New Approach for Interpretability of Neuro-Fuzzy Systems with Parametrized Triangular Norms

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Abstract. In this paper we proposed a new approach for interpretability of the neuro-fuzzy systems. It is based on appropriate use of parametric triangular norms with weights of arguments, which shape depends on values of their parameters and weights. The use of those norms as aggregation and inference operators increases precision of fuzzy system. Due to that, the rule base can be simpler and easier to interpretation. However, interpretation of parametric triangular norms is not that obvious as interpretation of nonparametric triangular norms such as algebraic or minimal norms. Proposed approach is based on choosing values of parameters from a set of values, where each value have its own interpretation. Additionally, a modified tuning algorithm for selection both the structure and structure parameters of fuzzy system with interpretability criteria under consideration is proposed. Proposed approach were tested on well-known nonlinear simulation problems.

Keywords: Nonlinear modeling · Fuzzy system · Interpretability criteria · Accuracy

1 Introduction

The fuzzy systems (see e.g. [12,15–21,29,30,34,42,43,48,80,81,85,94–99]) are based on fuzzy rules. In the past researchers paid attention to the accuracy of fuzzy systems while ignoring issues of their interpretability. However, in the 1990 s they started to notice the fact that a large number of rules or fuzzy sets in those rules is not conducive to the readability of the rule base. Nowadays fuzzy system designers are trying to reach an acceptable compromise between accuracy and interpretability [31,38,54]. In the literature a number of papers on the subject of interpretability of fuzzy systems can be found. Their authors have proposed among the others: (a) solutions aimed at reducing the number of fuzzy rules [2,4,31,38,54], reducing the number of fuzzy sets [35], reducing the number of system inputs [4,92] and reducing fuzzy system elements by merging [13,36], (b) solutions related to correct

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notation of fuzzy rules [4,49], correct activation of fuzzy rules [54] and distinguishability and interdependence of fuzzy sets [55,56,64], (c) solutions related to fuzzy systems construction aimed at interpretability, based on additional weights of importance of the rules, antecedences, consequences and system inputs [13,65,71], parameterized triangular norms [13,28,68] and precise defuzzification mechanism [13]. The literature abounds in numerous attempts to systematize solutions for interpretability (e.g. [3,32,82]).

The solutions proposed in this paper can be summarized as follows: (a) it is based on a use of parametric triangular norms with weights of arguments and on appropriate use of values of weights of fuzzy system elements. Proposed idea concerns choosing parameters values from a set of values, where each value have its own interpretation; (b) in this paper a new algorithm for selection the structure and parameters of a fuzzy system, constructed on the basis of the golden ball [60] algorithm is proposed. Moreover, the proposed algorithm takes into account all the interpretability criteria and it belongs to the methods based on populations [71]. The use of the learning algorithm also creates a good opportunity to find an appropriate trade-off between interpretability and accuracy. It is worth to note that many computational intelligence methods (see e.g. [1,5–9,22–27,33,39–41,44,50,58,61–63,66,67,69,76–79,83,86,87,93]) are successfully used in pattern recognition, modelling and optimization issues.

This paper is divided into following sections: in Sect. 2 a description of a fuzzy system is presented. In Sect. 3 a description of proposed learning algorithm is shown. The results of simulations are presented in Sect. 4, finally the conclusions are described in Sect. 5.

2 Description of a Neuro-Fuzzy System

In this paper a typical multi-input, multi-output flexible fuzzy system of the Mamdani-type is considered [13,14,70,71]. Neuro-fuzzy systems combine the natural language description of fuzzy systems and the learning properties of neural networks (see e.g. [11,46,47,84,88–91]). This system performs mapping $\mathbf{X} \to \mathbf{Y}$, where $\mathbf{X} \subset \mathbf{R}^n$ and $\mathbf{Y} \subset \mathbf{R}^m$. The rule base of this system consists of a collection of N fuzzy rules R^k, $k = 1, \ldots, N$. Each rule R^k takes the following form:

$$R^k : \left[\left(\begin{array}{c} \text{IF } \left(x_1 \text{ is } A_1^k\right) \left| w_{1,k}^A \right. \text{ AND ... AND } \left(x_n \text{ is } A_n^k\right) \left| w_{n,k}^A \right. \\ \text{THEN } \left(y_1 \text{ is } B_1^k\right) \left| w_{1,k}^B \right. \text{, ..., } \left(y_m \text{ is } B_m^k\right) \left| w_{m,k}^B \right. \end{array} \right) \left| w_k^{\text{rule}} \right. \right], \quad (1)$$

where n is the number of inputs, m is the number of outputs, $\bar{\mathbf{x}} = [\bar{x}_1, \ldots, \bar{x}_n] \in \mathbf{X}$ is a vector of input signals, $\mathbf{y} = [y_1, \ldots, y_m] \in \mathbf{Y}$ is a vector of output linguistic variables, $A_1^k, \ldots, A_n^k$ are input fuzzy sets characterized by membership functions $\mu_{A_i^k}(x_i)$ $(i = 1, \ldots, n)$, $B_1^k, \ldots, B_m^k$ are output fuzzy sets characterized by membership functions $\mu_{B_j^k}(y_j)$ $(j = 1, \ldots, m)$, $w_{k,i}^A \in [0,1]$ are weights of antecedents, $w_{j,k}^B \in [0,1]$ are weights of consequences, and $w_k^{\text{rule}} \in [0,1]$

are weights of rules. Fuzzy sets A_i^k and B_j^k represent linguistic variables (e.g. '*very low*', '*low*', '*medium*', '*high*', '*very high*', '*near* [value]'). In this paper we consider system based on Gaussian membership functions, which reflects well the industrial, natural, medical and social processes; however, our solutions may be related to any other membership function. The flexibility of the system is a result of using: (a) weights in the rule base, (b) precise aggregation operators of antecedences and rules (Sect. 2.2), (c) precise inference operators (Sect. 2.2), and (d) a precise defuzzification process (Sect. 2.1).

2.1 Defuzzification Process

Defuzzification is used to determine output signals $\bar{y}_j$ of fuzzy system for given input signals. This is carried out as follows (with center of area method):

$$\bar{y}_j = \frac{\sum\limits_{r=1}^{R_j} \bar{y}_{j,r}^{\text{def}} \cdot \mathop{\overset{\leftrightarrow}{S}{}^{*}}\limits_{k=1}^{N} \left\{ \overset{\leftrightarrow}{T}{}^{*} \left\{ \tau_k\left(\bar{\mathbf{x}}\right), \mu_{B_j^k}\left(\bar{y}_{j,r}^{\text{def}}\right); 1, w_{j,k}^{B}, p^{\text{imp}} \right\}; w_k^{\text{rule}}, p^{\text{agr}} \right\}}{\sum\limits_{r=1}^{R_j} \mathop{\overset{\leftrightarrow}{S}{}^{*}}\limits_{k=1}^{N} \left\{ \overset{\leftrightarrow}{T}{}^{*} \left\{ \tau_k\left(\bar{\mathbf{x}}\right), \mu_{B_j^k}\left(\bar{y}_{j,r}^{\text{def}}\right); 1, w_{j,k}^{B}, p^{\text{imp}} \right\}; w_k^{\text{rule}}, p^{\text{agr}} \right\}}, \quad (2)$$

where $\overset{\leftrightarrow}{T}{}^{*}$ and $\overset{\leftrightarrow}{S}{}^{*}$ are Aczél-Alsina parameterized triangular norms with weights of arguments (Sect. 2.2), $\tau_k\left(\bar{\mathbf{x}}\right)$ is the activation level of the rule k, p^{imp} is a shape parameter of t-norm used for inference, p^{agr} is a shape parameter of t-conorm used for aggregation of inferences from rules, and $\bar{y}_{j,r}^{\text{def}}$ $(r = 1, \ldots, R_j)$ are discretization points. In the system considered in this paper the number of discretization points R_j for any output j does not have to be equal to the number of rules N. It is creating a good opportunities for increasing the interpretability and accuracy of the fuzzy system. This issue was discussed in detail in our previous papers [13,14,51]. The activation level of the k-th rule $\tau_k\left(\bar{\mathbf{x}}\right)$ in the formula (2) is determined for the input signals vector $\bar{\mathbf{x}}$ and it is defined as follows:

$$\tau_k\left(\bar{\mathbf{x}}\right) = \mathop{\overset{\leftrightarrow}{T}{}^{*}}\limits_{i=1}^{n} \left\{ \mu_{A_i^k}\left(\bar{x}_i\right); w_{k,i}^{A}, p^{\tau} \right\}, \quad (3)$$

where p^{τ} is a shape parameter of t-norm used for aggregation of antecedences.

