An Adaptive Noisy Chaotic Neural Network Approach for Frequency Assignment in Satellite Communications Systems

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***Abstract*— We propose an adaptive noisy chaotic neural network (A-NCNN) to solve the frequency assignment problem (FAP) in the satellite communications systems. The objective of this NP-complete problem is to minimize the cochannel interference between two satellite systems by rearranging the frequency assignments. In the A-NCNN, the noisy chaotic neurondynamics manages the constraint satisfaction while the adaptive mapping scheme achieve the optimization of the problem. The performance of our A-NCNN is demonstrated through solving a set of benchmark problems, where the A-NCNN can find better solutions compared to the previous algorithms.**

I. Introduction

In satellite communication systems, cochannel interference seriously affects system design and operation [1]. Intersystem interference is an example of such cochannel interference. An effective way to deal with the interference reduction is the rearrangement of frequency assignments in practical situations [1]. The communications are assumed to operated between F_a and F_b as showed in Fig. 1. The cochannel interference can be evaluated by calculating the each pair of carriers using the same frequency, which is varied with different pairs. Because each carrier occupies different length of frequency. In order to avoid the nonlinearity, Mizuike *et al.* proposed the segmentation method of carriers, which uniformly divided the commonly shared frequency band into a number of discrete ”segments”. In each system, every carrier can be divided into consecutive unit segments. Thus the interference between the two systems (say M segments) in Fig. 1 is described by a $M \times M$ interference matrix $E = (e_{ij})$. The ijth element e_{ij} represents the cochannel interference when segment $\#i$ in system 2 use a common frequency with segment $\#j$ in system 1. Fig. 2(a) shows the segmentation of the system in Fig. 1 and the optimum assignment under the interference matrix in Fig. 2(b). In order to reduce the cochannel interference between the two adjacent systems, the frequency assignment in system 2 is rearranged but the assignments in system 1 is fixed. In this way, the FAP is equivalent to assign each segment in system 2 to a segment in system 1 of using a common frequency. The frequency reassignment can be formulated as a NP-Complete combinatorial optimization problem known as frequency assignment problem (FAP) [1]. Mizuike *et al.* introduced the segmentation of carriers and solved the problem using branch and bound. Funabiki *et al.* [2] used gradual neural network (GNN) which consists of $N \times M$ binary neurons for the N-carrier-M-segment system with gradual expansion scheme of activated neurons. Salcedo-Sanz *et al.* presented a hybrid method which combines Hopfield network and simulated annealing (HopSA) to tackle the FAP. The HopSA algorithm consists of a fast digital Hopfield neural network which manages the problem constraints and a simulated annealing which improves the solutions obtained.

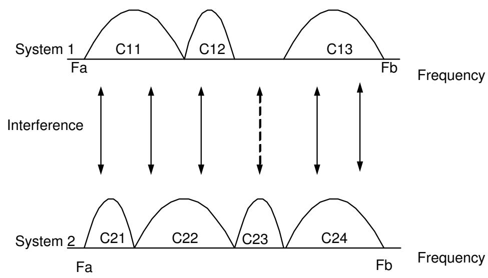
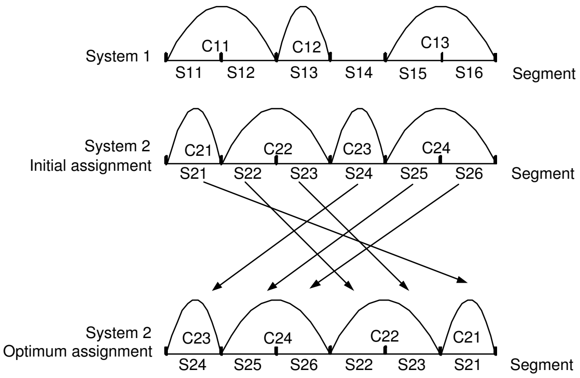


Fig. 1. Cochannel interference model for two adjacent satellite communications systems.

II. Problem Formulation

The primary objective of the FAP is to minimize the largest element of the interference matrix selected in the assignment. The second objective is to minimize the sum of interference of all the selected elements. The three constraints are that 1) every segment in system 2 must be assigned to a segment in system 1; 2) each segment in system 1 can assigned at most one segment in system 2; and 3) all segments of each carrier in system 2 should be assigned to consecutive segments in system 1 in the same order.

