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RESEARCH ARTICLE

Interactions between neural networks: a mechanism for tuning chaos and oscillations

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Abstract We show that chaos and oscillations in a higher-order binary neural network can be tuned effectively using interactions between neural networks. Our results suggest that network interactions may be useful as a means of adjusting the level of dynamic activities in systems that employ chaos and oscillations for information processing, or as a means of suppressing oscillatory behaviors in systems that require stability.

Keywords Chaos · Neural network · Bifurcation · Associate memory · Stability · Crisis

Introduction

The main purposes of neural network research include (1) understanding how the brain works and (2) building electronic systems with various features of a most powerful computational device—the human brain. Some parts of the brain, for example, the cerebral cortex, have well-defined layered structures with specific interconnections between different layers (Churchland and Sejnowski 1989). Wang and Ross (Wang and Ross 1990) proposed a system of two interacting neural networks as models for distraction and concentration; however, only the static retrieval properties of the system were investigated. Babloyantz and Lourenco (Babloyantz and Lourenco 1994) studied a neural network system with a similar architecture which has chaotic dynamics and performs pattern discrimination and motion detection. These studies have shown that interactions of neural networks are not only useful, but also important, in determining the cognitive capabilities of a neural system (Wang and Ross 1990; Babloyantz and Lourenco 1994).

In this paper we show that chaos and oscillations in a system of interacting neural networks can be tuned effectively by varying the interconnections between the networks. It is important to discover effective tuning mechanisms for chaotic and oscillatory activities because of the following two reasons. Firstly, *dynamical* behaviors may be used in the brain and man-made systems to efficiently process information, for example, store and retrieve memory. An effective tuning mechanism can be used to determine the level of chaotic or oscillatory behavior in the system, thereby facilitating information processing with dynamic states of the system. Secondly, oscillations and chaos have enjoyed many applications; however, stability is often desirable. For example, an electronic neural network designed to work under stable conditions may lose its functionalties should oscillatory or chaotic activities arise. A mechanism, such as interactions between networks proposed here, may be used to stabilize such a system.

Tuning dynamics with interactions of networks

The system considered here consists of two interconnected neural networks A and B as shown in Fig. 1 (Wang and Ross 1990), with N binary neurons in each

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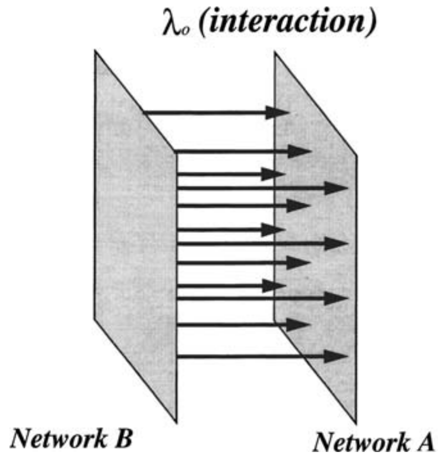

Fig. 1 Two interacting neural networks. In the present paper network B is a binary Hopfield network, whereas network A is a modified binary Hopfield network which includes higher-order weights and random synaptic disruption

network. In network A, neuron i updates its state according to

$$S_i(t+1) = \mathrm{sgn}(h_i(t)), \tag{1}$$

where $\mathrm{sgn}(x) = -1$ if $x \le 0$, $\mathrm{sgn}(x) = 1$ if $x > 0$, and $S_i(t)$ represents the state of neuron i at time t. The total input for neuron i is

$$h_i(t) = \gamma_1 \sum_{j=1}^{N} T_{ij} S_j(t) + \gamma_2 \sum_{j,k=1}^{N} T_{ijk} S_j(t) S_k(t) + \lambda_o S_i'(t) + \eta_i, \tag{2}$$

where

$$T_{ij} = C_{ij} \sum_{\mu=1}^{p} S_i^{\mu} S_j^{\mu} \text{ and } T_{ijk} = C_{ijk} \sum_{\mu=1}^{p} S_i^{\mu} S_j^{\mu} S_k^{\mu} \tag{3}$$

are the modified first-order and second-order Hebbian synaptic weights, $\vec{S}^{\mu}$ is the μ-th stored pattern, and p is the number of patterns stored. The coefficients γ_1 and γ_2 measure the relative strengths of first- and second-order interactions. We have introduced synaptic disruptions in the weights T_{ij} and T_{ijk} by choosing random variables C_{ij} and C_{ijk} as follows: C_{ij} is 1 with a probability (C/N), C_{ijk} is 1 with a probability $(2C/N^2)$, C_{ij} and C_{ijk} are zero otherwise. Hence after synaptic disruption, each neuron interacts with about C neurons with first-order interactions and C pairs of neurons with second-order interactions when the number of neurons N is large (Wang and Ross 1990). We consider a simple form of interaction between the networks, i.e., each neuron in network B sends a signal proportional to its own neuronal state to the corresponding neuron in network A (1), as represented by the third term in the right hand side of Eq.(2), where λ_o measures the strength of the interactions between the two networks and $S_i'(t)$ is the state of neuron i in network B (Russell 2003). We also include in Eq.(2) a background Gaussian noise η_i with a standard deviation σ_o in order to take into account the presence of signal transmission noise.

