Image Denoising Using Noisy Chaotic Neural Networks

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Abstract This paper uses the noisy chaotic neural network (NCNN) that we proposed earlier for image denoising as a constrained optimization problem. The experimental results show that the NCNN is able to offer good quality solutions.

Keywords Image processing · Neural networks · Chaos

1 Introduction

The objective of image denoising is to estimate the original image from a noisy image with some assumptions or knowledge of the image degradation process. There exist many approaches for image denoising [1, 2]. Here we adopt a Bayesian framework because it is highly parallel and it can decompound a complex computation into a network of simple local computations [2], which is important in hardware implementation of neural networks. This approach computes the maximum a posteriori (MAP) estimation of the original image given a noisy image. The MAP estimation requires the prior distribution of the original image and the conditional distribution of the data. The prior distribution of the original images imposes contextual constraints and can be modeled by Markov random field (MRF) or, equivalently, by Gibbs distribution. The MAP-MRF principle centers on applying MAP estimation on MRF modeling of images.

Li proposed the augmented Lagrange Hopfield method to solve the optimization problem [3]. He transformed a combinatorial optimization problem into real constrained optimization using the notion of continuous relaxation labeling. The HNN was then used to solve the real constrained optimization.

The neural network approaches have been shown to be a powerful tool for solving the optimization problems [4]. The HNN is a typical model of neural network

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R. Wang, F. Gu (eds.), *Advances in Cognitive Neurodynamics (II)*,
DOI 10.1007/978-90-481-9695-1_98,

with symmetric connection weights. It is capable of solving quadratic optimization problems. However, it suffers from convergence to local minima. To overcome the weakness, different simulated annealing techniques have been combined with the HNN to solve optimization problems [5]. The TCNN showed good performance in solving traveling salesman problem [6] . However CSA is deterministic and is not guaranteed to settle down at a global minimum. In view of this, Wang and Tian [7] proposed a novel algorithm called stochastic chaotic simulated annealing (SCSA) which combines both stochastic manner of SSA and chaotic manner of CSA. In this paper the NCNN, which performs SCSA algorithm, is applied to solve the constrained optimization in the MAP-MRF formulated image denoising. Experimental results show that the NCNN outperforms the HNN and the TCNN.

2 The Noisy Chaotic Neural Network for Image Denoising

Let $u_i(I)$ denote the internal state of the neuron (i, I) and $p_i(I)$ denote the output of the neuron (i, I). $p_i(I) \in [0, 1]$ represents the strength that the pixel at location i takes the value I. The NCNN is formulated as follows [7]:

$$p_i^{(t)}(I) = \frac{1}{1 + e^{-u_i^{(t)}(I)/\epsilon}} \tag{1}$$

$$\begin{aligned} u_i^{(t+1)}(I) = ku_i^{(t)}(I) - z^{(t)}(p_i^{(t)}(I) - I_o) + n^{(t)} + \\ \alpha(\textstyle\sum_{i'=1, i' \neq i}^{N} \sum_{I'=1}^{M} T_{iI;i'I'} p_{i'}^{(t)}(I') + \mathcal{I}_i(I)) \end{aligned} \tag{2}$$

$$z^{(t+1)} = (1 - \beta_z) z^{(t)} \tag{3}$$

$$A[n^{(t+1)}] = (1 - \beta_n) A[n^{(t)}] \tag{4}$$

where

$T_{iI;i'I'}$: connection weight from neuron (i', I') to neuron (i, I);
$\mathcal{I}_i(I)$: input bias of neuron (i, I);
k: damping factor of nerve membrane $(0 \leq k \leq 1)$
α: positive scaling parameter for inputs;
ϵ: steepness parameter of the output function $(\epsilon \geq 0)$;
z: self-feedback connection weight or refractory strength $(z \geq 0)$;
I_o: positive parameter;
n: random noise injected into the neurons;
β_z : positive parameter $(0 < \beta_z < 1)$;
β_n : positive parameter $(0 < \beta_n < 1)$;
$A[n]$: the noise amplitude.

When $n^{(t)}$ in (2) equals to zero, the NCNN becomes TCNN. When $z^{(t)}$ equals to zero, the TCNN becomes similar to the HNN with stable fixed point

