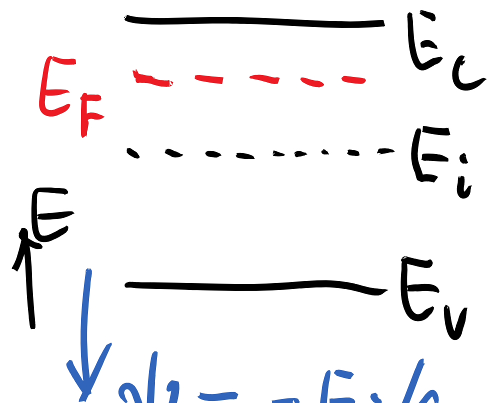
EE3013 - Summary

1. Basics.

$$\phi_F \equiv -E_F/q$$

$$n = n_i e^{(E_F - E_i)/kT} = n_i e^{(\psi - \phi_F)/V_{th}}$$

$$p = n_i e^{(E_i - E_F)/kT} = n_i e^{(\phi_F - \psi)/V_{th}}$$



$$\psi \equiv -E_i/q$$

$$n_0 p_0 = n_i^2 \quad \text{(equilibrium)}$$

$$p_0 + N_D - n_0 - N_A = 0 \quad \text{(charge neutrality)}$$

$$\Rightarrow n_0 \simeq N_D, \quad p_0 \simeq N_A \quad \text{("neutral")}$$

bands - flat

Nonequilibrium — external bias — Quasi-Fermi Level ("Imref")

$$n = n_i e^{(E_{Fn}-E_i)/kT} = n_i e^{(\psi-\phi_{Fn})/v_{th}}$$

$$\Rightarrow \phi_{Fn} \equiv -E_{Fn}/q = \psi - v_{th}\ln(n/n_i)$$

$$\Rightarrow \frac{d\phi_{Fn}}{dx} = \frac{d\psi}{dx} - v_{th}\frac{1}{n}\frac{dn}{dx}$$

$$\Rightarrow J_n = qn\mu_n\left(-\frac{d\phi_{Fn}}{dx}\right) = qn\mu_n\underbrace{\left(-\frac{d\psi}{dx}\right)}_{\mathcal{E}} + qn\underbrace{v_{th}\mu_n}_{D_n}\frac{dn}{dx}$$

"drift" + "diff"

$-qnv$

$(v = -\mu\mathcal{E})$

Basic Concept of **Fermi Level**: *Electrochemical Potential*

$$\boxed{E_F = \Phi - q\Psi}$$

The **Fermi level** (E_F) plays an important role in formulating ***equilibrium*** conditions when two different systems of different allowable energies are brought into contact. The combined system will be in thermal equilibrium only when E_F is the same in both parts. If E_F (relative to a common datum) in each system is not initially equal before contact, then on contact there will be a flow of electrons from the system with the higher initial E_F to the system with the lower initial E_F. This electron flow will continue until equality of the Fermi energies of the two systems is achieved.

$$\boxed{E_{F1} = E_{F2} \text{ or } \Phi_1 - q\psi_1 = \Phi_2 - q\psi_2}$$

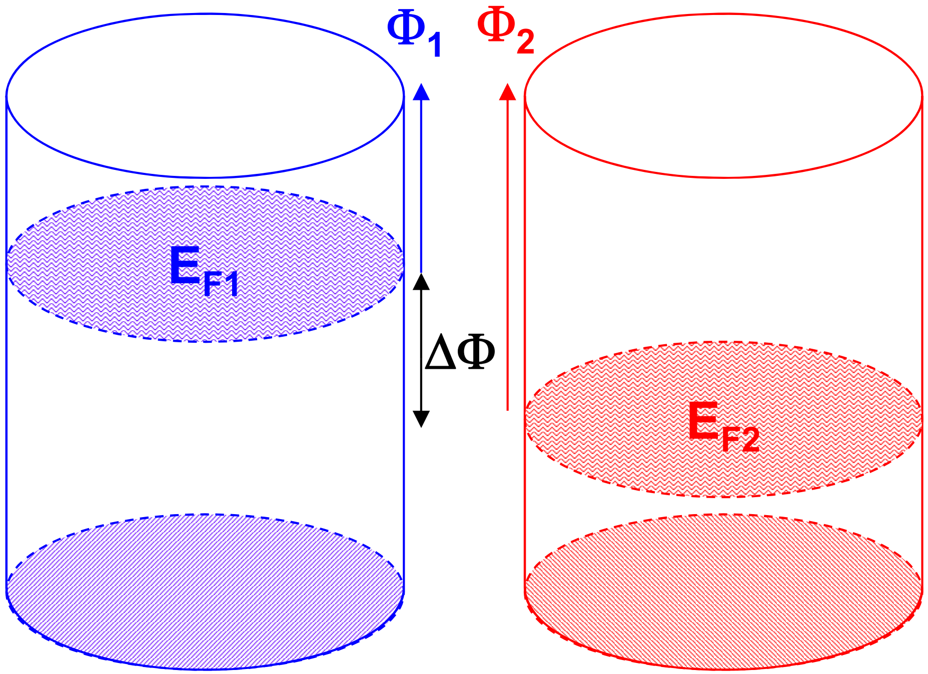
Let the work done in removing an electron from one material in isolation be Φ_1 and from the other, Φ_2, which are the *chemical potentials* (work functions in eV) of electrons in the two materials. Then, one might think equilibrium on contact would occur when $\Phi_1 = \Phi_2$. However, for charged particles like electrons, transfer is accompanied by *charging* of the materials. As a result, the two materials acquire potentials ψ_1 and ψ_2. The work done on transfer of an electron of charge (–q) now will be zero provided that $\boldsymbol{\Phi_1 - q\psi_1 = \Phi_2 - q\psi_2}$. In fact, this condition for equilibrium is identical to the condition of equality of ***Fermi levels*** (another name – ***electrochemical potentials***): $\mathbf{E_F = \Phi - q\psi}$

chemical potential *electrostatic potential*

Visualizing “Fermi Level” (Electrochemical Potential)

Φ: “Work function” (chemical potential) — amount of work to bring “water” out.