In the absence of interactions between the two networks ($\lambda_o = 0$), higher-order weights ($\gamma_2 = 0$), and synaptic disruption ($C_{ij} = 1$ for all i and j), network A is a binary Hopfield network (Hopfield 1982). For simplicity, we assume that network B is such a binary Hopfield network and does not receive any input from network A. In addition, we assume that $\vec{S}^1$ is the memory pattern to which the initial state of network A is closest and network B has already reached this memory state, i.e., $S_i'(t) = S^1{}_i$ in Eq.(2) for all i. We will thus concentrate on the dynamic behaviors of network A only. In the absence of interactions between the two networks ($\lambda_o = 0$) and higher-order weights ($\gamma_2 = 0$), network A is the sparsely-connected version of the binary Hopfield network studied by Derrida et al. (Derrida et al. 1987).

In the absence of synaptic disruption ($C_{ij} = C_{ijk} = 1$ for all i, j, and k), a similar system of two interacting networks was used by Wang and Ross (1990) to model distraction and concentration, and no oscillations or chaos were found. In the absence of interactions between the two networks ($\lambda_o = 0$), network A was studied by Wang, Pichler, and Ross (Wang et al. 1990). When each neuron in network A is connected with only a small fraction of all neurons in the network, i.e., $C \ll N$, the dynamics of network A in the presence of network interactions ($\lambda_o \neq 0$) is described by (the derivation is exactly the same as in [Wang et al. 1990] and is not repeated here):

$$m(t+1) = \mathrm{erf}\left\{\frac{\gamma_1 m(t) + \gamma_2 [m(t)]^2 + \lambda}{\sqrt{2}\sigma}\right\}. \tag{4}$$

Here $\lambda = \lambda_o / C$, $m(t) = <\vec{S}(t) \cdot \vec{S}^1 > /N$ is a statistical average of the overlap between the state of the network and the stored pattern to which the network is initially close. $\sigma \equiv \sqrt{(\gamma_1^2 + \gamma_2^2)(p-1)/C + (\sigma_o/C)^2}$ is a rescaled noise level that represents the combined effects of the random synaptic disruption, interference between stored patterns, and additional signal transmission noise. $\mathrm{erf}(x) \equiv 2/\sqrt{\pi} \int_0^x dz e^{-z^2}$ is the standard error function.

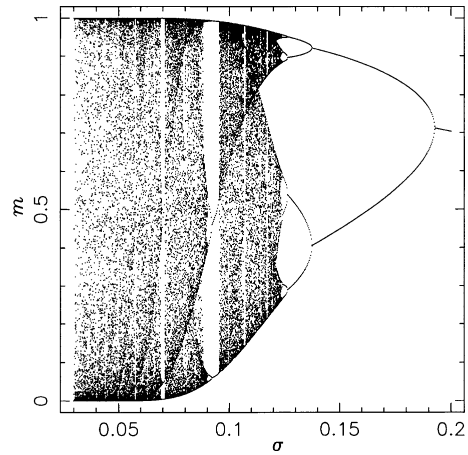

Fig. 2 The average overlap between the state of the network A and the initial attracting memory pattern, as a function of the rescaled noise level σ *in the absence of network interactions*, i.e., $\lambda = 0$. There is a wide spectrum of dynamic behaviors such as stable fixed points, period-doubling bifurcations, and chaos

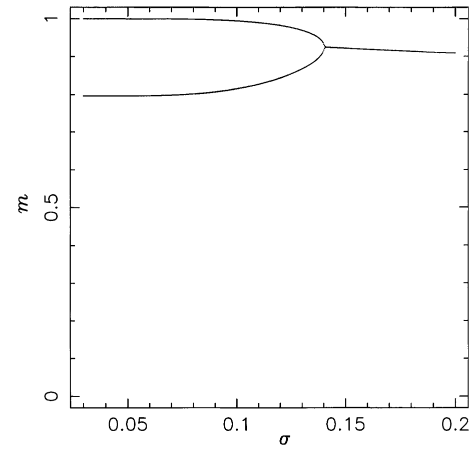

Fig. 4 The average overlap between the state of the network A and the initial attracting memory pattern, as a function of the rescaled noise level σ in the presence of *strong positive network interactions* ($\lambda = 0.9$): Chaos disappears. There are stable fixed points and period-2 oscillations only

