Proceedings - 19th International Conference - IEEE/EMBS Oct. 30 - Nov. 2, 1997 Chicago, IL. USA

Neural Networks under Periodic Operating Conditions: Transitions between Dynamic States

Lipo Wang

School of Computing and Mathematics
Deakin University
662 Blackburn Road
Clayton, Victoria 3168 Australia
Email: lwang@deakin.edu.au
http://www.cm.deakin.edu.au/~lwang

Abstract - We model the nonlinear dynamics of a neural system *under time-dependent operating conditions* by letting a system parameter vary periodically with time in a higher order neural network. We demonstrate explicitly that the varying operating environment can cause transitions between periodically oscillatory states and chaotic states. We discuss these results in relation to information processing in such systems.

Keywords - Neural networks, chaos, oscillations, nonlinear dynamics.

1. INTRODUCTION

Most existing research efforts on artificial neural networks (ANNs) focus on ANNs operating under *time-independent* conditions (e.g., [2] [11]). Effects of a varying environment should be investigated, since the brain and man-made systems are functioning in an ever-changing world. Markus *et al* [4] studied natural populations under varying growth conditions by introducing temporal perturbation in a bifurcation parameter. Ott *et al* [6] showed that chaos can be controlled by time-dependent perturbation of a system parameter. This method has been used to induce periodic oscillations in a nonlinear oscillator model [7] and a 9×9 network of oscillators [8]. We [10] studied effects of a temporal perturbation on a large network consisting of McCulloch-Pitts neurons [5], by calculating Lyapunov exponents in various parameter spaces and for various forms of varying environments. In the present paper, we demonstrate explicitly the temporal dynamics of the network and transitions between dynamic states under periodic operating conditions.

The present work is based on an exactly solvable higher order ANN which we proposed earlier [12]. In this system, both first and second order synapses are disconnected randomly at one time to model the sparse connectivity observed in real neural systems. Exact solutions are derived for the network dynamics and a variety of dynamical behaviors such as stable retrieving, oscillations, and chaos are revealed. Explicitly, we consider N binary neurons with the following updating rule:

$$S_i(t+1) = \begin{cases} +1 \text{ if } h_i(t) > 0 , \\ -1 \text{ if } h_i(t) \le 0 , \end{cases} \tag{1}$$

where $S_i(t)$ represents the state of neuron i at time t and the total input for neuron i is

$$h_i(t) = \gamma_1 \sum_{j=1}^{N} T_{ij} S_j(t) + \gamma_2 \sum_{j,k=1}^{N} T_{ijk} S_j(t) S_k(t) + \eta_i , \tag{2}$$

In eq.(2),

$$T_{ij} = C_{ij} \sum_{\mu=1}^{p} S_i^\mu S_j^\mu , \; T_{ijk} = C_{ijk} \sum_{\mu=1}^{p} S_i^\mu S_j^\mu S_k^\mu \tag{3}$$

are the modified Hebbian synaptic efficacies, $\vec{S}^\mu$ is the μ-th stored pattern, and p is the number of patterns stored. The coefficients γ_1 and γ_2 measure the relative strengths of first order and second order interactions. We have introduced synaptic disruptions in the efficacies T_{ij} and T_{ijk} by choosing random variables C_{ij} and C_{ijk} as follows: C_{ij} is 1 with a probability (C/N), C_{ijk} is 1 with a probability $(2C/N^2)$, C_{ij} and C_{ijk} are zero otherwise. We also include in eq.(2) a background random Gaussian noise η_i with a standard deviation σ_o.

