# Solving channel assignment problems using local search methods and simulated annealing

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## ABSTRACT

We solve the channel assignment problems (CAPs) with the main objective of minimizing the overall interference level while meeting the channel demand requirements. We use 3 methods, i.e., (1) local search (LS) with an acceptance ratio to re-initialize the search at a predefined threshold; (2) Simulated Annealing (SA); and (3) improve local search (ILS) with two control parameters, namely Restart (RS) and Stop (ST) thresholds. Simulation results on benchmarking CAPs show that these simple methods outperform other more complex heuristics on both the average and minimum cost solutions.

Keywords: Channel assignment, local search, simulated annealing

## 1. INTRODUCTION

The booming of the wireless communication industries in the area such as mobile communication (e.g., 3G mobile, Wireless LAN etc.), TV broadcasting, military communication and satellite communication projects have led to scarcity of usable frequencies in the radio spectrum. The usage of radio spectrum is controlled by the national governments and regulated worldwide by the International Telecommunication Union (ITU). Wireless service providers are charged for the licensing cost for the usage of the frequency bands and therefore lead to the need of developing frequency plans to minimize the interference level and licensing cost.

Various approaches have been introduced to maximize the usage of the available spectrum through different multiple access schemes (i.e. space, code and time and frequency division multiple access). In the commonly known mobile network, it is designed so that the same frequency can be used again after some re-use distance. In urban area each smaller cells is further divided into sectors through the use of directional antennae to increase the capacity of the system. While meeting the steadily increasing demand in the channel capacity requirement, the interference level at the receiving antennae must be kept low to a predefined limit. Therefore careful assignment of the channels to the incoming calls is necessary such that the signal-to-interference ratio (SIR) is maintained to an acceptable level.

There are three potential interferences namely *co-channel interference*; interference from using the same channel in other cells, *adjacent channel interference*; interference from adjacent cells using adjacent channels and finally *co-site interference*; interference from the same cell using different channels. For simplicity most channel assignment models ignore the environmental effects such as multi-path propagation and fade off rate of the radio wave etc and assume that omni directional antennae are used in the network.

In Static Channel Assignment (SCA, also known as Fixed Channel Assignment), fixed assignment of channels is done based on the forecast traffic to meet the immediate future demand. As for the Dynamic Channel Assignment (DCA), the process of assigning channels is adaptive and varies in accordance to the changing demands. The combination of the SCA and DCA lead to the Hybrid Channel Assignment (HCA) of which a number of channels are pre-assigned and some channels are reserved for online assignment upon request.

Great deal of research has been focusing on the SCA with various optimization techniques such as Simulated Annealing (SA), Neural Network, Genetic Algorithm, Tabu Search, etc. These techniques are used to solve for various Channel Assignment Problems (CAPs) with the objective of minimizing the overall interference level (Minimum-Interference CAP), the Blocking probability (Minimum Blocking CAP), the number of frequencies used (Minimum Order CAP) and the minimum and maximum used frequency (Minimum Span CAP).

In this paper, we deal with the channel assignment problems (CAPs) with the main objective of minimizing the overall interference level while meeting the channel demand requirements. The benchmark instances used by Smith & Palaniswami are used for comparison. We first use the traditional local search (LS) method in the convergence test (CT) to solve for the CAPs and an acceptance ratio is introduced as the controlling parameter to re-initialize the search at a predefined threshold. Three different perturbation functions namely one-exchange random (1-ER), one-exchange sequential (1-ES) and two-exchange random (2-ER) are proposed and tested in the CT. The 1-ER outperforms other perturbation functions and the minimum interference (cost) solutions obtained are superior to other heuristics. We then use Simulated Annealing (SA) to solve the CAPs. The SA performance is evaluated at different perturbation functions (1-ER & 1-ES) and various cooling schedules. Similarly, the results obtained using SA outperform other heuristics on both the average and minimum cost solutions. We then improve the local search (ILS) technique used in CT by introducing two control parameters, namely Restart (RS) and Stop (ST) thresholds, to increase the probability and consistency in searching for optimum solutions. Interestingly, this simple technique outperformed SA in some relatively simple test instances in terms of better average cost and faster convergence speed. Nevertheless the ILS is still inferior to SA in more difficult test problems. In general the ILS can still produce near optimum solutions and comparatively superior to results obtained in other heuristics as well.