In this paper, we used a two-dimensional neural network which consists of $N \times M$ neurons for the FAP of N carriers



(a)

		C11		C12		C14	
		S11	S12	S13	S14	S15	S16
C21	S21	20	20	40	0	25	25
C22	S22	50	10	30	0	55	*
	S23	*	50	30	0	15	55
C23	S24	30	30	45	0	35	35
C24	S25	45	5	25	0	50	*
	S26	*	45	25	0	10	50

(b)

Fig. 2. Segmentation and interference matrix for the system in Fig. 1. (a) Segmentation of the two systems. (b) the interference matrix for the M-segment system.

and M segments. The output of each neuron V_{ij} will be converted into the binary values V_{ij}^d. The value V_{ij}^d of neuron $\#ij$ represents whether the carrier $\#i$ is assigned to the segments $\#j - \#(j + c_i - 1), (i = 1, ..., N; j = 1, ..., M)$, where c_i indicates the length of carrier $\#i$. In section 2, we formulate the frequency assignment problem. In section 3, the adaptive noisy chaotic neural network model is proposed and the application to the FAP is discussed. Numerical results are stated and the performance is evaluated in section 4. In Section 5 we conclude the paper.

III. Adaptive Noisy Chaotic Neural Network

A. Model Definition

Since the Hopfield neural network (HNN) [3] [4] was proposed to solve the travelling salesman problem (TSP), there are many research efforts in combinatorial optimization using Hopfield-like neural networks. However, it was criticized by Wilson *et al.* [5] because of its low convergence and the poor solution quality. Simulated annealing (SA) has been combined with it [6] [7] and recently chaotic simulated annealing (CSA) proposed by Chen *et al.* [8] [9] is proved to be an effective in combinatorial optimization. CSA uses a transient chaotic neural network (TCNN) uses a deterministic dynamics to restrict search to a sub-space of the chaotic attracting set. The TCNN is shown to have global searching ability [10], however, it is not guaranteed to find a global minima no matter how slowly the annealing parameter is reduced [11]. Wang and Tian proposed the noisy chaotic neural network (NCNN) by adding the decaying stochastic noise into the TCNN. The NCNN performs stochastic searching both before and after chaos disappears, and is more likely to find optimal or sub-optimal solutions [12]:

$$x_{jk}(t) = \frac{1}{1 + e^{-y_{jk}(t)/\varepsilon}} , \quad (1)$$

$$\begin{aligned} y_{jk}(t+1) &= ky_{jk}(t) + -z(t)\left[x_{jk}(t) - I_0\right] + n(t) \\ &+\alpha \left\{ \sum_{i=1, i\neq j}^{N} \sum_{l=1, l\neq k}^{M} w_{jkil} x_{jk}(t) + I_{jk} \right\} , \end{aligned} \quad (2)$$

$$z(t+1) = (1 - \beta_1) z(t) , \quad (3)$$

$$n(t+1) = (1 - \beta_2) n(t) , \quad (4)$$

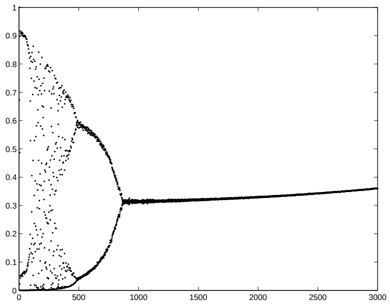
where the notations are:
x_{jk}, y_{jk} : output, input of neuron jk ;
w_{jkil}: connection weight from neuron jk to neuron il, with $w_{jkil} = w_{iljk}$ and $w_{jkjk} = 0$;

$$\sum_{\substack{i=1\\ i\neq j}}^{N} \sum_{\substack{l=1\\ l\neq k}}^{M} w_{jkil} x_{jk} + I_{ij} = -\partial E / \partial x_{jk} : \text{input to neuron jk} \quad (5)$$