When all neurons are updated simultaneously, the network obeys the following dynamic equation [12]:

$$m(t+1) = \mathrm{erf}\{\frac{\gamma_1 m(t) + \gamma_2[m(t)]^2)}{\sqrt{2}\sigma}\} . \quad (4)$$

Here $m(t)$ is the average overlap between the state of the system at time t and the initial attracting memory state. $\mathrm{erf}(y) = \frac{2}{\sqrt{\pi}} \int_0^y e^{-x^2} dx$ is the standard error function. σ is the rescaled noise deviation that represents the combined effects of the random synaptic disruption, interference between stored patterns, and additional background noise:

$$\sigma \equiv \sqrt{(\gamma_1^2 + \gamma_2^2)[(p-1)/C] + (\sigma_o/C)^2} \quad . \quad (5)$$

When $\sigma > 0.8$ and for any positive initial overlap $m(0) > 0$, the only non-negative fixed points are zeros. For $0.8 > \sigma > 0.2$, the system converges to a stable positive fixed point with any positive initial overlap. For example, the top panel of Fig.1 shows such a stable state at $\sigma = 0.3$. This corresponds to static retrievals. As σ decreases below 0.2, oscillations starts to appear and the network dynamics becomes chaotic through a sequence of period-doubling bifurcations as σ is further reduced. The second panel in Fig.1 shows a periodic oscillation with a period 2 at $\sigma = 0.14$, whereas the third panel of Fig.1 shows chaos at $\sigma = 0.12$. Amidst the largely chaotic behaviors, there are small "windows" of σ values in which the network dynamics exhibits periodic oscillations, for example, a *period-3* oscillation shown in the bottom panel of Fig.1. The bifurcation parameter σ is assumed to be *independent of time* in Fig.1.

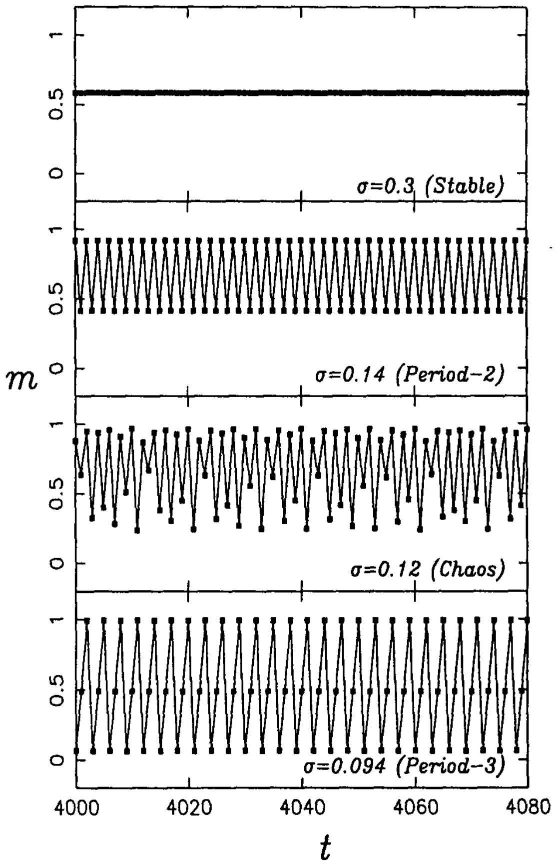

Figure 1: The overlap m between the state of the network and the initial attracting memory pattern plotted as a function of time t, for various choices of the rescaled noise level σ. The bifurcation parameter σ is independent of time.

2. PERIODIC OPERATING CONDITIONS

We now model a dynamic operating environment with a *time-dependent* system parameter σ. A simple choice of the time-dependence of σ is that σ alternates periodically between two values, for example, $\sigma = A$ at odd time steps and $\sigma = B$ at even time steps. The following interesting behaviors emerge as a result.

The top two panels of Fig.2 show chaotic dynamics of the network when σ is a *constant of time*, that is, $\sigma = A = 0.0893$ for all t in the top panel and $\sigma = B = 0.0879$ for all t in the second panel. However, when we let σ alternates periodically between $A = 0.0893$ and $B = 0.0879$, the network exhibits a periodic oscillation, as shown in the bottom panel of Fig.2. This demonstrates that temporal variations of the system parameter can induce order from chaotic behaviors, which is similar to the "noise-induced order" discussed by Matsumoto and Tsuda [3] in a physical system, as well as work on control of chaos by other authors [6], [7], [8] in various other systems.