#### 2. STATIC CHANNEL ASSIGNMENT

Suppose a mobile cellular network formed by *N cells* and the number of channels available in each cell is *M*. In each cell the channel demand is specified by a demand vector *D*, which  $D_i$  denotes channel demand in *cell i*. The minimum distance by which two channels must be separated in order to ensure low SIR is stored in the symmetric *Compatibility Matrix* suggested by Sivarajan et al [9] which is of dimension  $N \times N$ . Given the following  $4 \times 4$  compatibility matrix:

$$\mathbf{C} = \begin{pmatrix} 3 & 2 & 0 & 0 \\ 2 & 3 & 0 & 1 \\ 0 & 0 & 3 & 2 \\ 0 & 1 & 2 & 3 \end{pmatrix}$$

The diagonal terms  $C_{ii} = 3$  indicate that any two channels assigned to cell i must be at least three frequencies apart in order that no co-site interference exist.  $C_{12} = 2$  denotes that channels assigned to cells 1 and 2 must be at least two frequency apart. The off-diagonal element with  $C_{ij} = 1$  and  $C_{ij} = 2$  correspond to co-channel and adjacent channel constrain respectively. While  $C_{ij} = 0$  means the same frequency can be re-used.

For a given channel demand  $D^T = (1, 1, 1, 3)$ , it can be shown that the minimum channel M = 8 is required in order to obtain interference free solution as shown in the figure below.



Figure 1. Interference free assign for 4-cell and 8-channel network.

This type of channel assignment problem that determines the minimum channel (frequency) required for interference free solution is generally classified as Minimum Span CAPs. Smith & Palaniswami [1] denote this as CAP1. In practice the wireless service operators are supposed to pay for the full set of frequencies between the highest and lowest frequency used. Thus the span determines the cost, is therefore to be minimized.

For M = 7 there will be interference from the neighboring cell 3. Therefore M = 8 is the lower bound to achieve interference-free solution. Nevertheless, the number of channels available in most physical networks is far lesser than the

lower bound. Therefore the overall interference level should be kept to minimum. This type of channel assignment problem is classified as Minimum Interference CAP and denoted by Smith & Palaniswami [1] as CAP2.

The CAP2 is formulated as a zero and one generalized quadratic assignment problem with the objective function (cost) to be minimized. The objective function is reproduced as follow:

$$F(X) = \sum_{j=1}^{N} \sum_{k=1}^{M} X_{j,k} \sum_{i=1}^{N} \sum_{l=1}^{N} P_{j,i,(|k-l|+1)} X_{i,l}$$
(1)

 $X_{j,k}$  is a set of binary variables of which,

 $X_{j,k} = \begin{cases} 1 & \text{if cell j is assigned to channel k} \\ 0 & \text{otherwise} \end{cases}$ 

Subject to the channel demand

$$\begin{array}{l}
M\\ \Sigma\\ k=1
\end{array} \quad \forall j=1,\ldots,N$$
(2)

 $X_{j,k} \in \{0, 1\} \qquad \forall j = 1, ..., N, \text{ and } k = 1, ..., M.$ (3)

|k - l| is the distance between channel k and l in the channel domain.