2.2 Aggregation and Inference Operators

Use of parametrized-type triangular norms with weights of arguments considered in this paper contributes indirectly to an increase of the interpretability of the system (2). It results from high working precision of these operators, which allows for achieving the expected accuracy of the system (2) with a smaller number of rules N. In this paper a parametrized triangular norms with weights of arguments of Aczél-Alsina type are used. They are defined as follows:

$$\begin{cases} \overset{\leftrightarrow}{T}^{*}\{\mathbf{a};\mathbf{w},p\} = \begin{cases} \text{drastic t}-\text{norm} & \text{for} \quad p=0 \\ \text{minimum t}-\text{norm} & \text{for} \quad p=\infty \\ \exp\left(-\left(\sum\limits_{i=1}^{n}\left(-\ln\left(1-w_1\cdot\left(1-a_1\right)\right)\right)^p\right)^{\frac{1}{p}}\right) & \text{for } p\in(0,\infty) \end{cases} \\ \overset{\leftrightarrow}{S}^{*}\{\mathbf{a};\mathbf{w},p\} = \begin{cases} \text{drastic t}-\text{conorm} & \text{for} \quad p=0 \\ \text{maximum t}-\text{conorm} & \text{for} \quad p=\infty \\ 1-\exp\left(-\left(\sum\limits_{i=1}^{n}\left(-\ln\left(1-w_i\cdot a_i\right)\right)^p\right)^{\frac{1}{p}}\right) & \text{for } p\in(0,\infty) \end{cases} \end{cases}, \tag{4}$$

where p is a shape parameter of norm, $w_1 \dots, w_n \in [0,1]$ are weights of arguments $a_1, \dots, a_n \in [0,1]$.

3 Description of a Learning Algorithm

The proposed learning algorithm belongs to so-called population-based algorithms ([74]) and its purpose is to select the structure and the parameters of the fuzzy system (2). Population-based algorithms can be defined as search procedures based on the mechanisms of natural selection and inheritance and they use the evolutionary principle of survival of the fittest individuals. What differs population algorithms from traditional optimization methods, among others, is that they do not process task parameters directly, but their encoded form, they do not conduct a search starting from a single point, but from a population of points, they use only the objective function and not its derivatives, and they use probabilistic selection rules. It is worth to notice that, the gradient algorithms (see e.g. [72,73,75]) can also be applied to proposed interpretability criteria.

3.1 Encoding of Potential Solutions

Encoding of population of potential solutions used in the algorithm refers to the Pittsburgh approach [37]. A single individual of the population ($\mathbf{X}_{ch}$) is therefore an object that encodes the complete structure $\mathbf{X}_{ch}^{\text{str}}$ of the fuzzy system (2), its set parameters $\mathbf{X}_{ch}^{\text{set}}$ and real parameters $\mathbf{X}_{ch}^{\text{par}}$:

$$\mathbf{X}_{ch} = \left\{\mathbf{X}_{ch}^{\text{str}}, \mathbf{X}_{ch}^{\text{set}}, \mathbf{X}_{ch}^{\text{par}}\right\}. \tag{5}$$

Part $\mathbf{X}_{ch}^{\text{str}}$ of the individual $\mathbf{X}_{ch}$ encodes in a binary form the whole structure of the fuzzy system (2):

$$\mathbf{X}_{ch}^{\text{str}} = \left\{ \begin{array}{c} x_1, \dots, x_n, \\ A_1^1, ..., A_n^1, ..., A_1^{Nmax}, ..., A_n^{Nmax}, \\ B_1^1, ..., B_m^1, ..., B_1^{Nmax}, ..., B_m^{Nmax}, \\ \text{rule}_1, ..., \text{rule}_{Nmax}, \\ \bar{y}_{1,1}^{\text{def}}, ..., \bar{y}_{1,Rmax}^{\text{def}}, ..., \bar{y}_{m,1}^{\text{def}}, ..., \bar{y}_{m,Rmax}^{\text{def}} \end{array} \right\} = \left\{X_{ch,1}^{\text{str}}, ..., X_{ch,L^{\text{str}}}^{\text{str}}\right\}, \tag{6}$$

where $ch = 1, ..., Npop$ is the index of an individual in a population, $Npop$ is the number of individuals in a population, $Nmax$ is the maximum number of rules in the system (2), $Rmax$ is the maximum number of discretization points in the system (2) and L^{str} is the number of the individual components $\mathbf{X}_{ch}^{\text{str}}$ (referred as genes from now on) is determined as $L^{\text{str}} = Nmax \cdot (n + m + 1) + n + Rmax \cdot m$. The principle adopted in the encoding genes of $\mathbf{X}_{ch}^{\text{str}}$ is such that the gene with value 0 of the individual $\mathbf{X}_{ch}^{\text{str}}$ excludes the associated element from the system structure (2) and vice versa.

Part $\mathbf{X}_{ch}^{\text{set}}$ of the individual $\mathbf{X}_{ch}$ encodes the set parameters, which values have direct impact on interpretability. $\mathbf{X}_{ch}^{\text{set}}$ contains: (a) weights of antecedences, consequences and rules, and (b) parameters of triangular norms used for aggregation of antecedences (p^{agr}), inference of rules (p^{imp}) and aggregation of inference of rules (p^{τ}). Each of those parameters is chosen from a set of values. Each value from set have its own interpretation. Part $\mathbf{X}_{ch}^{\text{set}}$ takes the following form:

$$\mathbf{X}_{ch}^{\text{set}} = \left\{ \begin{array}{c} w_{1,1}^{A}, \ldots, w_{1,n}^{A}, \ldots, w_{Nmax,1}^{A}, \ldots, w_{Nmax,n}^{A}, \\ w_{1,1}^{B}, \ldots, w_{m,1}^{B}, \ldots, w_{1,Nmax}^{B}, \ldots, w_{m,Nmax}^{B}, \\ w_{1}^{\text{rule}}, \ldots, w_{Nmax}^{\text{rule}}, \\ p^{\tau}, p^{\text{imp}}, p^{\text{agr}}, \end{array} \right\} = \left\{ X_{ch,1}^{\text{set}}, \ldots, X_{ch,L^{\text{set}}}^{\text{set}} \right\}, \tag{7}$$

where $L^{\text{set}} = Nmax \cdot (n + m + 1) + 3$ is the number of components of individual $\mathbf{X}_{ch}^{\text{set}}$. The set of possible values for weights is defined as follows:

$$\text{set}^{w} = \{0.0, 0.5, 1.0\}, \tag{8}$$

where value 0.0 can be interpretable as not important, values 0.5 as important, and value 1.0 as very important. Additionally, when value 0.0 is chosen for an element, it its treat as reduced from a system (2). For parametrized triangular norms (4) the set of possible values was chosen to obtain similar behavior to the non-parametrized norms (see Table 1). The set of possible values is defined as follows:

$$\text{set}^{p} = \{0.00, 0.63, 1.00, 1.51, 10.00\}. \tag{9}$$

Table 1. The parameters that close behavior of triangular norm Aczél-Alsina to non-parametrical norms.