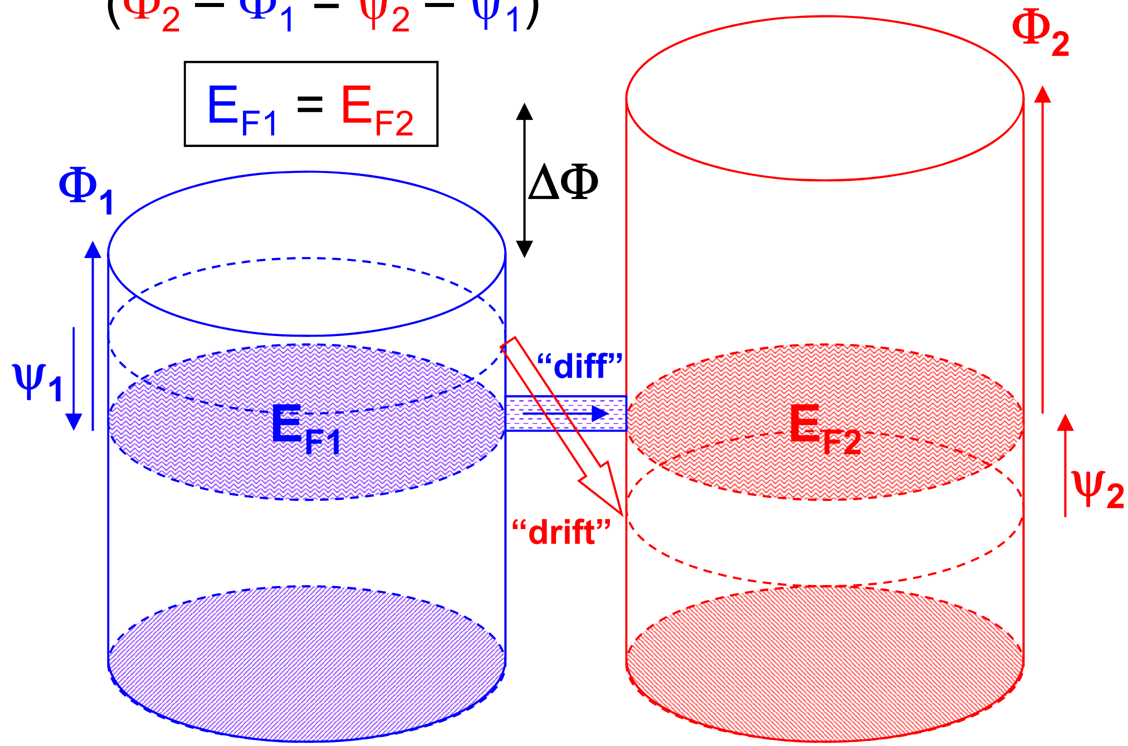
$\boxed{\Phi_1 < \Phi_2}$ $(E_{F1} > E_{F2})$



(“Open” System: separate equilibrium)

On connection: water flows from high level (E_{F1}) to low level; and water levels changing (“charging”) until $\Phi_1 - \psi_1 = \Phi_2 - \psi_2$.

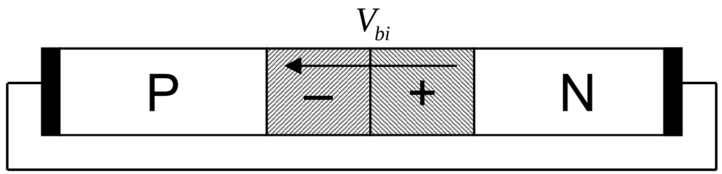
$(\Phi_2 - \Phi_1 = \psi_2 - \psi_1)$



(“Short” System: new equilibrium)

PN-junction : Built-in Potential

$$V_{bi} = \frac{kT}{q}\ln\frac{p_{op}^{\sim N_A}}{p_{on}\,\sim\frac{n_i^2}{N_D}} \approx \frac{kT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right)$$



$$\psi_{F,P} \equiv -\left(E_F - E_{i,P}\right)/q \qquad V_a = 0 \qquad \psi_{F,N} \equiv -\left(E_F - E_{i,N}\right)/q$$

$$= \phi_F - \psi_P \qquad\qquad = \phi_F - \psi_N$$

$$\boxed{\Phi_P - q\psi_P = \Phi_N - q\psi_N}$$

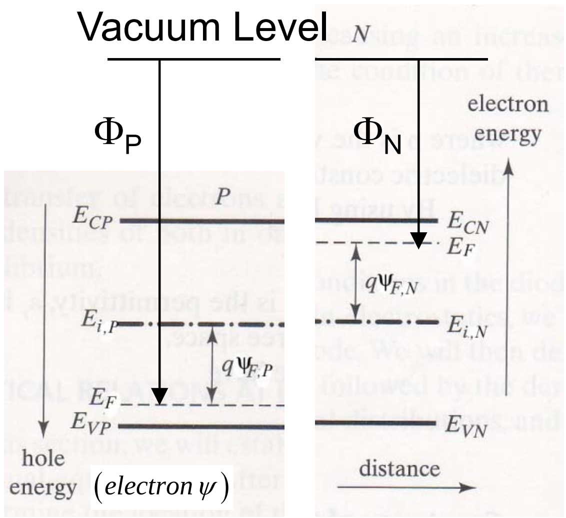
which leads to:

$$\boxed{-(\Phi_N - \Phi_P) = -q(\psi_N - \psi_P)}$$

Before contact ↓ $E_{F,N} - E_{F,P}$ || $kT\ln\left(\frac{N_D N_A}{n_i^2}\right)$

After contact ↓ $E_{i,N} - E_{i,P}$ || qV_{bi}

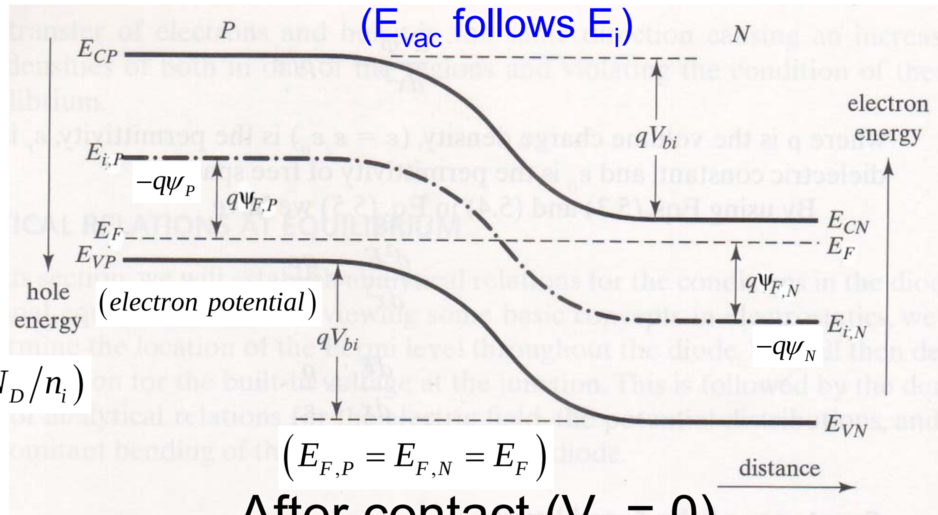
$$kT\ln\left(\frac{N_D N_A}{n_i^2}\right) = qV_{bi}$$



$$E_{F,P} = E_i - kT\ln\left(N_A/n_i\right) \qquad E_{F,N} = E_i + kT\ln\left(N_D/n_i\right)$$

$$\left(E_{i,P} = E_{i,N} = E_i\right)$$

Before contact ($V_a = \infty$)



$$\left(E_{F,P} = E_{F,N} = E_F\right)$$

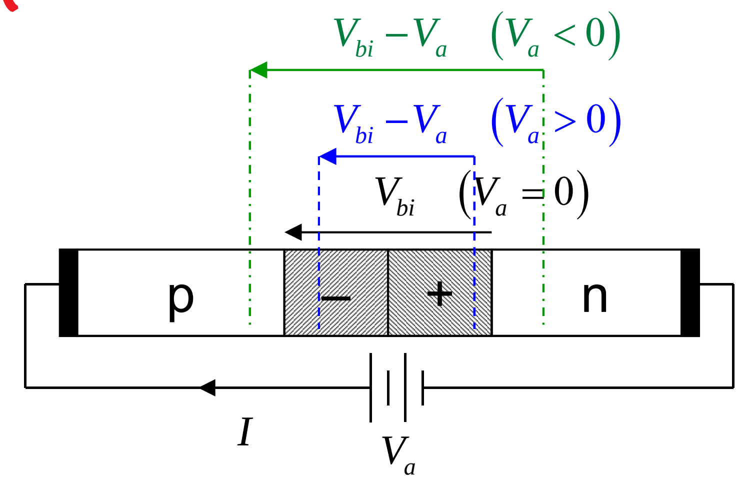
After contact ($V_a = 0$)