The cost tensor **P** is generated from the compatibility matrix according to the following recursive relation

$$P_{j,i,m+1} = \max(0, P_{j,i,m} - 1) \text{ for } m = 1, \dots, M-1$$
(4)
$$P_{j,i,m+1} = \max(0, P_{j,i,m} - 1) \text{ for } m = 1, \dots, M-1$$
(5)

$$P_{j, i, 1} = C_{ji} \qquad \text{for all } j, i = j \qquad (5)$$
$$P_{j, j, 1} = 0, \qquad \text{for all } j. \qquad (6)$$

The cost tensor **P** is a three-dimension matrix with its first and second dimension similar to the compatibility matrix except that the diagonal elements are overwritten to zero. The zero diagonal elements signify there should have no interference when k = l, which refer to the same channel in that particular cell.

The penalty is then decreased linearly in the third dimension of the cost tensor until it becomes equal to zero. Therefore the effective depth of the cost tensor is equivalent to the maximum diagonal value of the compatibility matrix.

The test benchmark instances used by Smith and Palaniswami  $[\underline{1}]$  are adopted in this paper for performance comparison. The data sets used are divided into three classes as follow:

Class 1: EX1 and EX2  
Class 2: HEX1 to HEX4  
Class 3: KUNZ1 to KUNZ4  
EX1: 
$$DI^{T} = (1, 1, 1, 3),$$
  
 $N=4,$   
 $M=11,$   
 $C^{(1)} = \begin{pmatrix} 5 & 4 & 0 & 0 \\ 4 & 5 & 0 & 1 \\ 0 & 0 & 5 & 2 \\ 0 & 1 & 2 & 5 \end{pmatrix}$   
EX2:  $D2^{T} = (2, 2, 2, 4, 3),$   
 $N = 5,$   
 $M = 17,$ 

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$$\boldsymbol{C}^{(2)} = \begin{pmatrix} 5 & 4 & 0 & 0 & 1 \\ 4 & 5 & 0 & 1 & 0 \\ 0 & 0 & 5 & 2 & 1 \\ 0 & 1 & 2 & 5 & 2 \\ 1 & 0 & 1 & 2 & 5 \end{pmatrix}$$

The Class 2 test instances are based on the 21-cell regular hexagonal network used by Sivarajan et al [9] as follow.



Figure 2. 21-cell hexagonal networks.

The two sets of demand are given by

 $D3^{T}$  = [2,6,2,2,2,4,4,13,19,7,4,4,7,4,9,14,7,2,2,4,2]

 $D4^{T} = [1,1,1,2,3,6,7,6,10,10,11,5,7,6,4,4,7,5,5,5,6]$ 

By considering the first two rings of cells around a particular chosen cell as the interferer, the compatibility matrix is generated by varying the adjacent channel constraints and co-site constraints for different network sizes according to Table 1. The number of available channels M is obtained by applying the lower bound rules of Gamst [7].

Table 1 Problem description for 21-cell hexagonal network.

Problem	Ν	Μ	D	<b>Co-channel</b>	Adjacent	C <sub>ii</sub>
HEX1	21	37	D3	Yes	No	2
HEX2	21	91	D3	Yes	Yes	3
HEX3	21	21	D4	Yes	No	2
HEX4	21	56	D4	Yes	Yes	3

Table 2 Problem Descriptions for KUNZ Test Problems.

Problem	Ν	М	С	D
KUNZ1	10	30	$[C^{(5)}]_{10}$	$[D5^{T}]_{10}$
KUNZ2	15	44	$[C^{(5)}]_{15}$	$[D5^{T}]_{15}$
KUNZ3	20	60	$[C^{(5)}]_{20}$	$[D5^{T}]_{20}$
KUNZ4	25	73	$C^{(5)}$	$D5^T$

For instance the compatibility matrix of the HEX2 and HEX4 can be generated as follow. Consider the cell 1 of the 21cell hexagonal network on Figure 2. The interfering cells for the first ring are given by cell 2, 7 and 8. The off diagonal elements of the compatibility matrix associated to these cells are  $C_{12}=C_{17}=C_{18}=2$ ; that is co-channel constrain equal to two and adjacent interference of one (the 3<sup>rd</sup> dimension of the cost tensor). The interfering cells in the second ring are given by cell 3, 6, 9, 14, 15 and 16. The corresponding off diagonal elements are  $C_{13}=C_{16}=C_{19}=C_{1.14}=C_{1.15}=C_{1.16}=1$ ; only co-channel interference of one. As the compatibility matrix is symmetrical, therefore  $C_{ij} = C_{ji}$  (e.g.  $C_{12}=C_{21}$ ). Kunz derived the Class 3 test instances from an actual 24 x 21-km area around Helsinki, Finland. The computer program GRAND [8] was used to obtain the expected traffic demand and interference relationship between the 25 regions around the base stations that are distributed unequally over the area.