Triangular norm	Drastic	Łukasiewicz	Algebraic	Hamacher	Minimum
Similarity parameter	0.00	0.63	1.00	1.51	10.00
Similarity level	Identical	Close	Identical	Close	Close

Part $\mathbf{X}_{ch}^{\text{par}}$ of the individual $\mathbf{X}_{ch}$ encodes the real parameters of the fuzzy system and it has the following form:

$$\mathbf{X}_{ch}^{\text{par}} = \left\{ \begin{array}{c} \bar{x}_{1,1}^{A}, \sigma_{1,1}^{A}, \ldots, \bar{x}_{n,1}^{A}, \sigma_{n,1}^{A}, \ldots \\ \bar{x}_{1,Nmax}^{A}, \sigma_{1,Nmax}^{A}, \ldots, \bar{x}_{n,Nmax}^{A}, \sigma_{n,Nmax}^{A}, \\ \bar{y}_{1,1}^{B}, \sigma_{1,1}^{B}, \ldots, \bar{y}_{m,1}^{B}, \sigma_{m,1}^{B}, \ldots \\ \bar{y}_{1,Nmax}^{B}, \sigma_{1,Nmax}^{B}, \ldots, \bar{y}_{m,Nmax}^{B}, \sigma_{m,Nmax}^{B}, \\ \bar{y}_{1,1}^{\text{def}}, \ldots, \bar{y}_{1,Rmax}^{\text{def}}, \ldots, \bar{y}_{m,1}^{\text{def}}, \ldots, \bar{y}_{m,Rmax}^{\text{def}} \end{array} \right\} = \left\{ X_{ch,1}^{\text{par}}, \ldots, X_{ch,L^{\text{par}}}^{\text{par}} \right\}, \tag{10}$$

where $\bar{x}_{i,k}^{A}, \sigma_{i,k}^{A}$ are membership function parameters of input fuzzy sets A_i^k, $\bar{y}_{j,k}^{B}, \sigma_{j,k}^{B}$ are membership function parameters of output fuzzy sets B_j^k, and $L^{\text{par}} = Nmax \cdot (2 \cdot n + 2 \cdot m + 1) + Rmax \cdot m$ is the number of components of individual $\mathbf{X}_{ch}^{\text{par}}$. Those parameters are significantly affecting interpretability of system (2), but there is no possibility to choose values of them from a set. In this case, the interpretability criteria presented in our previous paper [51] can be used.

3.2 Evaluation of Potential Solutions

The purpose of proposed algorithm is to minimize the value of the evaluation function specified for the individual $\mathbf{X}_{ch}$ in the following way:

$$\text{ff}\ (\mathbf{X}_{ch}) = T^* \left\{ \begin{array}{c} \text{ffacc}\ (\mathbf{X}_{ch}), \text{ffint}\ (\mathbf{X}_{ch}); \\ w_{\text{ffacc}}, w_{\text{ffint}} \end{array} \right\}, \tag{11}$$

where component ffacc $(\mathbf{X}_{ch})$ specifies the accuracy of the system (2), component ffint $(\mathbf{X}_{ch})$ specifies interpretability of the system (2) according to the adopted interpretability criteria, $w_{\text{ffacc}} \in [0,1]$ represents weight of the component ffacc $(\mathbf{X}_{ch})$, $w_{\text{ffint}} \in [0,1]$ represents weight of the component ffint $(\mathbf{X}_{ch})$ (values of weights w_{ffacc} and w_{ffint} result from expectations of the user regarding the ratio between the accuracy of the system (2) and its interpretability), and $T^*\{\cdot\}$ is algebraic triangular norm with weights of arguments defined as:

$$T^*\{\mathbf{a}; \mathbf{w}\} = \prod_{i=1}^{n} (1 + (a_i - 1) \cdot w_i). \tag{12}$$

Component ffacc $(\mathbf{X}_{ch})$ in formula (11) is determined as follows:

$$\text{ffacc}\,(\mathbf{X}) = \frac{1}{m} \sum_{j=1}^{m} \frac{\frac{1}{Z} \sum\limits_{z=1}^{Z} |d_{z,j} - \bar{y}_{z,j}|}{\max\limits_{z=1,\ldots,Z} \{d_{z,j}\} - \min\limits_{z=1,\ldots,Z} \{d_{z,j}\}}, \tag{13}$$

where Z is the number of rows of a learning sequence, $d_{z,j}$ is the desired output value of output j for input vector z $(z = 1, ..., Z)$, $\bar{y}_{z,j}$ is the real output value j calculated by the system for the input vector $\bar{\mathbf{x}}_z$. Equation (13) takes into account the normalization of errors at different outputs of the system (2) in order to eliminate significant differences between them.

The component ffint $(\mathbf{X}_{ch})$ represent the interpretability criteria, which apply mostly to the component $\mathbf{X}_{ch}^{\text{par}}$. Those criteria allows to obtain: (a) correct arrangement of fuzzy sets, (b) correct firing of the fuzzy rules, (c) cohesion of fuzzy set shapes, (d) appropriate fitting of fuzzy rules to data etc. The examples of interpretability criteria was considered in our previous papers (see e.g. [51]).

3.3 Processing of Potential Solutions

For selection the structure and parameters of the system (2) a modified golden ball algorithm (GB) [60] is proposed. The GB algorithm was chosen due to following advantages: (a) it allows for precise local search of the search space (due to using multiple populations), (b) it allows for precise global search of the search space (due to migration mechanism between populations), (c) it allows obtain high performance (it is achieved thanks to separate learning parameters of each population, which can be modified in case of giving bad results), (d) it allows to obtain good diversity of solutions (due to competition mechanism between populations).

The proposed algorithm works according to the following steps:

Step 1. Initialization. In this step a $Npop$ individuals (players) of population are randomly initiated and randomly assigned to $Nteam$ populations (teams). Each team obtains $Npla = Npop/Nteam$ players ($Npop$ should be multiplicity of $Nteam$). Each player is evaluated using fitness function (11). Moreover, each team gets randomly initiated set of parameters:

$$\mathbf{TEAM}_e = \{p_m, p_c, m_r\}, \tag{14}$$

where $p_m \in (0,1)$ is team mutation probability, $p_c \in (0,1)$ is team crossover probability, $m_r \in (0,1)$ is team mutation insensitivity, $e = 1, ..., Nteam$ stands for index of team.

Step 2. Teams traning. This step is carried out $Nstep$ times for each team separately. In the beginning, for each team a time variable t is set to 0.

Step 2.1. New players creation. In this step a $Npla$ new players are created for each team, according to evolutionary strategy $(\mu + \lambda)$ [71]. Those players are created by cloning the players chosen via roulette wheel method [71] from actual players of the team. If the condition $\mathbf{TEAM}_e\{p_c\} < \mathrm{U}_r(0,1)$ (where $\mathrm{U}_r(a,b)$ stands for random value from range [a,b]) is met, those genes are additionally crossovered with genes randomly chosen via roulette wheel method players from players of the team. $\mathbf{TEAM}_e\{p_c\}$ stands for using field p_c of team $\mathbf{TEAM}_e$.

Step 2.2. New players modification. In this step each gene $X_{ch,g}^{\text{par}}$ of newly created players is mutated (when condition $\mathbf{TEAM_e}\{p_m\} < \mathrm{U}_r(0,1)$ is met) according to following equation:

$$X_{ch,g}^{\text{par}} := X_{ch,g}^{\text{par}} + \left(\bar{X}_{ch,g}^{\text{par}} - \underline{X}_{ch,g}^{\text{par}}\right) \cdot \mathrm{U}_r(-1,1) \cdot \mathbf{TEAM}_e\{m_r\} \cdot \frac{Nstep - t}{Nstep}, \tag{15}$$

where range $[\underline{X}^{\text{par}}_{ch,g}, \overline{X}^{\text{par}}_{ch,g}]$ stands for minimum and maximum allowed value of gene $X^{\text{par}}_{ch,g}$, $Nstep$ stands for maximum number of steps of teams training. It is easy to notice that, the range of mutation is decreasing with each step of teams training (due to increasing value t). In turn, for each gene $X^{\text{str}}_{ch,g}$ (when condition $X^{\text{str}}_{ch,g}$ is $\mathbf{TEAM}_e\{p_m\} < \mathrm{U}_r(0,1)$ is met) a random value from set $\{0,1\}$ is assigned. Genes $X^{\text{set}}_{ch,g}$ are modified analogically to genes $X^{\text{str}}_{ch,g}$. The new values of genes coding weights are randomly chosen from set (8), and the new values of triangular norms parameters (4) are randomly chosen from set (9).