PN-junction - with bias (V_a)

$$V_j = V_{bi} - V_a \xrightarrow{V_a=0} V_{bi}$$

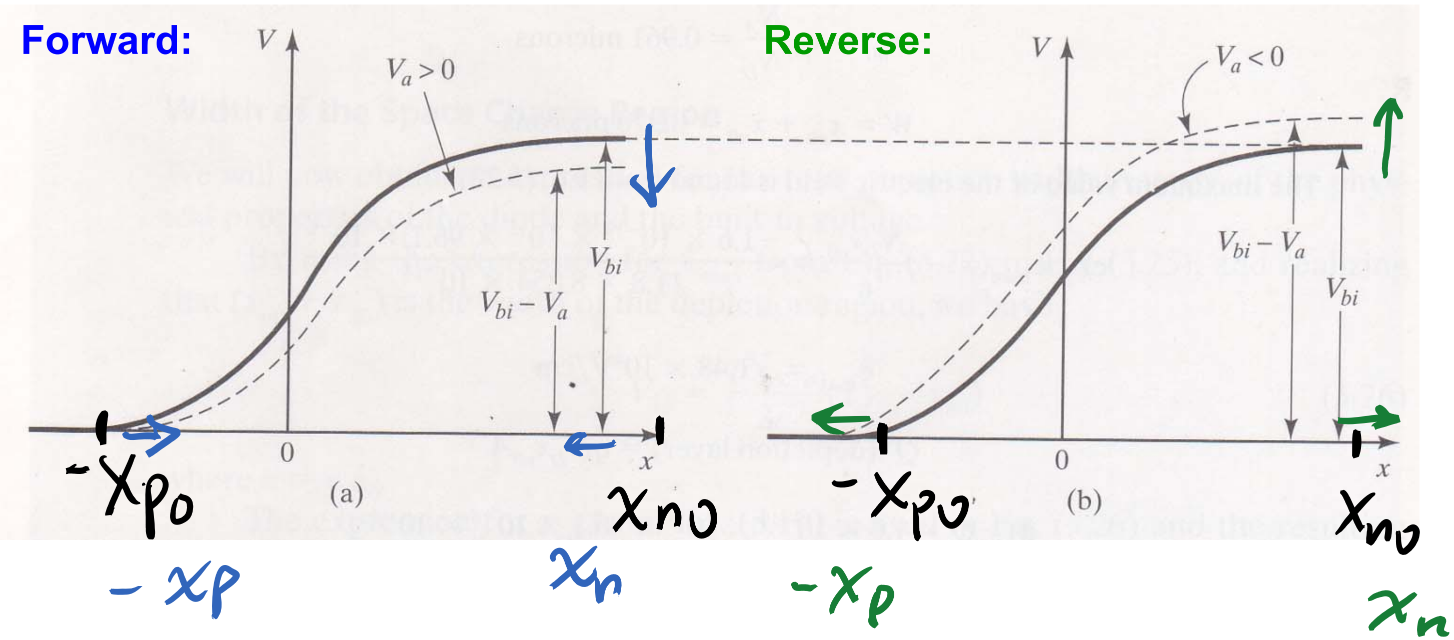
Fwd ($V_a > 0$): $V_j \downarrow$, $W \downarrow$

Rev ($V_a < 0$): $V_j \uparrow$, $W \uparrow$



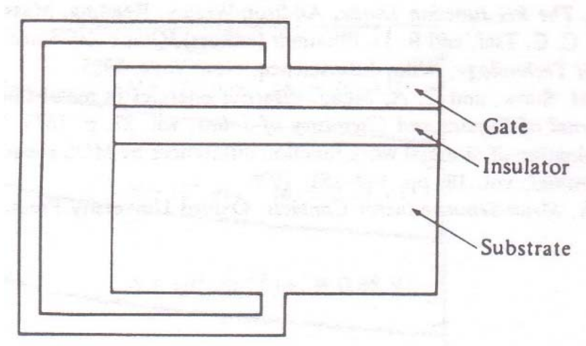
V_j: "Junction voltage"

"barrier height"



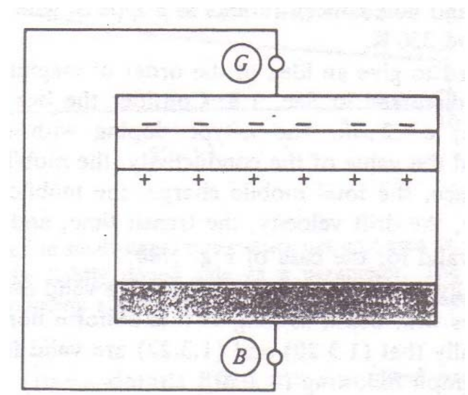
MOS: Concept of the Flatband Voltage

(a)



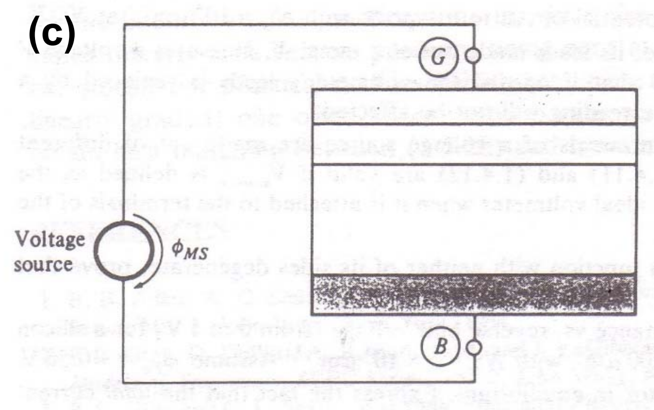
A 2-terminal MOS structure with gate, substrate (bulk), and short-circuiting external connection all made out of the same semiconductor material.