The opposite process is also possible. The top panel of Fig.3 shows the temporal dynamics of the network, i.e., a period-3 oscillation, when σ is a *constant of time*, that is, $\sigma = A = 0.0946$ for all t. Similarly, the network oscillates with a period 6 when the bifurcation parameter is kept at $\sigma = B = 0.0907$ all the time (see the second panel of Fig.3). However, when we let σ alternates periodically between $A = 0.0946$ and $B = 0.0907$, the network exhibits chaotic behavior as shown in the bottom panel of Fig.3.

Time-dependent operating conditions can therefore

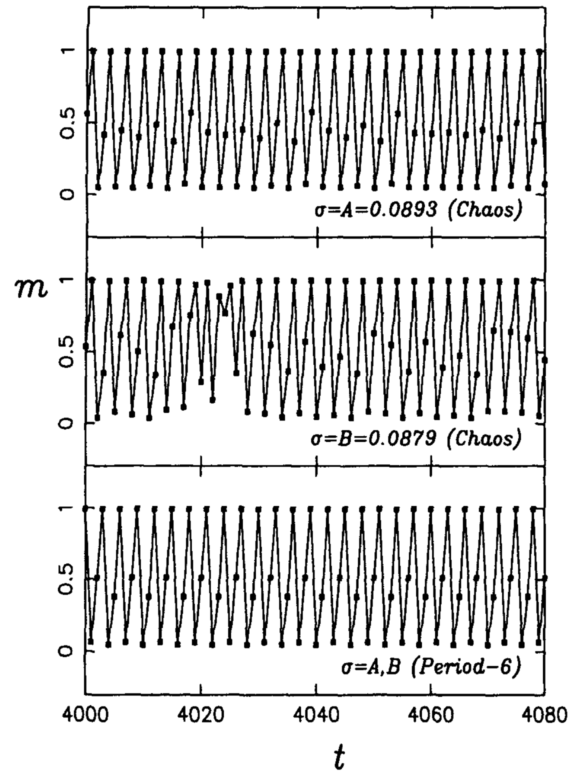

Figure 2: Transition from chaotic to periodic states under periodic operating conditions. The overlap m between the state of the network and the initial attracting memory pattern plotted as a function of time t, for various choices of the rescaled noise level σ. The bifurcation parameter σ is independent of time in the top panel ($\sigma = A = 0.0893$) and the second panel ($\sigma = B = 0.0879$), whereas σ alternates periodically between two values $A = 0.0893$ and $B = 0.0879$.

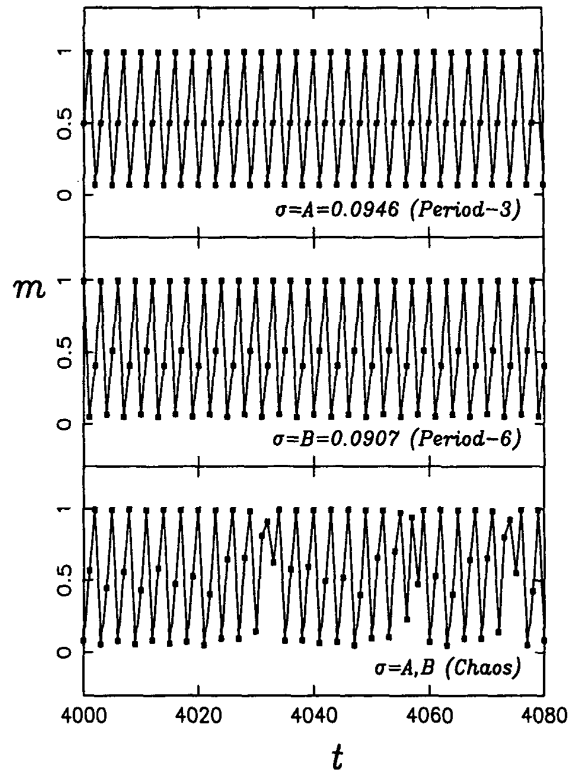

Figure 3: Transition from periodic to chaotic states under periodic operating conditions. The overlap m between the state of the network and the initial attracting memory pattern plotted as a function of time t, for various choices of the rescaled noise level σ. The bifurcation parameter σ is independent of time in the top panel ($\sigma = A = 0.0946$) and the second panel ($\sigma = B = 0.0907$), whereas σ alternates periodically between two values $A = 0.0946$ and $B = 0.0907$.