Figures 2–5 show the network dynamics for various strengths of network interaction, i.e., different choices of λ, in the case where $\gamma_1 = 1$ and $\gamma_2 = -1$. Fig. 2 shows the case without network interactions ($\lambda = 0$) [Wang et al. 1990], where abundant oscillatory and chaotic activities exist. When a small amount of *positive* network interactions ($\lambda > 0$) is introduced, chaos is first suppressed (Fig.3). As the strength of the network interactions λ increases, chaos disappears and periodic oscillations also become suppressed (Fig.4). Oscillatory behaviors disappear and the system becomes completely stable for $\lambda \geq 2.4$. When *negative* network interactions ($\lambda < 0$) are present, the system shows crisis: for $\sigma < \sigma_{\text{crisis}}(\lambda)$, chaotic and oscillatory behaviors disappear and network A settles at a stable fixed point near $m = -1$ even if the initial overlap is positive ($m(t = 0) > 0$). The dashed line in Fig. 5 represents unstable fixed points at $m = 0$ and serves as the attraction boundary for positive and negative attractors in cases where $\sigma \geq \sigma_{\text{crisis}}$. $\sigma_{\text{crisis}}(\lambda = 0) = 0$ and $\sigma_{\text{crisis}}(\lambda)$ increases as $|\lambda|$ increases.

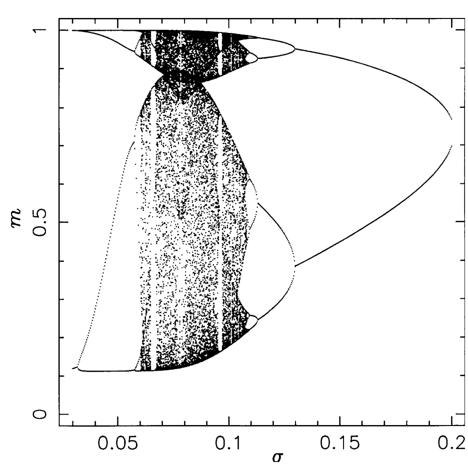

Fig. 3 The average overlap between the state of the network A and the initial attracting memory pattern, as a function of the rescaled noise level σ in the presence of *weak positive network interactions* ($\lambda = 0.1$): suppressed chaotic region

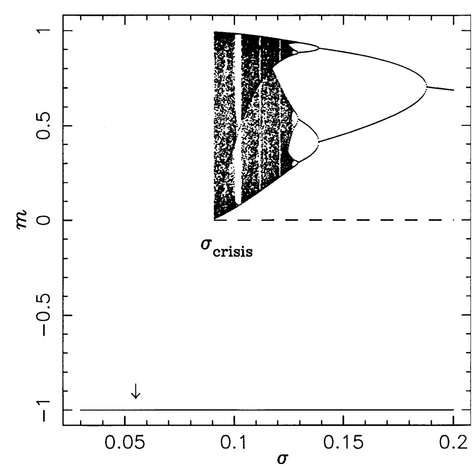

Fig. 5 The average overlap between the state of the network A and the initial attracting memory pattern, as a function of the rescaled noise level σ in the presence of *negative network interactions* ($\lambda = -0.05$): existence of crisis (abrupt disappearance of chaos and oscillations for $\sigma < \sigma_{\text{crisis}}(\lambda = -0.05) = 0.909$). The solid line represents stable fixed points near $m = -1$. The dashed line indicates unstable fixed points at $m = 0$ and represents the boundary of attraction for positive and negative attractors of the system. These fixed points (the solid and the dashed lines) also exist for $\lambda \geq 0$ but are omitted in Figs. 2–4

Summary and discussions

We have demonstrated that interactions between neural networks may serve as an effective means to tune dynamic activities in a higher order neural network. When *dynamic* attractors, instead of *static* ones, are used to store and retrieve information, the level of chaos and oscillations in a nonlinear dynamic system may be adjusted by varying the strength of interaction between the networks. In addition, network interactions can also be used to suppress chaos and oscillations in systems where stability is desired (Wang et al. 2004; Wang and Kate 1998; Wang 1996; Wang 1996). For instance, if one starts with chaos and gradually stabilizes the system by tuning the interaction, it will be similar to chaotic annealing proposed by Chen and Aihara (Chen and Aihara 1995) and Nozawa (Nozawa 1992), and may be used in solving optimization problems (e.g., Haixiang and Lipo 2005; Lipo et al. 2006)).

