Accelerating Numerical Electromagnetic Code Computation by Using the Wavelet Transform

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Abstract— **The Numerical Electromagnetic Code (NEC) is one of the most popular tools for electromagnetic simulation of wireframe structures. The application of NEC is often limited to small to medium sized problems due to its full matrix nature. In this paper, an approach by using the wavelet transform to increase the efficiency and capability of NEC is presented. In the approach, a sparse moment matrix equation can be produced and solved by efficient sparse solver instead of solving full matrix equation in the original NEC. Under close examination, structures with less singularities are found to have much better accuracy and higher compression rates.**

Index terms— **Numerical analysis, wavelet transforms, wire antennas, wire scatters.**

I. Introduction

The Numerical Electromagnetic Code (NEC) [1] is probably still the most popular tool for modelling and analysis of electromagnetic (EM) response of complex wireframe metallic structures. As NEC uses the method of moment (MoM) and full matrix equation solver so that NEC is extremely hungry for memory and computational time for large problems. It is still formidable for NEC to treat problems with unknowns more than 3000 on most high-end personal computers, workstations, and low-end supercomputers. In engineering application, many problems are large and complicated with unknowns much more than this. To solve large problems by NEC has always been a challenging task for computational electromagnetics researchers.

In recent years, there has been growing interest in the application of wavelet analysis to EM problems. The wavelet transform method (WTM) [2] has been developed using the translating and dilating of a suitable basis function, known as the mother wavelet. This mother wavelet then undergoes the decomposition and reconstruction algorithms producing the wavelet transform matrix.

In this study, we tried to apply the WTM to NEC so as to transform the full impedance matrix into a highly sparse matrix which can be solved by an efficient sparse solver.

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The modified version of NEC (NEC-WTM) is tested by a number of examples. The compression rate (sparsity) and accuracy are compared with the results obtained from the original version 2 of NEC (NEC-2) [1].

II. The Wavelet Matrix Transformation

In the NEC, like many other MoM based codes, the most time consuming part in the computation is to solve the moment matrix equation

$$[Z]\cdot[I]=[V], \tag{1}$$

where $[Z]$ is a full moment matrix, $[I]$ is a current distribution related unknown column vector to be solved, and $[V]$ is the known source related column vector. In NEC, the LU decomposition method which forms the bulk of the computational time required to solve the full matrix equation is $O(N^3)$ for large matrix of order N. For large problem, due to the memory required for the storage of all the elements in the full matrix, the out-of-core operation is performed where the $[Z]$ matrix has to be stored in 4 sequential access files each of the size of the full matrix. Thus making this an extremely storage and computational time demanding process.

In order to increase the efficiency and capability for larger problems, the recently developed wavelet transform method (WTM) [2] is applied into NEC. The wavelet matrix transformation produces a sparse moment matrix similar to that obtained by using basis expansion in MoM. By using the sparse wavelet transform matrix $[\tilde{W}]$, the wavelet matrix transformation can be carried out on (1) as follows:

$$[\tilde{W}][Z][\tilde{W}]^T\cdot([\tilde{W}]^T)^{-1}[I]=[\tilde{W}][V], \tag{2}$$

where $[\tilde{W}]^T$ is the transpose of the wavelet transform matrix $[\tilde{W}]$. The detailed procedure for the formulation of the wavelet transform matrix can be found in [2]. After this transformation, we now obtain a new matrix equation.

$$[Z']\cdot[I']=[V'], \tag{3}$$

where, $[Z']=[\tilde{W}][Z][\tilde{W}]^T$, $[I']=([\tilde{W}]^T)^{-1}[I]$, and $[V']=[\tilde{W}][V]$. Next, after choosing a threshold value τ, we can discard the elements in the matrix $[Z']$ whose magnitudes are smaller than $\tau\cdot m$, where m is the largest magnitude of the matrix elements. The threshold value τ need to be

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well chosen so as to balance the computational efficiency and accuracy of the approximate solutions.

After this process, a sparse matrix can be obtained and solved much more efficiently by a sparse solver. Solving a sparse matrix requires $O(NlogN)$ operations, where N is the number of unknowns. This is much more efficient as compared to $O(N^3)$ for a full matrix solution in the original version of NEC. Once the $[I']$ is solved, matrix $[I]$ can then be reconstructed by

$$[I] = [\tilde{W}]^T \cdot [\tilde{W}][I] = [\tilde{W}]^T[I']. \quad (4)$$

This process would therefore be an efficient method of solution which considerably reduces the computational time and storage requirements.