Step 2.3. New players evaluation. After modification of genes from Step 2.2, all new players are evaluated according to fitness function (11).

Step 2.4. Selection of team players. The selection of team players is independent for each team and it lies on choosing $Npla$ best players from both the actual teams players and the newly created players from Step 2.1.

Step 2.5. Stop condition of teams traning. In this step a value t is incremented. After that, the condition $t < Nstep$ is checked. If this condition is met, algorithm goes back to step 2.1, otherwise algorithm goes to next step (Step 3).

Step 3. League competition. In this step each $\mathbf{TEAM}_e$ compete (playing matches) $Nmatch$ times with all teams. Each match consist of $Natt$ attacks. Each attack relies on comparing values of fitness function of randomly chosen players from both teams. The player with better value of fitness function scores one point for its team. The team with more (or equal) points gets a league point. On the basis of league points the teams are sorted (from best to worst). It is worth to mention that, the results of competition are determined by random factor, which ensure appropriate migration between teams from next step.

Step 4. Players transfer. Players migration (transfer) between teems is based on moving players between better and worst teams. Best team from best half of the teams is transferring $Nrep$ ($Nrep < Npla$) worst players with $Nrep$ best players from best team from worst half of the teams, etc. Thus, the last part of this step will concern transfer between worst team from best half of the teams with worst team from worst half of the teams.

Step 5. Changing training plans. In this step, a parameters (14) of worst half of the teams are changed by averaging them with parameters from better half of the teams (the parameters of best team from worst half of the teams are averaged with parameters of best team from best half of teams, etc.).

Step 6. Stop condition. In this step a number of iterations of the algorithm is checked. If this number reached value of $Niter$ algorithm stops and best found solution is presented, otherwise algorithm goes back to Step 2.

4 Simulations

The set of nonlinear issues (see e.g. [52,53]) examined in the simulations is shown in Table 2. The purpose of the simulations was to obtain systems of the forms (2) characterized by the lowest values of elements of the form (13). In this paper the following interpretability criteria were considered: (a) complexity criterium, (b) reducing overlapping of fuzzy sets criterium, (c) increasing integrity of shape criterium, and (d) increases complementarity criterium (for details see our previous paper [51]). In the simulations the algorithm described in Sect. 3 to select its structure and parameters were used. The simulations were performed for five different variants of weights of the evaluation function (11): from the one focused on accuracy (W1) to the one focused on interpretability (W5) (see Table 3). A set of the proposed algorithm parameters was selected experimentally as following: number of iterations $Niter = 50$, number of individual training steps $Nstep = 20$, number of players $Npop = 100$, number of teams $Nteam = 10$, number of matches $Nmatch = 2$, number of attacks $Natt = 20$ and number of transfered players $Nrep = 2$. A set of parameters of the fuzzy system was selected as following: maximum number of rules $Nmax = 7$, and maximum number of discretization points $Rmax = 21$.

Each simulation (for each variant W1...W5) was repeated 100 times. The obtained results were averaged and presented in Table 4 and in Fig. 1. The learn-

Table 2. Simulation problems discussed.

No	Test set name	Number of input attributes	Number of output attributes	Number of sets	Problem label
1	Nelson function [57]	2	1	128	NF
2	Yacht Hydrodynamics [59]	5	1	308	YH
3	Concrete Slump [10]	7	3	103	CS

Table 3. A set of variants of the weights of the evaluation function (11).

Variant	w_{ffacc}	w_{ffint}	Description
W1	0.90	0.10	focused on high accuracy
W2	0.70	0.30	focused on accuracy
W3	0.50	0.70	intermediate between W2 and W4
W4	0.30	0.70	focused on interpretability
W5	0.10	0.90	focused on high interpretability

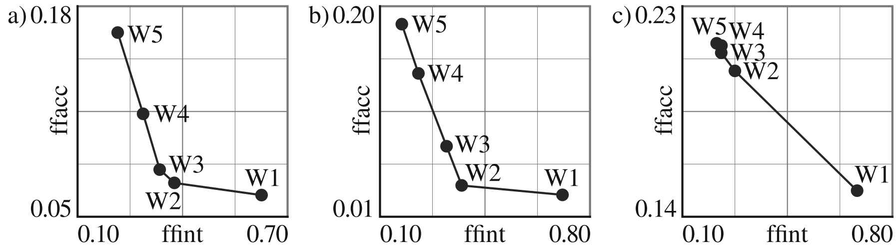

Fig. 1. Obtained trade-off between accuracy and interpretability for problem: (a) NF, (b) YH, (c) CS.

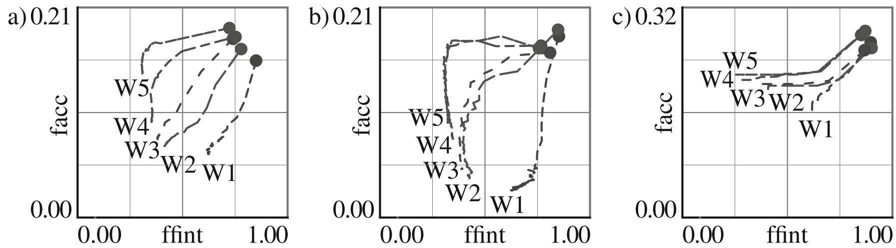

Fig. 2. Averaged learning process for problem: (a) NF, (b) YH, (c) CS. Filled circles stands for best solutions from first iteration of proposed algorithm.

Table 4. Averaged simulation results for considered problems.

Simulation problem	Evaluation function	case					Other authors results [45,51]
		W1	W2	W3	W4	W5	
NF	ffacc (·)	0.063	0.069	0.078	0.095	0.144	n/a
	ffint (·)	0.643	0.386	0.360	0.322	0.270	n/a
	RMSE	1.348	1.446	1.636	2.046	3.263	1.104 - 2.653
YH	ffacc (·)	0.027	0.040	0.049	0.076	0.095	n/a
	ffint (·)	0.652	0.416	0.365	0.321	0.287	n/a
	RMSE	2.614	3.617	4.281	6.996	8.629	0.820 - 2.236
CS	ffacc (·)	0.153	0.183	0.190	0.198	0.202	n/a
	ffint (·)	0.646	0.346	0.310	0.288	0.268	n/a
	RMSE	14.563	16.864	18.104	19.152	19.302	11.941 - 16.668

ing process was presented in Fig. 2. Typical examples of rules obtained for case W3 (which represent balanced trade-off between accuracy and interpretability) were presented in Fig. 3 and in Table 5. The notation of fuzzy rules examples obtained for case W3 and shown in Fig. 3 were presented in Table 5.

