZ2 problem with interference 31.

Base Station No.	# Channels	Assigned channels
1	10	1,3,8,12,21,24,26,32,34,42
2	11	5,8,12,20,24,26,31,34,36,38,42
3	9	2,11,13,15,17,19,27,30,44
4	5	3,8,26,34,42
5	9	7,14,25,29,33,37,39,41,43
6	4	21,31,34,36
7	5	14,25,29,33,43
8	7	7,9,23,25,37,39,41
9	4	18,20,28,40
10	8	2,4,6,10,16,19,22,35
11	8	1,5,12,21,24,32,36,38
12	9	5,11,13,15,17,21,32,38,44
13	10	4,10,16,18,20,22,28,31,35,40
14	7	7,9,23,25,37,41,43
15	7	11,13,15,17,27,30,44

Table 13. Channel assignment for KUNZ3 problem with interference 13.

Base Station No.	# Channels	Assigned channels
1	10	6,11,18,21,26,28,39,41,46,51
2	11	1,4,9,13,23,36,44,47,49,54,58
3	9	10,15,20,23,25,32,36,53,56
4	5	1,4,6,41,44
5	9	14,19,27,30,40,42,48,50,55
6	4	9,18,22,26
7	5	7,24,30,50,57
8	7	14,27,38,40,42,48,55
9	4	16,31,45,49
10	8	3,8,10,12,17,32,43,46
11	8	9,26,28,47,51,54,58,60
12	9	2,11,22,33,35,37,47,54,60
13	10	5,12,16,22,29,31,39,45,52,59
14	7	7,24,29,34,38,52,57
15	7	2,15,20,25,35,37,53
16	6	1,4,33,41,44,49
17	4	11,48,54,58
18	5	3,10,19,40,42
19	5	8,23,27,32,55
20	7	14,17,21,30,36,43,46

Table 14. Channel assignment for KUNZ4 problem with interference 0.

Base Station No.	# Channels	Assigned channels
1	10	5,9,18,24,28,42,49,57,61,65
2	11	4,16,22,26,38,40,44,53,59,63,67
3	9	1,12,21,30,34,39,43,45,66
4	5	22,28,42,49,61
5	9	8,15,23,27,33,37,46,62,72
6	4	11,16,24,73
7	5	8,23,31,37,56
8	7	15,33,46,54,60,62,72
9	4	7,29,48,58
10	8	2,6,13,19,25,51,64,70
11	8	5,16,20,24,32,38,55,57
12	9	10,14,20,32,36,41,47,55,68
13	10	3,7,11,29,48,50,58,69,71,73
14	7	17,31,35,52,54,56,60
15	7	1,10,12,34,41,45,66
16	6	9,28,39,44,59,63
17	4	5,13,15,20
18	5	21,25,33,43,62
19	5	6,40,67,70,72
20	7	2,13,15,22,36,49,51
21	6	21,28,33,37,46,62
22	4	50,54,61,65
23	5	19,27,31,58,71
24	7	8,12,21,26,36,41,47
25	5	6,13,25,46,64