(b)



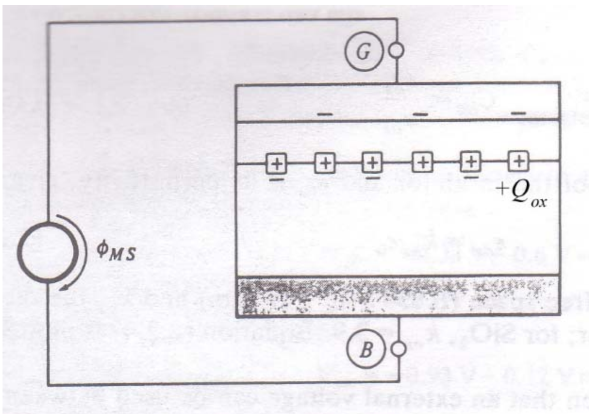
A MOS (with gate and bulk made of different materials) with zero oxide charge and with gate–bulk terminals short-circuited.

(c)



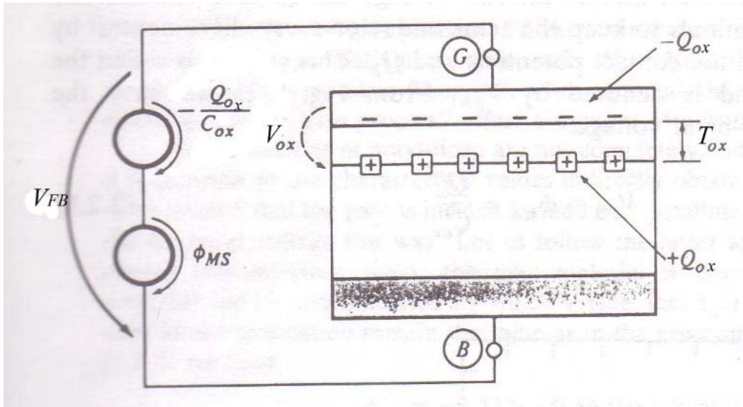
The structure of (b) with a voltage source (ϕ_{MS}) so that the surface charge becomes zero.

(d)



Effect of non-zero effective oxide charge.

(e)



The structure of (d) with additional external bias ($-Q_{ox}/C_{ox}$) so that the surface charge becomes zero.

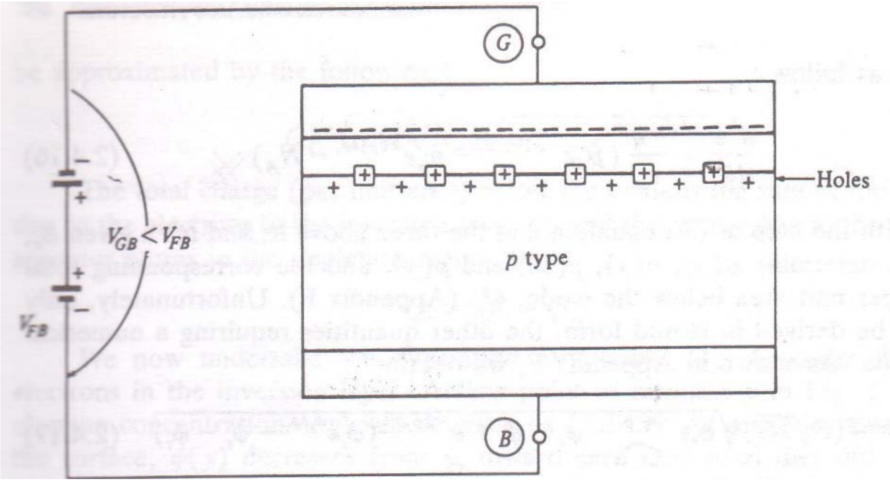
Flatband voltage:

$$\boxed{V_{FB} = \phi_{MS} - Q_{ox}/C_{ox}}$$

With an external bias V_{FB}, the MOS structure becomes "ideal" (bands become flat).

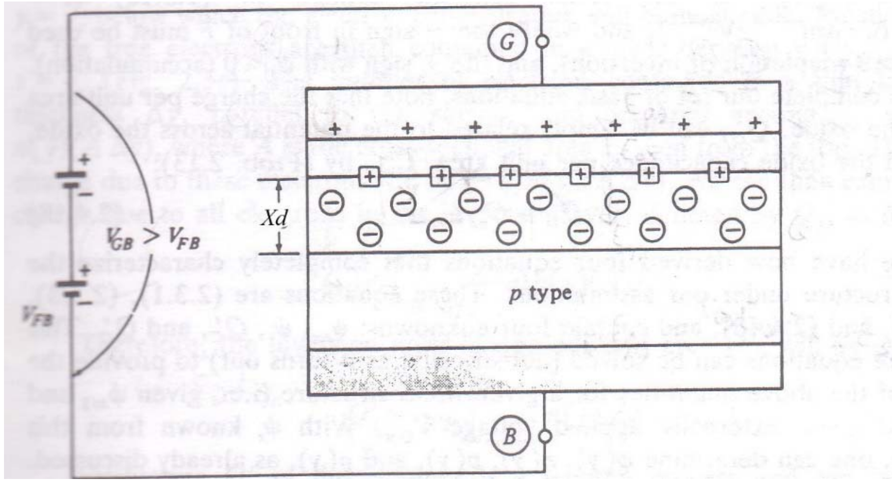
MOS: Accumulation and Depletion

Accumulation:



A negative bias is applied on top of V_{FB} (so the total gate–bulk voltage is below flatband voltage). The negative charge on the gate induces (attracts) the mobile holes towards the (p-type) Si surface, forming a thin layer of positively-charged holes "accumulated" at the surface. The induced hole layer is very thin since hole concentration is an exponential function of the surface potential.

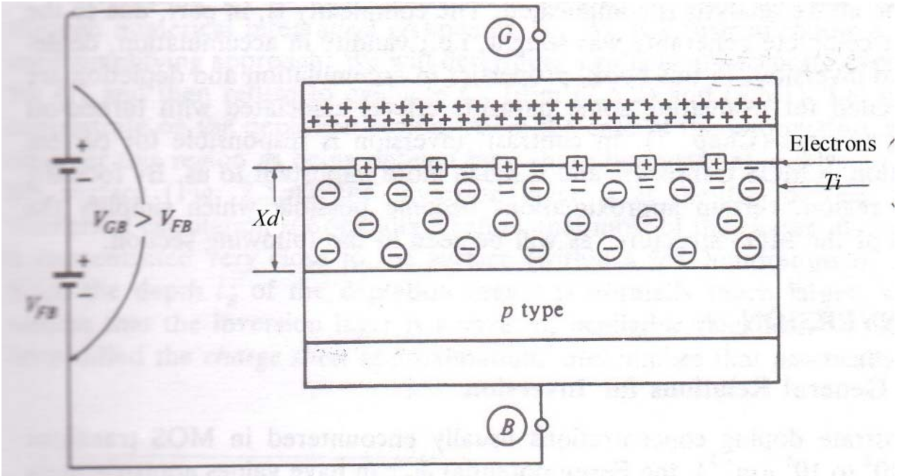
Depletion:



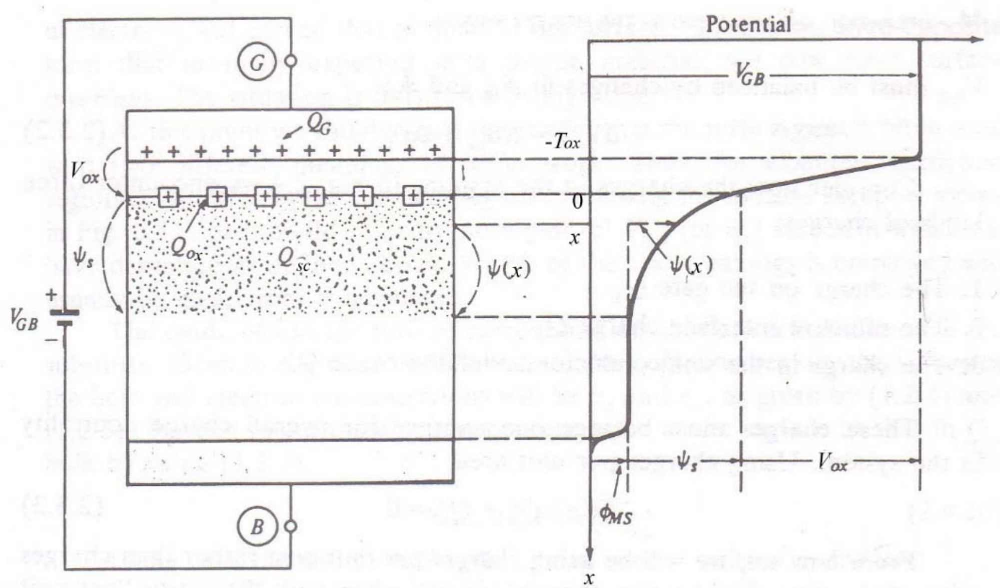
A positive bias is applied on top of V_{FB} (so the total gate–bulk voltage is above flatband voltage but still below the "threshold voltage"). The positive charge on the gate repels the mobile holes towards the Si substrate, leaving behind negatively-charged ionized acceptors, forming a depletion layer ("depleted" of holes) with its thickness increasing as V_{GB} is increased for balancing the gate charge.

MOS: Strong Inversion and Potential Distribution

<u>Strong inversion:</u>



As the positive bias is further increased much larger than flatband voltage, the positive charge on the gate starts to attract electrons towards the Si surface (while expanding the depletion layer at the same time), forming an inversion layer ("inverted" charge of its original majority carrier). Beyond the onset of strong-inversion (called "threshold voltage"), electrons are plenty to screen the gate charge in a very thin layer, and depletion layer is reaching a maximum.



- The total gate–bulk voltage (V_{GB}) is the sum of the voltage drop across the oxide (V_{ox}), across the induced space-charge layer in Si (ψ_s), and the contact potential due to work function difference (ϕ_{MS}) — **potential balance** (or KVL: total voltage around a loop is zero).
- The total charge on the gate (Q_g) and inside the oxide (Q_{ox}) are always balanced by the induced charge in Si ($-Q_{sc}$), which consists of bulk/depletion charge ($-Q_b$) and inversion charge ($-Q_i$) — **charge balance** (or charge neutrality: total charge in MOS is zero).

MOS Charge and Surface-Potential Relations

Surface/bulk charge density

$\left(np = n_i^2\right)$

General carrier-potential relations (any x):

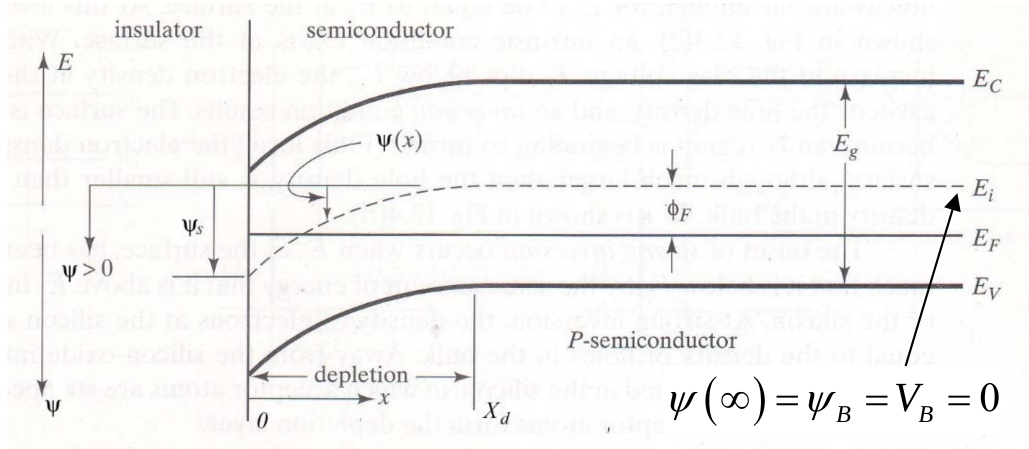
$$p = n_i e^{(E_i - E_F)/kT} = n_i e^{-(\psi - \phi_F)/v_{th}} \qquad n = n_i e^{(E_F - E_i)/kT} = n_i e^{(\psi - \phi_F)/v_{th}}$$

At surface (x = 0):

$$p_s = p(0) = n_i e^{-(\psi_s - \phi_F)/v_{th}} \qquad n_s = n(0) = n_i e^{(\psi_s - \phi_F)/v_{th}}$$

In neutral bulk ($x > X_d$):

$$p_0 = p\big|_{x \geq X_d} = n_i e^{\phi_F / v_{th}} \qquad n_0 = n\big|_{x \geq X_d} = n_i e^{-\phi_F / v_{th}}$$



$$\psi(\infty) = \psi_B = V_B = 0$$

Regions of operation in terms of surface potential

Surface potential	Terminal bias	Surface condition	Surface carrier density
$\psi_s < 0$	$V_{GB} < V_{FB}$	*Accumulation*	$p_s > p_0 = N_A$
$\psi_s = 0$	**$V_{GB} = V_{FB}$**	**Flatband**	$p_s = p_0 = N_A$
$0 < \psi_s < \phi_F$	$V_{FB} < V_{GB} < V_L$	*Depletion*	$n_s < p_s < p_0 = N_A$
$\psi_s = \phi_F$	$V_{GB} = V_L$	Intrinsic	$n_s = p_s = n_i$
$\phi_F < \psi_s < 2\phi_F$	$V_L < V_{GB} < V_T$	Weak inversion	$p_s < n_s < p_0 = N_A$
$\psi_s = 2\phi_F$	**$V_{GB} = V_T$**	**Threshold**	$n_s = p_0 = N_A$
$\psi_s > 2\phi_F$	$V_{GB} > V_T$	*Strong inversion*	$n_s > p_0 = N_A$

- In ***accumulation***, holes (p_s) dominate
- In ***depletion*** (including weak-to-moderate inversion), depletion charge (N_A) dominates
- In ***strong inversion***, electrons (n_s) dominate, in addition to depletion charge (N_A)

3. MOS

- Potential balance:

$$V_{GB} = \phi_{MS} + V_{ox} + \psi_s$$

- Charge balance:

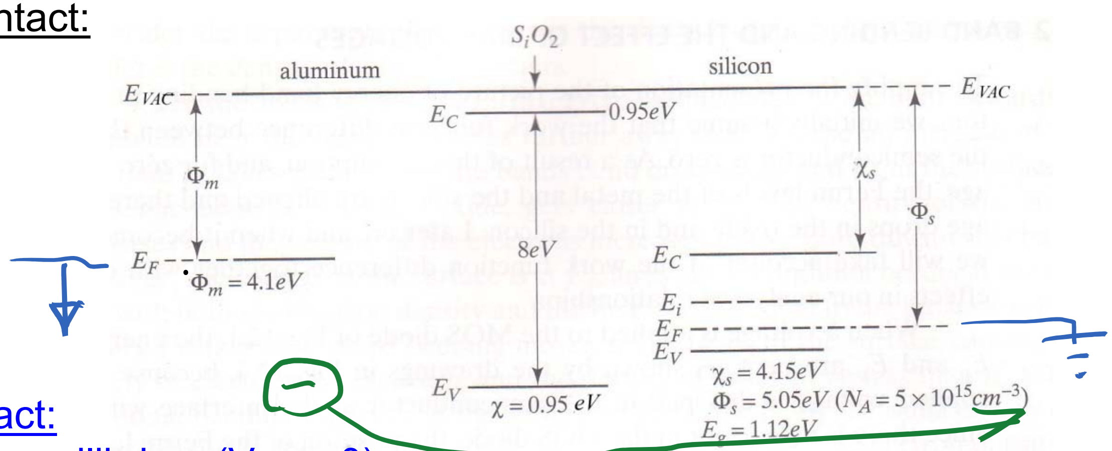
$$Q_g + Q_{ox} + Q_{sc} = 0$$

- Gauss Law:

$$Q_g = \epsilon_{ox} \mathcal{E}_{ox} = C_{ox} V_{ox} \quad \left(\mathcal{E}_{ox} = \frac{V_{ox}}{T_{ox}} \right)$$

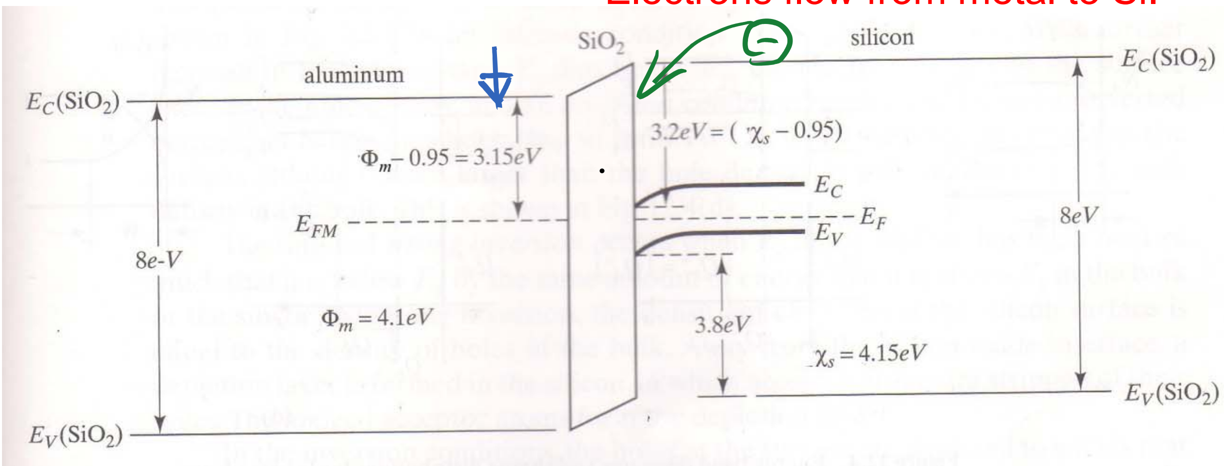
$$Q_{sc} = -\epsilon_{Si} \mathcal{E}_s \quad \left(C_{ox} = \frac{\epsilon_{ox}}{T_{ox}} \right)$$

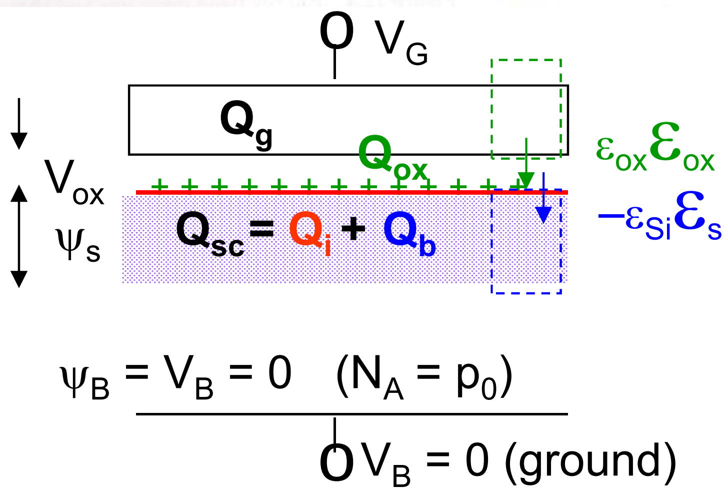
Before contact:



After contact:
at thermal equilibrium (V_{GB} = 0)

Electrons flow from metal to Si.





$$V_{GB} = \phi_{MS} + \frac{Q_g}{C_{ox}} + \psi_s \qquad Q_g = -(Q_{ox} + Q_{sc})$$

$$= \underbrace{\phi_{MS} - \frac{Q_{ox}}{C_{ox}}}_{V_{FB}} - \frac{Q_{sc}}{C_{ox}} + \psi_s$$