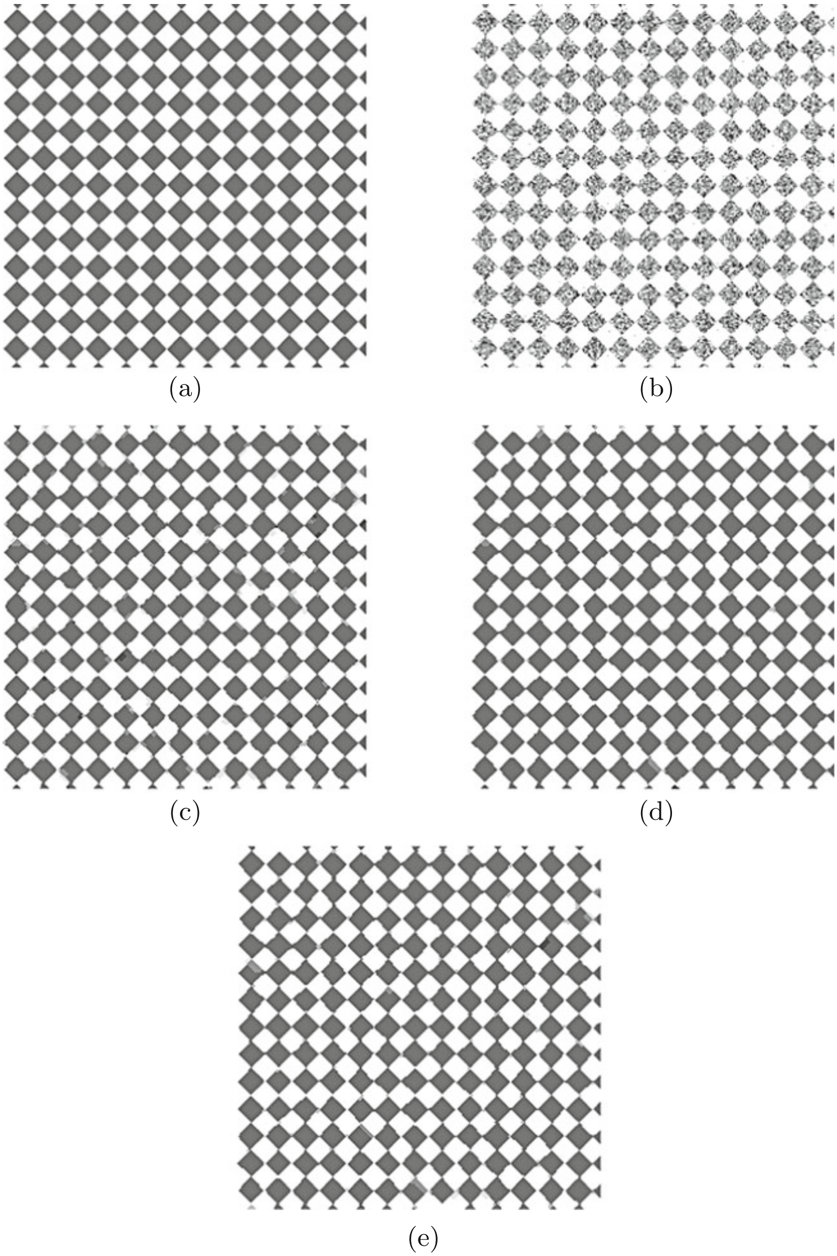

Fig. 1 Denoising of the square image with noise level $\sigma = 0.5$: (**a**) Original image. (**b**) Noisy image. (**c**)–(**e**) Denoised images using the HNN, the TCNN and the NCNN, respectively

dynamics. The basic difference between the HNN and the TCNN is that a nonlinear term $z^{(t)}(p_i^{(t)}(I) - I_o)$ is added to the HNN. Since the "temperature" $z^{(t)}$ tends toward zero with time evolution, the updating equations of the TCNN eventually reduce to those of the HNN. In (2) the variable $z^{(t)}$ can be interpreted as the strength of negative self-feedback connection of each neuron, the damping of $z^{(t)}$ produces successive bifurcations so that the neurodynamics eventually converge from strange attractors to a stable equilibrium point [5].

CSA is deterministic and is not guaranteed to settle down to a global minimum. In view of this, Wang and Tian [7] added a noise term $n^{(t)}$ in (2). The noise term continues to search for the optimal solution after the chaos of the TCNN disappears.

The dynamics of the NCNN for image denoising is described by (more details will be discussed elsewhere):

$$u_i^{(t+1)}(I) = ku_i^{(t)}(I) - z^{(t)}(p_i^{(t)}(I) - I_o) + n^{(t)} + \alpha q_i^{(t)}(I) \tag{5}$$

where

$$q_i(I) = -\frac{\partial L_\beta(p,\gamma)}{\partial p_i(I)}$$

Note that the Lagrange multipliers are updated with neural outputs according to $\gamma_k^{(t+1)} = \gamma_k^{(t)} + \beta C_i(p^{(t)})$. Experimental results are shown in Fig. 1.

3 Conclusion

We have implemented the noisy chaotic neural network (NCNN) to address the MAP-MRF formulated image denoising problem. NCNN effectively overcomes the local minimum problem because of its SCSA characteristics. We have shown that the NCNN gives better quality solutions compared to the HNN and the TCNN.

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