The conclusions from the simulations can be summarized as follows: (a) choosing specified values from set allow to obtain interpretable values of weights and values of parameters of triangular norms, (b) obtained results are similar (in a field of accuracy) to results presented by other authors, (c) use of variants

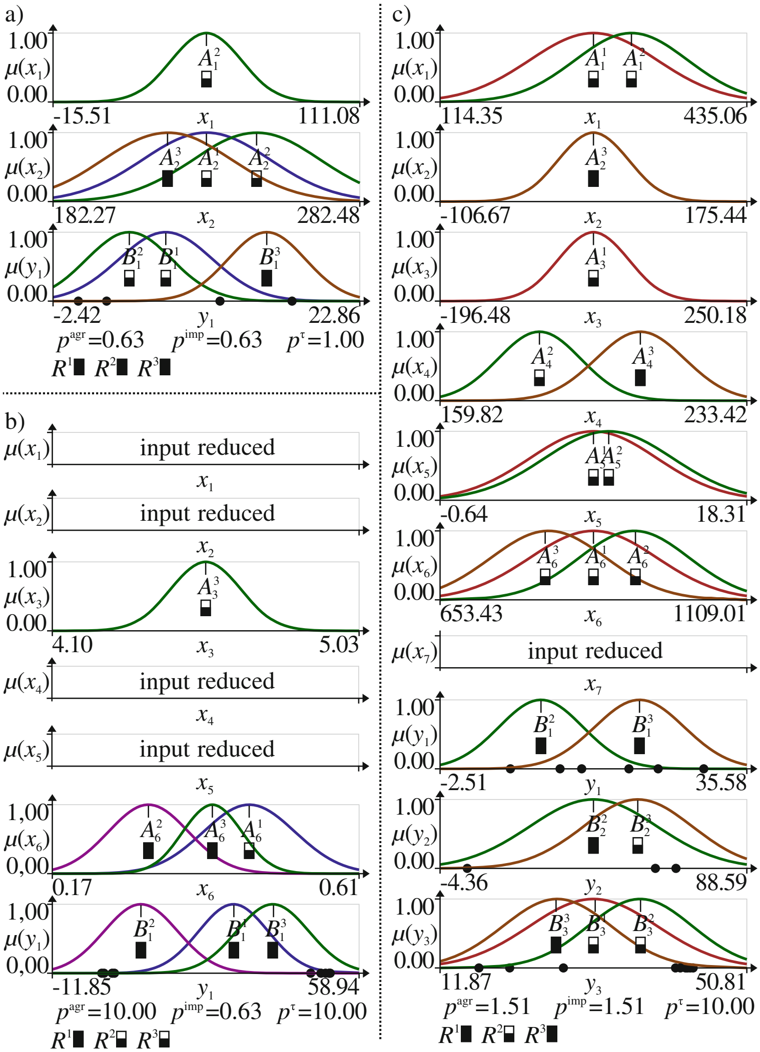

Fig. 3. Examples of obtained fuzzy sets (case W3) for problem: (a) NF, (b) YH, (c) CS. Rectangles stands for weights of fuzzy sets and rules (filled rectangle - very important value, half-filled rectangle - important value, empty rectangle - not important value). Circles stands for discretization points.

W1-W5 allows to obtain diversified solutions (in a field of expected trade-off between accuracy and interpretability).

Table 5. Summary with examples of fuzzy rules in the form of (1) of the fuzzy system (2) for variant W3 (Fig. 3).

NF	R^1 : IF (x_2 is *medium* $\lvert m$) THEN (y is *medium* $\lvert m$) $\lvert h$ R^2 : IF (x_1 is *near* (47.78) $\lvert m$ AND x_2 is *high* $\lvert m$) THEN (y is *low* $\lvert m$) $\lvert h$ R^3 : IF (x_2 is *low* $\lvert h$) THEN (y is *high* $\lvert h$) $\lvert h$
YH	R^1 : IF (*froude number* is *high* $\lvert m$) THEN (*resistance* is *medium* $\lvert h$) $\lvert h$ R^2 : IF (*froude number* is *low* $\lvert h$) THEN (*resistance* is *low* $\lvert h$) $\lvert m$ R^3 : IF (*l.displ.* is *near* (7.57) $\lvert m$ AND *froude number* is *medium* $\lvert h$) THEN (*resistance* is *high* $\lvert h$) $\lvert m$
CS	R^1 : IF (*cement* is *low* $\lvert m$ AND *fly ash* is *near* (26.85) $\lvert m$ AND *sp* is *low* $\lvert m$ AND *coarse aggr.* is *medium* $\lvert m$) THEN (*str.* is *medium* $\lvert m$) $\lvert h$ R^2 : IF (*cement* is *high* $\lvert m$ AND *water* is *low* $\lvert m$ AND *sp* is *high* $\lvert m$ AND *coarse aggr.* is *high* $\lvert m$) THEN (*slump* is *low* $\lvert h$ AND *flow* is *low* $\lvert h$ AND *str.* is *high* $\lvert m$) $\lvert m$ R^3 : IF (*slag* is *near* (34.38) $\lvert h$ AND *water* is *high* $\lvert h$ AND *coarse aggr.* is *low* $\lvert m$) THEN (*slump* is *high* $\lvert h$ AND *flow* is *high* $\lvert m$ AND *str.* is *low* $\lvert h$) $\lvert h$

5 Conclusions

In this paper a new approach for interpretability of neuro-fuzzy systems with parametrized triangular norms was presented. In this approach, it is assumed that, a part of parameters are selected from set of defined values, where each of those values have its own interpretation. Those sets concerns weights (of antecedences, consequences and rules) and parameters of parametrized triangular norms with weights of arguments. This approach required use of proper learning algorithm. Therefore, we proposed modified golden ball algorithm, which allows to select parameters from set of values, select real values of parameters, and select binary parameters. Proposed learning algorithm can be used to learning all types of systems, where both the parameters and the structure have to be found. Obtained simulations results can be considered as good in a both fields of accuracy and interpretability.

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References

1. Abbas, J.: The bipolar Choquet integrals based on ternary-element sets. J. Artif. Intell. Soft Comput. Res. **6**(1), 13–21 (2016)
2. Alcal, R., Ducange, P., Herrera, F., Lazzerini, B., Marcelloni, F.: A multi-objective evolutionary approach to concurrently learn rule and data base sof linguistic fuzzy rule-based systems. IEEE Trans. Fuzzy Syst. **17**, 1106–1122 (2009)
3. Alonso J. M.: Modeling Highly Interpretable Fuzzy Systems, European Centre for Soft Computing (2010)
4. Alonso, J.M., Magdalena, L.: HILK++: an interpretability-guided fuzzy modeling methodology for learning readable and comprehensible fuzzy rule-based classifiers. Soft Computing **15**(10), 1959–1980 (2011)
5. Bartczuk, Ł.: Gene expression programming in correction modelling of nonlinear dynamic objects. Adv. Intell. Syst. Comput. **429**, 125–134 (2016)
6. Bartczuk, Ł., Przybył, A., Koprinkova-Hristova, P.: New method for nonlinear fuzzy correction modelling of dynamic objects. In: Rutkowski, L., Korytkowski, M., Scherer, R., Tadeusiewicz, R., Zadeh, L.A., Zurada, J.M. (eds.) ICAISC 2014, Part I. LNCS, vol. 8467, pp. 169–180. Springer, Heidelberg (2014)
7. Bartczuk, Ł., Rutkowska, D.: Type-2 fuzzy decision trees. In: Rutkowski, L., Tadeusiewicz, R., Zadeh, L.A., Zurada, J.M. (eds.) ICAISC 2008. LNCS (LNAI), vol. 5097, pp. 197–206. Springer, Heidelberg (2008)
8. Bartczuk, Ł., Rutkowska, D.: Medical diagnosis with type-2 fuzzy decision trees, computers in medical activity. Adv. Intell. Soft Comput. **65**, 11–21 (2009)
9. Bruździński, T., Krzyżak, A., Fevens, T., Jeleń, Ł.: Web-based framework for breast cancer classification. J. Artif. Intell. Soft Comput. Res. **4**(2), 149–162 (2014)
10. Cheng, Y.I.: Modeling slump flow of concrete using second-order regressions and artificial neural networks. CCC **29**(6), 474–480 (2007)
11. Cierniak, R., Rutkowski, L.: On image compression by competitive neural networks and optimal linear predictors. Signal Process. Image Commun. **156**, 559–565 (2000)
12. Cpalka, K.: A method for designing flexible neuro-fuzzy systems. In: Rutkowski, L., Tadeusiewicz, R., Zadeh, L.A., Żurada, J.M. (eds.) ICAISC 2006. LNCS (LNAI), vol. 4029, pp. 212–219. Springer, Heidelberg (2006)
13. Cpałka, K.: A new method for design and reduction of neuro-fuzzy classification systems. IEEE Trans. Neural Networks **20**, 701–714 (2009)
14. Cpałka, K.: On evolutionary designing and learning of flexible neuro-fuzzy structures for nonlinear classification. Nonlinear Anal. Series A. Theor. Methods Appl. **71**(2009), e1659–e1672 (2009). Elsevier
15. Cpałka, K., Łapa, K., Przybył, A., Zalasiński, M.: A new method for designing neuro-fuzzy systems for nonlinear modelling with interpretability aspects. Neurocomput. **135**, 203–217 (2014)
16. Cpałka, K., Rebrova, O., Nowicki, R., Rutkowski, L.: On design of flexible neuro-fuzzy systems for nonlinear modelling. Int. J. General Syst. **42**(6), 706–720 (2013)
17. Cpałka, K., Rutkowski, L.: Flexible Takagi-Sugeno neuro-fuzzy structures for nonlinear approximation. WSEAS Trans. Syst. **4**(9), 1450–1458 (2005)
18. Cpałka K., Rutkowski L.: Flexible Takagi-Sugeno fuzzy systems, Neural Networks. In: Proceedings of the 2005 IEEE International Joint Conference on IJCNN 2005, vol. 3, pp. 1764–1769 (2005)
19. Cpałka, K., Zalasiński, M.: On-line signature verification using vertical signature partitioning. Expert Syst. Appl. **41**(9), 4170–4180 (2014)