$$\therefore\ V_{GF} \equiv V_{GB} - V_{FB} = -\frac{Q_{sc}}{C_{ox}} + \psi_s$$

$$V_T \equiv V_{GB}\Big|_{\psi_s = 2\phi_F}$$

$$= V_{FB} - \frac{Q_b(\psi_s = 2\phi_F)}{C_{ox}} + 2\phi_F$$

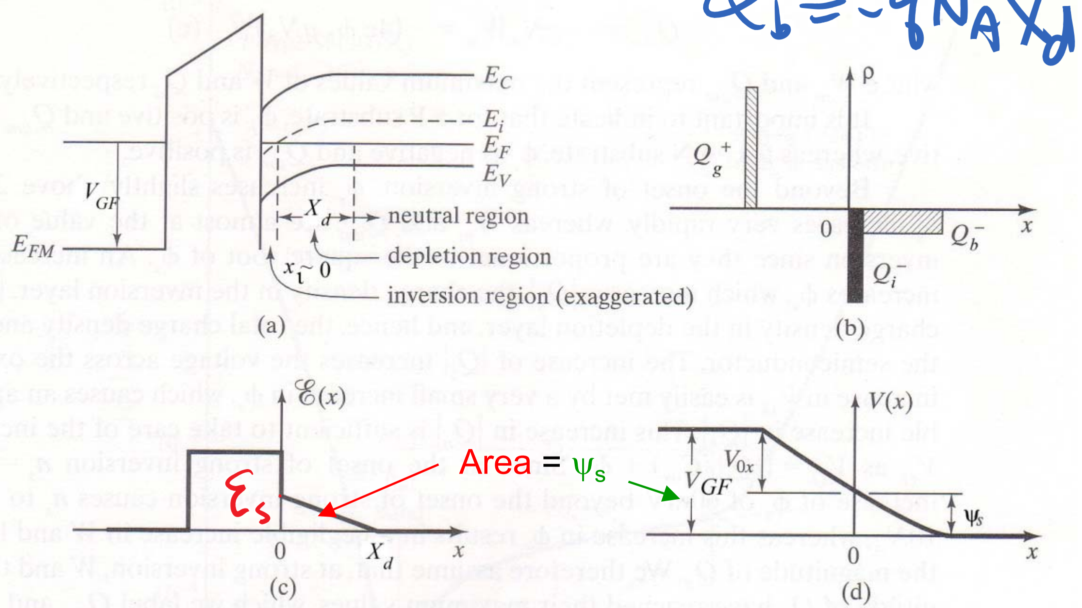
$$= V_{FB} + \underline{\gamma}\sqrt{2\phi_F} + 2\phi_F$$

(body factor: $\gamma = \sqrt{2q\epsilon_{Si}N_A}/C_{ox}$)

<u>CSA</u>: $Q_{sc} = Q_b + Q_i$

<u>Full depletion</u>: $Q_i \approx 0$

$Q_b = -qN_A X_d$

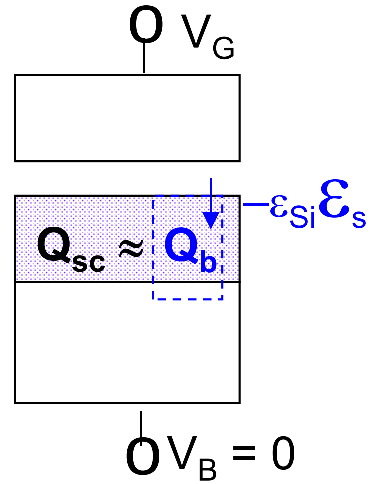


$$\psi_s = \frac{1}{2} X_d \cdot \mathcal{E}_s$$

$$= \frac{1}{2} X_d \frac{qN_A X_d}{\epsilon_{Si}}$$

$$\Downarrow$$

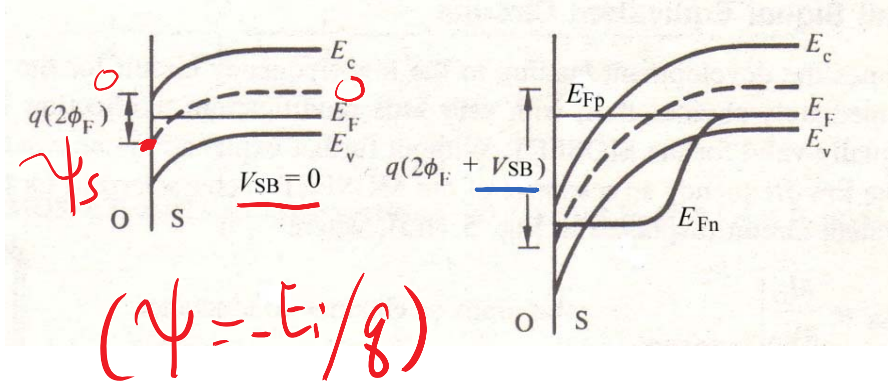
$$\boxed{X_d = \sqrt{2\varepsilon_{Si}\psi_s / qN_A}}$$

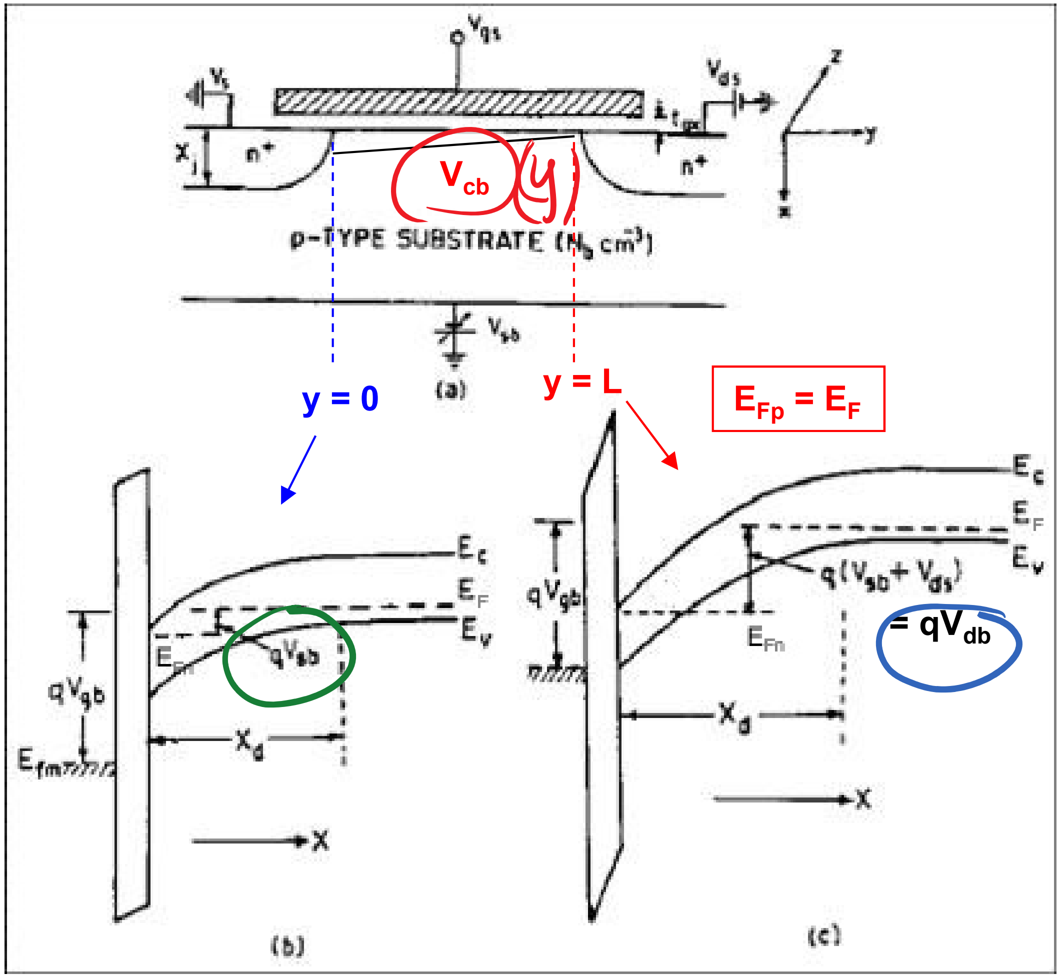


$$-\epsilon_{Si}\mathcal{E}_s = Q_b = -qN_A X_d$$

4. MOSFET

Quasi-Fermi level difference ("imref-split") = external bias





$V_{cb}(y)$ ⟨ $V_{sb}\,(y=0)$; $V_{db}\,(y=L)$

$\quad V_{db} = V_{sb} + V_{ds}$

$V_{ds} \neq 0 \Rightarrow I_{ds} \neq 0$ (nonequilibrium)

$$V_T \equiv V_{GS}\Big|_{\psi_s = 2\phi_F + V_{SB}} = V_{FB} + \gamma\sqrt{2\phi_F + V_{SB}} + 2\phi_F$$