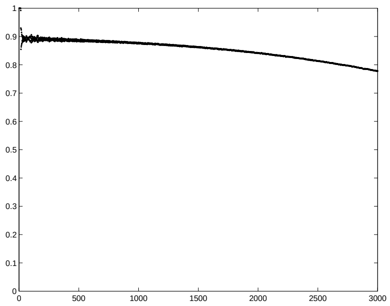
E : energy function;
I_{jk} : input bias of neuron jk ;
k : damping factor of nerve membrane $(0 \leq k \leq 1)$;
α : positive scaling parameter for inputs ;
β_1 : damping factor for neuronal self-coupling $(0 \leq \beta_1 \leq 1)$;
β_2 : damping factor for stochastic noise $(0 \leq \beta_2 \leq 1)$;
$z(t)$: self-feedback connection weight or refractory strength $(z(t) \geq 0)$, $z(0)$ is a constant;
I_0 : positive parameter;
ε : steepness parameter of the output function $(\varepsilon > 0)$;
$n(t)$: random noise injected into the neurons, in $[-A, A]$ with a uniform distribution;
$A[n]$: amplitude of noise n.

The single neuron dynamics of NCNN by varying the parameter I_0 is showed in Fig. 3 (the x-axis is the time steps t, the y-axis is the output of neuron $x_{ij}(t)$). We can see that larger biases (I_0) result in larger firing probabilities and vice versa. Thus we proposed a novel neural network called adaptive noisy chaotic neural network described as follows:

$$\begin{aligned} y_{jk}(t+1) = ky_{jk}(t) + \alpha &\left\{ \sum_{\substack{i=1\\ i\neq j}}^{N} \sum_{\substack{l=1\\ l\neq k}}^{M} w_{jkil} x_{jk}(t) + I_{jk} \right\} \\ &- z(t)\left[x_{jk}(t) - I_{jk}^b\right] + n(t) . \end{aligned} \quad (6)$$



(a) $I_0 = 0.3$



(b) $I_0 = 0.9$

Fig. 3. The single neuron dynamics of adaptive noisy chaotic neural network with different value of I_0.

where I^b_{jk} is not a constant parameter but a variable which determines the selection of firing of each neuron. The value of I_{jk} is related to the problem optimization term. In this problem, the objective is to minimize the largest element of the interference matrix selected in the assignment and at the same time minimize the sum of interference of all the selected elements. Thus we define the choose of I^b_{jk} as follows:

$$I^b_{jk} = 1 - \frac{d_{jk} - d_{j,min}}{d_{j,max} - d_{j,min}} \ . \quad (7)$$

where d_{jk} is the jk element in the cost matrix $D = (d_{ij}, i = 1, ..., N; j = 1, ..., M)$, and $d_{j,max}$ is the maximum value in the line j of matrix D and $d_{j,min}$ is the minimum value the line j of matrix D. The cost d_{ij} for neuron $\#ij$ is given by the largest element in interference matrix among c_i elements $e_{kj}, e_{k+1,j+1}, ..., e_{k+c_i-1,j+c_i-1}$, if assign the carrier $\#i$ to segments $\#j - \#(j + c_i - 1)$, where k is the first segment number of carrier $\#i$ in the interference matrix and c_i is the length of carrier $\#i$ [2].

From the linear mapping in eqn (7), it can be seen that the neuron $\#ij$ with smaller cost c_{ij} will have bigger probability to fire while the ones with larger cost will be inhibited to fire. Through the mapping in eqn. (7), we can not only achieve the objectives of FAP but also separate the objective from the formulation of energy function, which will make the tuning of weighting coefficients in energy function more easier without the need to balance the optimization term and constraint term in one energy function. Moreover, it will improve the convergence speed of the noisy chaotic neural network as shown in the result discussion section.

B. A-NCNN for FAP

We formulate the energy function of A-NCNN as follows:

$$E = \frac{W_1}{2} \sum_{i=1}^{N} \sum_{j=1}^{M} \sum_{\substack{p=1 \\ p \neq i}}^{N} \sum_{q=j-c_p+1}^{j+c_i-1} V_{ij} V_{pq} + \frac{W_2}{2} \sum_{i=1}^{N} (\sum_{q=1}^{M} V_{iq} - 1)^2 + \frac{W_3}{2} \sum_{i=1}^{N} \sum_{j=1}^{M} V_{ij}(1 - V_{ij}) \ . \quad (8)$$

where W_1 and W_2 are weighting coefficients, W_1 is the second constraint that each segment in system 1 can be assigned to at most one segment in system 2, W_2 re[resents the first constraint that each first segment of N carriers in system 2 must be assigned to one segment among M segments in system 1.