This paper has considered only one simple type of interactions between neural networks, that is, we considered only two sparsely connected neural networks and we assumed that one of the networks is initially close to a memory pattern and the other network has already reached this memory state. We also assumed that the interaction is uni-directional. These special assumptions enabled us to derive certain results and carry out some discussions. There can be many other cases of interactions between two neural networks, for example, when one of the networks is initially close to a memory pattern, but the other network has settled to a different memory pattern (Wang and Ross 1990). There can also be more than two neural networks interacting and the interactions can be bi-directional. These types of network interactions will be subject to future studies.

Sparse synaptic connections or synaptic disruptions considered in this paper have biological motivations. For example, the human brain has about 10^{11} neurons, but each neuron has, on average, only 10^4 synaptic connections to other neurons. Therefore the synaptic connections in the human brain are far from fully connected and are actually quite sparse. Synaptic disruptions may result from aging or certain diseases (Russell 2003), such as Alzheimer's disease (Bruce and J. Wesson Ashford 2002; Tsai et al. 2004). Some chemicals may also induce synaptic disruptions (Joshua T. Trachtenberg and Wesley J. Thompson 1997; Dan et al. 2003).

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References

Babloyantz A, Lourenco C (1994) Computation with chaos: a paradigm for cortical activity. Proc Natl Acad Sci USA 91:9027–9031

Bruce Teter, J. Wesson Ashford (2002) Neuroplasticity in Alzheimers disease. J Neurosci Res 70:402–437

Chen L, Aihara K (1995). Chaotic simulated annealing by a neural network model with transient chaos. Neural Networks 8(6):915–930

Churchland PS, Sejnowski TJ (1989) The Computational Brain. MIT Press, Cambridge MA

Dan Qiao, Frederic J. Seidler, Charlotte A. Tate, Mandy M. Cousins, Theodore A. Slotkin (2003) Fetal chlorpyrifos exposure: Adverse effects on brain cell development and cholinergic biomarkers emerge postnatally and continue into adolescence and adulthood. Environ Health Persp 111(4):536–544

Derrida B, Gardner E, Zippelius A (1987) An exactly solvable asymmetric neural network model. Europhys Lett 4(2):167–173

Haixiang Shi, Lipo Wang (2005) Broadcast scheduling in wireless multihop networks using a neural-network-based hybrid algorithm. Neural Networks 18:765–771

Hopfield JJ (1982) Neural networks and physical systems with emergent collective computational abilities. Proc Natl Acad Sci USA 79:2554–2558

Joshua T. Trachtenberg, Wesley J. Thompson (1997) Nerve terminal withdrawal from rat neuromuscular junctions induced by neuregulin and schwann cells. J Neurosci 17(16):6243–6255

Lipo Wang and Haixiang Shi, "A gradual noisy chaotic neural network for solving the broadcast scheduling problem in packet radio networks," IEEE Trans Neural Networ, accepted, 2006

Nozawa H (1992) A neural network model as a globally coupled map and applications based on chaos. Chaos 2(3):377–86

Russell Blaylock (2003) Interaction of cytokines, excitotoxins, and reactive nitrogen and oxygen species in autism spectrum disorders. J Am Nutraceutical Assoc 6(4):21–35

Tsai J, Grutzendler J, Duff K, Gan WB (2004) Fibrillar amyloid deposition leads to local synaptic abnormalities and breakage of neuronal branches. Nat Neurosci 7:1181–1183

Wang LP (1996) Oscillatory and chaotic dynamics in neural networks under varying operating conditions. IEEE Trans Neural Networ 7(6):1382–1388

Wang LP (1996) Suppressing chaos with hysteresis in a higher order neural network. IEEE Trans Circuit and Syst-II 43(12):845–846

Wang LP, Kate Smith (1998) On chaotic simulated annealing. IEEE Trans Neural Networ 9(4):716–718

Wang LP, Pichler EE, Ross J (1990) Oscillations and chaos in neural networks: an exactly solvable model. Proc Natl Acad Sci USA 87:9467–9471

Wang LP, Ross J (1990) Interactions of neural networks: models for distraction and concentration. Proc Natl Acad Sci USA 87:7110–7114

Wang LP, Sa Li, Fuyu Tian, Fu XJ (2004) A noisy chaotic neural network for solving combinatorial optimization problems: Stochastic chaotic simulated annealing. IEEE Trans System, Man, Cybern, Part B - Cybernetics 34(5):2119–2125