The demand vector is given by

 $D5^{T} = [10, 11, 9, 5, 9, 4, 5, 7, 4, 8, 8, 9, 10, 7, 7, 6, 4, 5, 5, 7, 6, 4, 5, 7, 5]$ 

These test problems are obtained by considering different cells size of the network as shown in the Table 2. The subscript on the compatibility and demand vectors denote the number of rows and columns to be considered when generating the matrix.

For instance  $[C^{(5)}]_{15}$  denotes the compatible matrix is generated by consider the first 15 rows and columns of the  $C^{(5)}$  given below.

		1	2	I	1	0	0	1	1	1	1	0	0	0	0	I	1	1	0	0	0	0	0
		(	1	2	1	1	0	0	1	1	1	1	0	0	0	0	1	1	1	0	0	0	0
			1	1	2	1	1	0	0	1	1	1	1	0	0	0	0	1	1	1	0	0	0
			0	1	1	2	1	0	0	0	1	1	1	1	0	0	0	0	1	1	0	0	0
			0	0	1	1	2	0	0	0	0	1	1	1	0	0	0	0	0	1	0	0	0
			1	0	0	0	0	2	1	1	0	0	0	0	1	1	1	0	0	0	0	0	0
			1	1	0	0	0	1	2	1	1	0	0	0	1	1	1	1	0	0	1	0	0
			1	1	1	0	0	1	1	2	1	1	0	0	0	1	1	1	1	0	1	1	0
			1	1	1	1	0	0	1	1	2	1	1	0	0	0	1	1	1	1	1	1	1
			0	1	1	1	1	0	0	1	1	2	1	1	0	0	0	1	1	1	0	1	1
$C^{(5)}$	=		0	0	1	1	1	0	0	0	1	1	2	1	0	0	0	0	1	1	0	0	1
			0	0	0	1	1	0	0	0	0	1	1	2	0	0	0	0	0	1	0	0	0
			0	0	0	0	0	1	1	0	0	0	0	0	2	1	1	0	0	0	0	0	0
			1	0	0	0	0	1	1	1	0	0	0	0	1	2	1	1	0	0	1	0	0
			1	1	0	0	0	1	1	1	1	0	0	0	1	1	2	1	1	0	1	1	0
			1	1	1	0	0	0	1	1	1	1	0	0	0	1	1	2	1	1	1	1	1
			0	1	1	1	0	0	0	1	1	1	1	0	0	0	1	1	2	1	1	1	1
			0	0	1	1	1	0	0	0	1	1	1	1	0	0	0	1	1	2	0	1	1
			0	0	0	0	0	0	1	1	1	0	0	0	0	1	1	1	1	0	2	1	1
			0	0	0	0	0	0	0	1	1	1	0	0	0	0	1	1	1	1	1	2	1 /
			0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	1	1	1	1	1	2

## 3. SIMULATIONS AND RESULTS

This Section presents simulations and results. The simulations are performed in several stages. Due to limited spaces, more details on the simulation results will be discussed elsewhere.

#### **3.1** Convergence Test (Simple Local Search)

Part of the SA algorithm is implemented in this stage without a cooling schedule. Instead, for the reason of simplicity we introduced a parameter called acceptance ratio to randomly re-assign the channel assignment matrix when the search of the solution reaches a certain predefined threshold. The new solution will only be accepted when the overall cost (interference) is lower or equal to the current solution, which is equivalent to SA at freezing temperature or a local search.