20. Cpałka, K., Zalasiński, M., Rutkowski, L.: New method for the on-line signature verification based on horizontal partitioning. Pattern Recogn. **47**, 2652–2661 (2014)
21. Cpałka, K., Zalasiński, M., Rutkowski, L.: A new algorithm for identity verification based on the analysis of a handwritten dynamic signature. Appl. Soft Comput. **43**, 47–56 (2016)
22. Das, S., Kar, S., Pal, T.: Group decision making using interval-valued intuitionistic fuzzy soft matrix and confident weight of experts. J. Artif. Intell. Soft Comput. Res. **4**(1), 57–77 (2014)
23. Duda, P., Hayashi, Y., Jaworski, M.: On the Strong convergence of the orthogonal series-type kernel regression neural networks in a non-stationary environment. In: Korytkowski, M., Scherer, R., Tadeusiewicz, R., Zadeh, L.A., Zurada, J.M., Rutkowski, L. (eds.) ICAISC 2012, Part I. LNCS, vol. 7267, pp. 47–54. Springer, Heidelberg (2012)
24. Duda, P., Jaworski, M., Pietruczuk, L.: On pre-processing algorithms for data stream. In: Rutkowski, L., Korytkowski, M., Scherer, R., Tadeusiewicz, R., Zadeh, L.A., Zurada, J.M. (eds.) ICAISC 2012, Part II. LNCS, vol. 7268, pp. 56–63. Springer, Heidelberg (2012)
25. Dziwiński, P., Avedyan, E.D.: A new approach to nonlinear modeling based on significant operating points detection. In: Rutkowski, L., Korytkowski, M., Scherer, R., Tadeusiewicz, R., Zadeh, L.A., Zurada, J.M. (eds.) ICAISC 2015. LNCS, vol. 9120, pp. 364–378. Springer, Heidelberg (2015)
26. Dziwiński, P., Bartczuk, Ł., Przybył, A., Avedyan, E.D.: A new algorithm for identification of significant operating points using swarm intelligence. In: Rutkowski, L., Korytkowski, M., Scherer, R., Tadeusiewicz, R., Zadeh, L.A., Zurada, J.M. (eds.) ICAISC 2014, Part II. LNCS, vol. 8468, pp. 349–362. Springer, Heidelberg (2014)
27. Er, M.J., Duda, P.: On the weak convergence of the orthogonal series-type kernel regresion neural networks in a non-stationary environment. In: Wyrzykowski, R., Dongarra, J., Karczewski, K., Waśniewski, J. (eds.) PPAM 2011, Part I. LNCS, vol. 7203, pp. 443–450. Springer, Heidelberg (2012)
28. Farahbod, F., Eftekhari, M.: Comparsion of different T-norm operators in classification problems. Int. J. Fuzzy Logic Syst. **2**(3), 33–41 (2012)
29. Gabryel, M., Cpałka, K., Rutkowski, L.: Evolutionary strategies for learning of neuro-fuzzy systems. In: Proceeding of the I Workshop on Genetic Fuzzy Systems, Granada, pp. 119–123 (2005)
30. Gabryel, M., Korytkowski, M., Scherer, R., Rutkowski, L.: Object detection by simple fuzzy classifiers generated by boosting. In: Korytkowski, M., Scherer, R., Tadeusiewicz, R., Zadeh, L.A., Zurada, J.M., Rutkowski, L. (eds.) ICAISC 2013, Part I. LNCS, vol. 7894, pp. 540–547. Springer, Heidelberg (2013)
31. Gacto, M.J., Alcal, R., Herrera, F.: Integration of an index to preserve the semantic interpretability in the multi-objective evolutionary rule selection and tuning of linguistic fuzzy systems. IEEE Trans. Fuzzy Syst. **18**, 515–531 (2010)
32. Gacto, M.J., Alcal, R., Herrera, F.: Interpretability of linguistic fuzzy rule-based systems: an overview of interpretability measures. Inf. Sci. **181**(20), 4340–4360 (2011)
33. Gałkowski, T., Rutkowski, L.: Nonparametric recovery of multivariate functions with applications to system identification. Proc. IEEE **73**(5), 942–943 (1985)
34. Grycuk, R., Gabryel, M., Korytkowski, M., Scherer, R., Voloshynovskiy, S.: From single image to list of objects based on edge and blob detection. In: Korytkowski, M., Scherer, R., Tadeusiewicz, R., Zadeh, L.A., Zurada, J.M., Rutkowski, L. (eds.) ICAISC 2014, Part II. LNCS, vol. 8468, pp. 605–615. Springer, Heidelberg (2014)