induce transitions from ordered states to chaotic states and vice versa. (Similar transitions between chaotic and periodic states were also observed in the study of natural populations under time-dependent growth conditions [4].) If *dynamical* behaviors are used to efficiently store memory, as suggested by recent physiological and theoretical studies [9], [1], [13], a varying environment can be used to facilitate information processing, e.g., memory retrieving, by inducing transitions between various dynamic network states, which will be a subject of future investigations.

ACKNOWLEDGEMENTS

The author would like to thank Tim Pearson of the California Institute of Technology for making a wonderful plotting software PGPLOT available free of charge and for helping create the figures in this paper using PGPLOT.

References

[1] W.J. Freeman, "The physiology of perception." *Scientific American*, February, pp. 78-85, 1991.

[2] R. Hecht-Nielsen, *Neurocomputing*, Reading, MA: Addison-Wesley, 1990.

[3] K. Matsumoto and I. Tsuda, "Noise-induced order," *J. Stat. Phys.*, vol. 31, pp. 87-106, 1983.

[4] M. Markus, B. Hess, J. Rössler and M. Kiwi, "Populations under periodically and randomly varying growth conditions." in *Chaos in Biological Systems*, H. Degn, A.V. Holden, and L.F. Olsen (eds.), New York:Plenum Press, 1987, pp. 267 - 277.

[5] W.S. McCulloch and W.H. Pitts, "A logical calculus of ideas immanent in nervous activity," *Bull. Math. Biophys.*, vol. 5, pp. 115-133, 1943.

[6] E. Ott, C. Grebogi, and J.A. Yorke, "Controlling chaos," *Phys. Rev. Lett.*, vol. 64, no. 11, pp. 1196-1199, March, 1990.

[7] S. Rajasekar and M. Lakshmanan, *Int. J. Bifurc. Chaos*, vol. 2, pp. 201, 1990.

[8] J. A. Sepulchre and A. Babloyantz, "Controlling chaos in a network of oscillators", *Phys. Rev. E*, vol. 48, no. 2, pp. 945-950, August, 1993.

[9] C. A. Skarda and W. J. Freeman, "How brains make chaos in order to make sense of the world," *Brain and behavioral Science*, vol. 10, pp. 161-195, 1987.

[10] L. Wang, "Oscillatory and chaotic dynamics in neural networks under varying operating conditions," *IEEE Trans. Neural Networks*, vol. 7, no. 6, pp. 1382-1388, November, 1996.

[11] L. Wang and D.L. Alkon, Eds. *Artificial Neural Networks: Oscillations, Chaos, and Sequence Processing*, Los Alamitos, California: IEEE Computer Press, 1993.

[12] L. Wang, E. E. Pichler, and J. Ross, "Oscillations and chaos in neural networks: an exactly solvable model," *Proc. Natl. Acad. Sci. USA*, vol. 87, pp. 9467 - 9471, December, 1990.

[13] Y. Yao and W. J. Freeman, "Model of biological pattern recognition with spatially chaotic dynamics," *Neural Networks*, vol. 3, pp. 153-170, 1990.

Lipo Wang received his PhD in physics from Louisiana State University in 1988. Since then he has been interested in the theory and applications of neural networks. In 1989 he worked at Stanford University, CA, as a postdoctoral fellow. In 1990 he was a lecturer in electrical engineering at the University College of University of New South Wales, Australia. From 1991 to 1994 he worked at the National Institutes of Health as a Fogarty Associate. He is now a lecturer at the School of Computing and Mathematics, Deakin University, Rusden Campus, Australia. His research interests include temporal behavior, pattern recognition, fuzzy control, and biological networks. He is the author or co-author of over 35 journal publications, holds a U.S. patent on neural networks, and is the principal author/editor of *Artificial Neural Networks: Oscillations, Chaos, and Sequence Processing* (Los Alamitos, CA: IEEE Computer Press, 1993).