We record the assignment result if any improve solution has been found. Several perturbation functions were investigated to compare the performance and the best perturbation method was then used in the SA algorithm implementation. These results obtained were then used as a reference to fine-tune the SA cooling schedule.

#### 3.2 Simulated Annealing (SA)

In this section instead of merely accepting solution with improvement in cost, a probabilistic acceptance function is added to allow hill climbing (escape from local minima) and the allowable deterioration of the cost level is controlled by



Figure 3. Basic functional flow of convergence test.

a cooling schedule. The best cooling schedule was then determined by considering the Convergence Speed, Average Cost and Minimum Cost Achievable for various cooling schedules.

#### 3.3 Improved Local Search

The simple local search technique was modified to use some fixed thresholds (namely re-start and stop thresholds) instead of the acceptance ratio to re-initialize the search at regular intervals. The results obtained were quite competitive and perform better for KUNZs (faster convergence) and HEX1 test problems than the SA.

	GAMS	SD	SSA	HN	HCHN	SONN	СТ
Problem	Min	Ave Min	Ave Min	Ave Min	Ave Min	Ave Min	Min
EX1	2	0.6 0	0.0 0	0.2 0	0.0 0	0.4 0	0
EX2	3	1.1 0	0.1 0	1.8 0	0.8 0	2.4 0	0
HEX1	54	56.8 55	50.7 49	49.0 48	48.7 48	53.0 52	46
HEX2	27	28.9 25	20.4 19	21.2 19	19.8 19	28.5 24	13
HEX3	89	88.6 84	82.9 79	81.6 79	80.3 78	87.3 84	73
HEX4	31	28.2 26	21.0 20	21.6 20	18.9 17	29.1 22	11
KUNZ1	28	24.4 22	21.6 21	22.1 21	21.1 20	22.0 21	20
KUNZ2	39	38.1 36	33.2 32	32.8 32	31.5 30	33.4 33	29
KUNZ3	13	17.9 15	13.9 13	13.2 13	13.0 13	14.4 14	13
KUNZ4	7	5.5 3	1.8 1	0.4 0	0.1 0	2.2 1	0

Table 3. Lowest cost comparison for various heuristics with convergence test.

Legend:

GAMS - commercial optimization package with non-linear solver (MINOS-5)

SD - Steepest Decent

SSA - Static Simulated Annealing

HN - Hopfield Network

HCHN - Hill Climbing Hopfield Network

SONN - Self-Organizing Neural Network

CT - Convergence Test

# 4. CONCLUSION

In this paper, we have solved the quadratic channel assignment problems using the Simulated Annealing (SA) and Improved Local Search (ILS) techniques. Both techniques achieve reasonably good solutions as compared to other heuristics. Between the two techniques used, the SA generally performs better than the ILS due to its general applicability to all test instances (less dependent to network sizes and demands) and its ability to achieve good quality solutions more consistently. Our experiment results show that the performance of the SA depends greatly on the selection of the cooling schedules and the perturbation functions. Among the three perturbations methods proposed, the One-Exchange Random (1-ER) perturbation function has been determined to be superior in the quadratic channel assignment formulation for both the SA and ILS.

The effect of various cooling schedules on the performance of SA has been evaluated by considering such performance factors as convergence speed, average and minimum costs. Our experiment results shown that too high of an initial temperature in the cooling schedule will cause excessive processing overhead whereas higher initial temperature is required to escape from local minima. The optimum initial temperature of the SA has been determined through experiment in this project for the various test instances and more consistent results are obtained by increasing the  $L_k$  and  $\alpha$ . Nevertheless in general, the optimum initial temperature changes for different network sizes, demands and interference constrains.

Finally an application program has been developed based on the SA algorithm that can be used to solve the channel assignments for various network sizes and demands. The acceptance ratio technique proposed by Kirkpatrick [11] has been adopted in the program as a provision to dynamically determine the initial temperature for various network conditions.

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