35. Guillaume, S., Charnomordic, B.: Generating an interpretable family of fuzzy partitions from data. IEEE Trans. Fuzzy Syst. **12**(3), 324–335 (2004)
36. Icke, I., Rosenberg, A.: Multi-objective genetic programming for visual analytics. In: Silva, S., Foster, J.A., Nicolau, M., Machado, P., Giacobini, M. (eds.) EuroGP 2011. LNCS, vol. 6621, pp. 322–334. Springer, Heidelberg (2011)
37. Ishibuchi, H., Nakashima, T., Murata, T.: Comparsion of the Michigan and Pittsburgh approaches to the design of fuzzy classification systems. Electr. Commun. Japan, Part 3 **80**(12), 379–387 (1997)
38. Ishibuchi, H., Nojima, Y.: Analysis of interpretability-accuracy tradeoff of fuzzy systems by multiobjective fuzzy genetics-based machine learning. Int. J. Approximate Reasoning **44**, 4–31 (2007)
39. Jaworski, M., Er, M.J., Pietruczuk, L.: On the application of the parzen-type kernel regression neural network and order statistics for learning in a non-stationary environment. In: Rutkowski, L., Korytkowski, M., Scherer, R., Tadeusiewicz, R., Zadeh, L.A., Zurada, J.M. (eds.) ICAISC 2012, Part I. LNCS, vol. 7267, pp. 90–98. Springer, Heidelberg (2012)
40. Jaworski, M., Pietruczuk, L., Duda, P.: On resources optimization in fuzzy clustering of data streams. In: Rutkowski, L., Korytkowski, M., Scherer, R., Tadeusiewicz, R., Zadeh, L.A., Zurada, J.M. (eds.) ICAISC 2012, Part II. LNCS, vol. 7268, pp. 92–99. Springer, Heidelberg (2012)
41. Kitajima, R., Kamimura, R.: Accumulative information enhancement in the self-organizing maps and its application to the analysis of mission statements. J. Artif. Intell. Soft Comput. Res. **5**(3), 161–176 (2015)
42. Korytkowski, M., Nowicki, R., Scherer, R.: Neuro-fuzzy rough classifier ensemble. In: Alippi, C., Polycarpou, M., Panayiotou, C., Ellinas, G. (eds.) ICANN 2009, Part I. LNCS, vol. 5768, pp. 817–823. Springer, Heidelberg (2009)
43. Korytkowski, M., Rutkowski, L., Scherer, R.: From ensemble of fuzzy classifiers to single fuzzy rule base classifier. In: Rutkowski, L., Tadeusiewicz, R., Zadeh, L.A., Zurada, J.M. (eds.) ICAISC 2008. LNCS (LNAI), vol. 5097, pp. 265–272. Springer, Heidelberg (2008)
44. Korytkowski, M., Rutkowski, L., Scherer, R.: Fast image classification by boosting fuzzy classifiers. Inf. Sci. **327**, 175–182 (2016)
45. Kummer, N., Najjaran, H.: Adaboost.MRT: boosting regression for multivariate estimation. Artif. Intell. Res. **3**(4), 64–76 (2014)
46. Laskowski, Ł., Laskowska, M.: Probing of synthesis route. J. Solid State Chem. **220**, 221–226 (2014)
47. Laskowski, Ł., Laskowska, M., Bałanda, M., Fitta, M., Kwiatkowska, J., Dziliński, K., Karczmarska, A.: Raman and magnetic analysis. Microporous Mesoporous Mater. **200**, 253–259 (2014)
48. Li, X., Er, M.J., Lim, B.S., Zhou, J.H., Gan, O.P., Rutkowski, L.: Fuzzy regression modeling for tool performance prediction and degradation detection. Int. J. Neural Syst. **2005**, 405–419 (2010)
49. Liu, F., Quek, C., Ng, G.S.: A novel generic hebbian ordering-based fuzzy rule base reduction approach to Mamdani neuro-fuzzy system. Neural Comput. **19**, 1656–1680 (2007)
50. Ludwig, S.A.: Repulsive self-adaptive acceleration particle swarm optimization approach. J. Artif. Intell. Soft Comput. Res. **4**(3), 189–204 (2014)
51. Łapa K.: Algorithms for extracting interpretable expert knowledge in nonlinear modeling issues, PhD Thesis (in polish), Czestochowa University of Technology (2015)

52. Łapa, K., Przybył, A., Cpałka, K.: A new approach to designing interpretable models of dynamic systems. In: Rutkowski, L., Korytkowski, M., Scherer, R., Tadeusiewicz, R., Zadeh, L.A., Zurada, J.M. (eds.) ICAISC 2013, Part II. LNCS, vol. 7895, pp. 523–534. Springer, Heidelberg (2013)
53. Łapa, K., Zalasiński, M., Cpałka, K.: A new method for designing and complexity reduction of neuro-fuzzy systems for nonlinear modelling. In: Rutkowski, L., Korytkowski, M., Scherer, R., Tadeusiewicz, R., Zadeh, L.A., Zurada, J.M. (eds.) ICAISC 2013, Part I. LNCS, vol. 7894, pp. 329–344. Springer, Heidelberg (2013)
54. Marquez A. A., Marquez F. A., Peregrin A.: A multi-objective evolutionary algorithm with an interpretability improvement mechanism for linguistic fuzzy systems with adaptive defuzzification. In: IEEE International Conference on Fuzzy Systems, pp. 1–7 (2010)
55. Mencar, C., Castellano, G., Fanelli, A.M.: On the role of interpretability in fuzzy data mining. Int. J. Uncertainty Fuzziness Knowl. Based Syst. **15**(5), 521–537 (2007)
56. Mencar, C., Castiello, C., Cannone, R., Fanelli, A.M.: Interpretability assessment of fuzzy knowledge bases: a cointension based approach. Int. J. Approximate Reasoning **52**(4), 501–518 (2011)
57. Nelson, W.: Analysis of performance-degradation data. IEEE Trans. Reliab. **2**(2), 149–155 (1981)
58. Nobukawa, S., Nishimura, H., Yamanishi, T., Liu, J.-Q.: Chaotic states induced by resetting process in Izhikevich Neuron Model. J. Artif. Intell. Soft Comput. Res. **5**(2), 109–119 (2015)
59. Ortigosa I., Lopez R., Garcia J.: A neural networks approach to residuary resistance of sailing yachts prediction. In: Proceedings of the International Conference on Marine Engineering MARINE 2007 (2007)
60. Osaba, E.: Golden Ball: a novel meta-heuristic to solve combinatorial optimization problems based on soccer concepts. Appl. Intell. **12**(2013), 145–166 (2013)
61. Patgiri, C., Sarma, M., Sarma, K.K.: A class of neuro-computational methods for assamese fricative classification. J. Artif. Intell. Soft Comput. Res. **5**(1), 59–70 (2015)
62. Pietruczuk, L., Duda, P., Jaworski, M.: A new fuzzy classifier for data streams. In: Rutkowski, L., Korytkowski, M., Scherer, R., Tadeusiewicz, R., Zadeh, L.A., Zurada, J.M. (eds.) ICAISC 2012, Part I. LNCS, vol. 7267, pp. 318–324. Springer, Heidelberg (2012)
63. Pietruczuk, L., Duda, P., Jaworski, M.: Adaptation of decision trees for handling concept drift. In: Rutkowski, L., Korytkowski, M., Scherer, R., Tadeusiewicz, R., Zadeh, L.A., Zurada, J.M. (eds.) ICAISC 2013, Part I. LNCS, vol. 7894, pp. 459–473. Springer, Heidelberg (2013)
64. Pulkkinen, P., Koivisto, H.: A dynamically constrained multiobjective genetic fuzzy system for regression problems. IEEE Trans. Fuzzy Syst. **18**(1), 161–177 (2010)
65. Riid, A., Rüstern, E.: Interpretability, interpolation and rule weights in linguistic fuzzy modeling. In: Fanelli, A.M., Pedrycz, W., Petrosino, A. (eds.) WILF 2011. LNCS, vol. 6857, pp. 91–98. Springer, Heidelberg (2011)
66. Rutkowska, A.: Influence of membership functions shape on portfolio optimization results. J. Artif. Intell. Soft Comput. Res. **6**(1), 45–54 (2016)
67. Rutkowski, L.: Identification of MISO nonlinear regressions in the presence of a wide class of disturbances. IEEE Trans. Inform. Theory **37**(1), 214–216 (1997)
68. Rutkowski, L., Cpaka, K.: Flexible neuro fuzzy systems. IEEE Trans. Neural Networks **14**, 554–574 (2003)