$(Q_i \ll Q_b)$

Strong Inversion: $\psi_s = \psi_s(0) + V_{CB}(y) = 2\phi_F + V_{SB} + V(y)$

$(0 < V < V_{ds})$

$$V_{GB} - V_{FB} = -\frac{Q_i + Q_b}{C_{ox}} + \psi_s$$

$$\Rightarrow Q_i = -C_{ox}(V_{GB} - V_{FB} - \psi_s) - Q_b$$

$Q_b \approx -\gamma C_{ox}\sqrt{2\phi_F + V_{SB}}$

Linear (drift) current: $J_{n,drift} = q n v = \mu Q_i\left(-\frac{d\psi}{dy}\right)$

$$I_{DS} = \mu_n\left(\frac{W}{L}\right)\int_0^{V_{ds}}(-Q_i)\,dV = \mu_n C_{ox}\frac{W}{L}\left(V_{GS} - V_T - \frac{1}{2}V_{DS}\right)V_{DS}$$

Saturation current: ("pinch-off": $V_{DS} \geq V_{Dsat} = V_{GS} - V_T$)

$$I_{DS} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_T)^2$$

(or: $V_{GD} \leq V_T$)

Summary of Ideal (Long-Channel) MOSFET Equations

DC current

(For PMOS, all taking absolute values.)

$$I_{DS} = \begin{cases} 0 & V_{GS} \le V_T & \text{Cut-off} \\ K_n\left(V_{GS} - V_T - \tfrac{1}{2}V_{DS}\right)V_{DS} & V_{GS} > V_T ; V_{DS} \le V_{Dsat} & \text{Linear} \\ \tfrac{1}{2}K_n\left(V_{GS} - V_T\right)^2 & V_{GS} > V_T ; V_{DS} > V_{Dsat} & \text{Saturation} \end{cases}$$

Transconductance

$$g_m \equiv \frac{dI_{DS}}{dV_{GS}}$$

$$g_m = \begin{cases} 0 \\ K_n V_{DS} \\ K_n\left(V_{GS} - V_T\right) \end{cases}$$

Gain factor

$$K_n = \kappa\left(W/L\right) = \mu_n C_{ox}\left(W/L\right)$$

$\kappa = \mu_n C_{ox}$ Process-dependent transconductance parameter

Saturation voltage

$$V_{Dsat} = V_{GS} - V_T$$

Depletion width / charge

$$X_d = \sqrt{\frac{2\varepsilon_{Si}\psi_s}{qN_A}} \qquad Q_b = -qN_A X_d = -\Upsilon C_{ox}\sqrt{\psi_s}$$

Threshold voltage

$$V_T = V_{FB} + \Upsilon\sqrt{2\phi_F + V_{SB}} + 2\phi_F = V_{FB} - Q_{bm}/C_{ox} + 2\phi_F$$

Strong-inversion:

$$\psi_s = 2\phi_F + V_{SB} \qquad X_d = X_{dm} \qquad Q_b = Q_{bm}\left(X_{dm}\right)$$

Body factor

$$\Upsilon = \frac{\sqrt{2q\varepsilon_{Si}N_A}}{C_{ox}}$$

$$C_{ox} = \varepsilon_{ox}/T_{ox}$$

Flatband voltage

$$V_{FB} = \phi_{MS} - Q_{ox}/C_{ox}$$

$$Q_{ox} = Q_m + Q_f + Q_{ot}$$

$$\phi_{MS} = \Phi_M - \Phi_S = \Phi_M - \left(\chi + E_g/2q + \phi_F\right)$$

Bulk Fermi potential:

$$\phi_F = \frac{kT}{q}\ln\left(\frac{N_A}{n_i}\right)$$

EE4613: Summary of Short-Channel MOSFET Equations

- **Threshold voltage**
  - ***Long-channel (1D theoretical model)***

$$\boxed{V_t \equiv V_{gs}\Big|_{\psi_s=2\phi_F+V_{sb}} = V_{FB} + \Upsilon\sqrt{2\phi_F + V_{sb}} + 2\phi_F}$$

$$\phi_F = \frac{kT}{q}\ln\left(\frac{N_A}{n_i}\right) \qquad \Upsilon = \sqrt{2q\varepsilon_{Si}N_A}\Big/C_{ox} \qquad C_{ox} = \varepsilon_{ox}/T_{ox}$$

$$V_{FB} \equiv \phi_{MS} - Q_{ox}/C_{ox} = \Phi_M - \left(\chi + E_g/2 + \phi_F\right) - Q_{ox}/C_{ox}$$

  - ***Short-channel (triangle charge-sharing model)***

$$\boxed{V_{t0}\left(L_g\right) \equiv V_{t0\_long} - \Delta V_{t0} = V_{t0\_long} - \frac{4\varepsilon_{Si}\phi_F}{\varepsilon_{ox}}\frac{T_{ox}}{L_g - 2\sigma X_j}}$$

  - ***Short-channel DIBL***

$$\boxed{\Delta V_{DIBL}\left(L_g\right) \equiv V_{t0}\left(L_g\right) - V_{ts}\left(L_g\right)}$$

- **Drain current**
  - ***Linear***

$$\boxed{I_{ds} = \mu_0 C_{ox}\frac{W}{L}\left(V_{gs} - V_t - \frac{1}{2}A_b V_{ds}\right)V_{ds}}$$

  - ***Subthreshold***

$$\boxed{I_{ds} = \mu_0 C_d v_{th}^2 \frac{W}{L} e^{\left(V_{gs}-V_t\right)/\left(n v_{th}\right)}\left(1 - e^{-V_{ds}/v_{th}}\right)}$$

$$n = 1 + C_d/C_{ox}$$

$$C_d = \varepsilon_{Si}/X_{dm} = \frac{\Upsilon C_{ox}}{2\sqrt{2\phi_F + V_{sb}}}$$

  - ***Saturation***

$$\boxed{I_{dsat} = W v_{sat} C_{ox}\frac{\left(V_{gs} - V_t\right)^2}{V_{gs} - V_t + A_b E_{sat} L_{eff}}} \Rightarrow \propto \begin{cases} \left(V_{gs} - V_t\right)^2 & \left(L_{eff} \to \infty;\ \text{long-channel: quadratic}\right) \\ \left(V_{gs} - V_t\right) & \left(L_{eff} \to 0;\ \text{short-channel: linear}\right) \end{cases}$$