From equations (2), (6) and (8), the dynamic equation for the A-NCNN is:

$$y_{jk}(t+1) = k y_{jk}(t) + \alpha[-W_2(\sum_{q=1}^{M} V_{iq} - 1) - W_1(\sum_{\substack{p=1 \\ p \neq i}}^{N} \sum_{q=j-c_p+1}^{j+c_i-1} V_{pq}) - \frac{W_3}{2}(1 - 2V_{jk})] - z(t) \left[x_{jk}(t) - I^b_{jk}\right] + n(t) \ . \quad (9)$$

The neuron output is continuous between 0 and 1, we convert the continuous output V_{ij} of neuron $\#ij$ to discrete neuron output V^d_{ij}, as follows [8]:

$$V^d_{ij} = \begin{cases} 1 & \text{if } V_{ij} > \sum_{k=1}^{N} \sum_{l=1}^{M} V_{kl}(t)/(N \times M) \\ 0 & \text{otherwise} \end{cases}$$

The NCNN is updated cyclically and asynchronously. The new state information of a neuron is immediately available for the other neurons in the next iteration. The iteration is terminated once a feasible assignment is obtained.

IV. Simulation Results

The A-NCNN contains two type of parameters, i.e., model parameters and weighting coefficients. We choose the model parameters based on the rules that the set parameters will produces richer and more flexible neuron dynamics, the value of these parameters are similar to those used in other optimization problems [12] - [13] . The selection of weighting coefficients is based on the rules that all terms in the energy function should be comparable in magnitude, so that none of them dominates. The values of model parameters shows as follows:

$$k = 0.9, \alpha = 0.015, \beta_1 = 0.001, \beta_2 = 0.0002$$
$$\varepsilon = 1/250, I_0 = 0.65, z_0 = 0.08, A[n(0)] = 0.009 \ . \quad (10)$$

Note that the parameters listed here is only a empirical value, tuning on these parameters is needed when applied to different problems, especially the weighting coefficients.

Table I summarizes the main characteristics of the benchmark problems in this paper. Benchmark problems BM $\#1$

TABLE I

Specification of FAP instances.

Instance	Number of carriers N	Number of segments M	Range of carrier length	Range of interference
BM #1	4	6	1 - 2	5 - 55
BM #2	4	6	1 - 2	1 - 9
BM #3	10	32	1 - 8	1 - 10
BM #4	10	32	1 - 8	1 - 100
BM #5	10	32	1 - 8	1 - 1000
Case #6	18	60	1 - 10	1 - 100
Case #7	30	100	1 - 10	1 - 100

TABLE II

Comparisons of the simulation results (largest interference and total interference) obtained by A-NCNN with GNN and HopSA.

Instance	GNN[2]		HopSA[14]		A-NCNN	
	largest	total	largest	total	largest	total
BM #1	30	100	30	100	30	100
BM #2	4	13	4	13	4	13
BM #3	7	85	9	97	7	88
BM #4	64	880	64	888	64	880
BM #5	640	8693	817	6910	640	7246
Case #6	49	1218	45	1080	37	1052
Case #7	100	4633	98	3396	61	2824

to BM $\#5$ are taken from [2] and cases Case $\#6$ and Case $\#7$ are from [14]. We simulate our A-NCNN on P4 2.4GHZ workstation and we run each instance 50 times with different initial conditions of neural network.

Table II shows the results obtained by A-NCNN and a comparison with other previous methods. For the benchmark problems from $BM\#1$ to $BM\#5$, the A-NCNN algorithm equals or improves the results of other existing algorithms. Our method van obtain better results when the problems become more complicated as showed in Case $\#6$ and Case $\#7$ in Table II. Table III shows the iteration steps and computation time of our A-NCNN for all the problems. We also list the converge rate of our method in this table. One iteration step in A-NCNN stands for one loop that all neurons are updated once. From Table III we can see that our A-NCNN achieves at lease 82% convergence rate in 7 instances. Furthermore, it can converge to a solution in nearly constant iteration steps and the runtime grows slower compared with the increase of the problem size.