69. Rutkowski, L.: Adaptive probabilistic neural networks for pattern classification in time-varying environment. IEEE Trans. Neural Networks **15**(4), 811–827 (2004)
70. Rutkowski, L.: Flexible Neuro-Fuzzy Systems. Kluwer Academic Publishers, Dordrecht (2004)
71. Rutkowski, L.: Computational Intelligence. Springer, Heidelberg (2008)
72. Rutkowski, L., Cpałka, K.: Flexible structures of neuro-fuzzy systems. In: Sincak, P., Vascak, J. (eds.) Quo Vadis Computational Intelligence. Studies in Fuzziness and Soft Computing, vol. 54, pp. 479–484. Springer, Heidelberg (2000)
73. Rutkowski, L., Cpałka, K.: Compromise approach to neuro-fuzzy systems. In: Sincak, P., Vascak, J., Kvasnicka, V., Pospichal, J. (eds.) Intelligent Technologies - Theory and Applications, vol. 76, pp. 85–90. IOS Press (2002)
74. Rutkowski L., Cpałka K.: Flexible weighted neuro-fuzzy systems. In: Proceedings of the 9th Internationa; Conference on Neural Information Processing (ICONIP-02), Orchid Country Club, Singapore, 18–22 November 2002. CD
75. Rutkowski L., Cpałka K.: Neuro-fuzzy systems derived from quasi-triangular norms. In: Proceedings of the IEEE International Conference on Fuzzy Systems, Budapest, 26–29 July 2004, vol. 2, pp. 1031–1036 (2004)
76. Rutkowski, L., Jaworski, M., Pietruczuk, L., Duda, P.: Decision trees for mining data streams based on the gaussian approximation. IEEE Trans. Knowl. Data Eng. **26**(1), 108–119 (2014)
77. Rutkowski, L., Jaworski, M., Pietruczuk, L., Duda, P.: The CART decision tree for mining data streams. Inf. Sci. **266**, 1–15 (2014)
78. Rutkowski, L., Pietruczuk, L., Duda, P., Jaworski, M.: Decision Trees for mining data streams based on the McDiarmid's bound. IEEE Trans. Knowl. Data Eng. **25**(6), 1272–1279 (2013)
79. Rutkowski, L., Przybył, A., Cpałka, K., Er, M.J.: Online speed profile generation for industrial machine tool based on neuro-fuzzy approach. In: Rutkowski, L., Scherer, R., Tadeusiewicz, R., Zadeh, L.A., Zurada, J.M. (eds.) ICAISC 2010, Part II. LNCS, vol. 6114, pp. 645–650. Springer, Heidelberg (2010)
80. Scherer, R.: Neuro-fuzzy systems with relation matrix. In: Rutkowski, L., Scherer, R., Tadeusiewicz, R., Zadeh, L.A., Zurada, J.M. (eds.) ICAISC 2010, Part I. LNCS, vol. 6113, pp. 210–215. Springer, Heidelberg (2010)
81. Scherer R., Rutkowski L.: Relational equations initializing neuro-fuzzy system. In: Proceeding of the 10th Zittau Fuzzy Colloquium, Zittau, Germany, pp. 18–22 (2002)
82. Shukla, P.K., Tripathi, S.P.: A new approach for tuning interval type-2 fuzzy knowledge bases using genetic algorithms. J. Uncertainty Anal. Appl. **2**(4), 1–15 (2014)
83. Starczewski, J.T., Bartczuk, Ł., Dziwiński, P., Marvuglia, A.: Learning methods for type-2 FLS based on FCM. In: Rutkowski, L., Scherer, R., Tadeusiewicz, R., Zadeh, L.A., Zurada, J.M. (eds.) ICAISC 2010, Part I. LNCS, vol. 6113, pp. 224–231. Springer, Heidelberg (2010)
84. Szarek, A., Korytkowski, M., Rutkowski, L., Scherer, R., Szyprowski, J.: Application of neural networks in assessing changes around implant after total hip arthroplasty. In: Rutkowski, L., Korytkowski, M., Scherer, R., Tadeusiewicz, R., Zadeh, L.A., Zurada, J.M. (eds.) ICAISC 2012, Part II. LNCS, vol. 7268, pp. 335–340. Springer, Heidelberg (2012)
85. Szarek, A., Korytkowski, M., Rutkowski, L., Scherer, R., Szyprowski, J.: Forecasting wear of head and acetabulum in hip joint implant. In: Rutkowski, L., Korytkowski, M., Scherer, R., Tadeusiewicz, R., Zadeh, L.A., Zurada, J.M. (eds.) ICAISC 2012, Part II. LNCS, vol. 7268, pp. 341–346. Springer, Heidelberg (2012)

86. Szczypta, J., Przybył, A., Cpałka, K.: Some aspects of evolutionary designing optimal controllers. In: Rutkowski, L., Korytkowski, M., Scherer, R., Tadeusiewicz, R., Zadeh, L.A., Zurada, J.M. (eds.) ICAISC 2013, Part II. LNCS, vol. 7895, pp. 91–100. Springer, Heidelberg (2013)
87. Szczypta, J., Przybył, A., Wang, L.: Evolutionary approach with multiple quality criteria for controller design. In: Rutkowski, L., Korytkowski, M., Scherer, R., Tadeusiewicz, R., Zadeh, L.A., Zurada, J.M. (eds.) ICAISC 2014, Part I. LNCS, vol. 8467, pp. 455–467. Springer, Heidelberg (2014)
88. Smyczyńska, J., Hilczer, M., Smyczyńska, U., Stawerska, R., Tadeusiewicz, R., Lewiński, A.: Artificial neural models - a novel tool for predictying the efficacy of growth hormone (GH) therapy in children with short stature. Neuroendocrinology Lett. **36**(4), 348–353 (2015). (ISSN: 0172-780X, ISSN-L: 0172-780X)
89. Smyczyńska, U., Smyczyńska, J., Hilczer, M., Stawerska, R., Lewiński, A.: Artificial neural networks - a novel tool in modelling the effectiveness of growth hormone (GH) therapy in children with GH deficiency. Pediatric Endocrinology **14**(2(51)), 9–18 (2015)
90. Tadeusiewicz, R.: Neural networks as a tool for modeling of biological systems. Bio-Algorithms Med-Syst. **11**(3), 135–144 (2015)
91. Tadeusiewicz, R.: Neural networks in mining sciences - general overview and some representative examples. Archivum Min. Sci. **60**(4), 971–984 (2015)
92. Vanhoucke, V., Silipo, R.: Interpretability in multidimensional classification. In: Casillas, J., Cordón, O., Herrera, F., Magdalena, L. (eds.) Interpretability Issues in Fuzzy Modeling. Studies in Fuzziness and Soft Computing, vol. 128, pp. 193–217. Springer, Heidelberg (2003)
93. Yamamoto, Y., Yoshikawa, T., Furuhashi, T.: Improvement of performance of Japanese P300 speller by using second display. J. Artif. Intell. Soft Comput. Res. **5**(3), 221–226 (2015)
94. Zalasiński M., Cpałka K.: A new method of on-line signature verification using a flexible fuzzy one-class classifier, pp. 38–53. Academic Publishing House EXIT (2011)
95. Zalasiński, M., Cpałka, K.: New approach for the on-line signature verification based on method of horizontal partitioning. In: Rutkowski, L., Korytkowski, M., Scherer, R., Tadeusiewicz, R., Zadeh, L.A., Zurada, J.M. (eds.) ICAISC 2013, Part II. LNCS, vol. 7895, pp. 342–350. Springer, Heidelberg (2013)
96. Zalasiński, M., Cpałka, K., Er, M.J.: New method for dynamic signature verification using hybrid partitioning. In: Rutkowski, L., Korytkowski, M., Scherer, R., Tadeusiewicz, R., Zadeh, L.A., Zurada, J.M. (eds.) ICAISC 2014, Part II. LNCS, vol. 8468, pp. 216–230. Springer, Heidelberg (2014)
97. Zalasiński, M., Cpałka, K., Er, M.J.: A new method for the dynamic signature verification based on the stable partitions of the signature. In: Rutkowski, L., Korytkowski, M., Scherer, R., Tadeusiewicz, R., Zadeh, L.A., Zurada, J.M. (eds.) ICAISC 2015. LNCS, vol. 9120, pp. 161–174. Springer, Heidelberg (2015)
98. Zalasiński, M., Cpałka, K., Hayashi, Y.: New method for dynamic signature verification based on global features. In: Rutkowski, L., Korytkowski, M., Scherer, R., Tadeusiewicz, R., Zadeh, L.A., Zurada, J.M. (eds.) ICAISC 2014, Part II. LNCS, vol. 8468, pp. 231–245. Springer, Heidelberg (2014)
99. Zalasiński, M., Cpałka, K., Hayashi, Y.: New fast algorithm for the dynamic signature verification using global features values. In: Rutkowski, L., Korytkowski, M., Scherer, R., Tadeusiewicz, R., Zadeh, L.A., Zurada, J.M. (eds.) ICAISC 2015. LNCS, vol. 9120, pp. 175–188. Springer, Heidelberg (2015)