TABLE III

Iteration steps, computation time (second), and converge rate for A-NCNN on P4 2.4GHz Workstation.

Instance	Iteration Steps	Time (Sec.)	Convergence Rate
#1	378.3	0.02	100%
#2	478.1	0.02	100%
#3	44.6	0.01	100%
#4	2157.5	2.1	96%
#5	1819.8	1.8	98%
#6	2128.8	11.8	82%
#7	2890.7	78.3	86%

V. Conclusion

In this paper, we propose a novel method called the adaptive noisy chaotic neural network and apply it to solve the frequency assignment problem in satellite communication systems. The A-NCNN consists of $N \times M$ neurons for the N-carrier-M-segment system. We use the linear mapping function which maps the objectives of the FAP into the variable parameter I_i, thus simplified the formulation of FAP and tuning of weighting coefficients. The simulation results of 7 instances show that the A-NCNN can find better solution of cochannel interference compared to the previous methods with very low computational cost.

References

[1] T. Mizuike and Y. ITO, "Optimization of frequency assignment," *IEEE Trans. Communications*, vol. 37, pp. 1031–1041, 1989.

[2] N. Funabiki and S. Nishikawa, "A gradual neural-network approach for frequency assignment in satellite communication systems," *IEEE Trans. Neural Networks*, vol. 8, pp. 1359–1370, 1997.

[3] J. J. Hopfield, "Neurons with graded response have collective computational properties like those of two-state neurons," in *Proc. Natl. Acad. Sci. USA*, vol. 81, pp. 3088–3092, 1984.

[4] J. J. Hopfield and D. W. Tank, "Neural computation of decisions in optimization problems," *Biological Cybernetics*, vol. 52, pp. 141–152, 1985.

[5] G. Wilson and G. Pawley, "On the stalibility of the travelling salesman problem algorithm of hopfield and tank," *Biol. Cybern.*, vol. 58, pp. 63–70, 1988.

[6] C. Peterson and J. R. Anderson, "A mean field theory algorithm for neural networks," *Complex Systems*, vol. 1, pp. 995–1019, 1987.

[7] L. Wang and K. Smith, "On chaotic simulated annealing," *IEEE Transactions on Neural Networks*, vol. 9, pp. 716–718, 1998.

[8] L. Chen and K. Aihara, *Transient chaotic neural networks and chaotic simulated annealing*, pp. 347–352. Amsterdam: Elsevier Science Publishers, 1994.

[9] L. Chen and K. Aihara, "Chaotic simulated annealing by a neural network model with transient chaos," *Neural Networks*, vol. 8, pp. 915–930, 1995.

[10] L. Chen and K. Aihara, "Global searching ability of chaotic neural networks," *IEEE Trans. Circuits and Systems - I: Fundamental Theory and Applications*, vol. 46, no. 8, pp. 974–993, 1999.

[11] I. Tokuda, K. Aihara, and T. Nagashima, "Adaptive annealing for chaotic optimization," *Phys. Rev. E*, vol. 58, pp. 5157–5160, 1998.

[12] L. P. Wang and F. Tian, "Noisy chaotic neural networks for solving combinatorial optimization problems," in *Proc. International Joint Conference on Neural Networks*, vol. 4, pp. 37–40, 2000.

[13] L. P. Wang, S. Li, F. Y. Tian, and X. J. Fu, "A noisy chaotic neural network for solving combinatorial optimization problems: Stochastic chaotic simulated annealing," *IEEE Trans. System, Man, Cybern, Part B - Cybernetics*, vol. 34, pp. 2119–2125, 2004.

[14] S. Salcedo-Sanz, R. Santiago-Mozos, and C. B. no Calzón, "A hybrid hopfield network-simulated annealing approach for frequency assignment in satellite communications systems," *IEEE Trans. Systems, Man, and Cybernetics-Part B: Cybernetics*, vol. 34, pp. 1108–1116, 2004.