III. Implementation and Experiment

We have implemented the wavelet transform method into the NEC code and have used the NEC-WTM code to test various examples and to study its effectiveness. The results obtained by the NEC-WTM code are compared with the solutions by the original version of NEC code.

The first example is the structure of a monopole with four ground simulation wires mounted on a high metallic pole. The geometry of the considered problem is shown in Fig. 1(a). The structure shows a quarter-wavelength monopole mounted on a 40 wavelength high mounting pole. The frequency used in the simulation is 1 GHz and the excitation is a 1 V voltage source.

The sparsity pattern of the $[Z']$ matrix with a threshold value $\tau = 10^{-9}$ is shown in Fig. 1(b). The black dots shows the remaining nonzero elements. In this particular case, only 93264 elements are left out of the total 1048576 elements. This is 8.89% of the original full 1024 by 1024 matrix.

Fig. 2 compares the current distribution and gain pattern solutions from the original NEC and the NEC-WTM with a threshold of $\tau = 10^{-9}$, respectively.

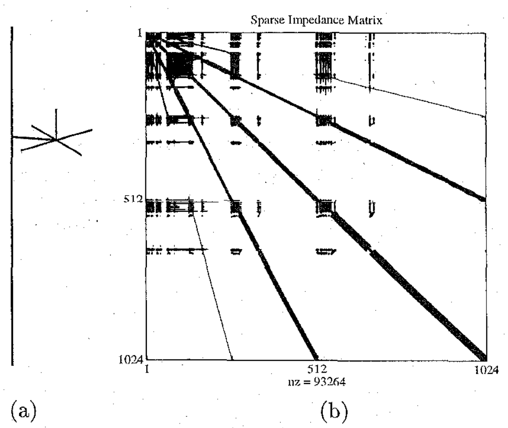

Fig. 1. (a) The geometrical structure of the test example; (b) the sparsity pattern of the matrix $[Z']$ after the wavelet transform and a threshold value of $\tau = 10^{-9}$.

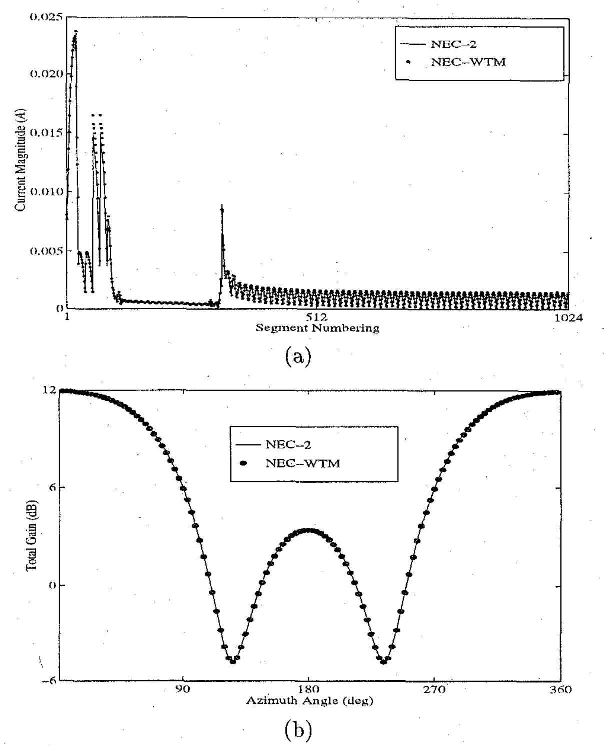

Fig. 2. Comparison of the sparse matrix solution using the NEC-WTM ($\tau = 10^{-9}$, compression rate 8.89%) and the full matrix solution by the original NEC. (a) Current distribution; (b) total gain pattern.

From the comparison, it can be seen that the approximate solutions by the NEC-WTM are very close to those by the original NEC, showing a high accuracy even when the compression rate is high. In this case, only 8.89% of the elements in the $[Z']$ are left. With this high compression rate, computational time would be drastically reduced, showing that the matrix equation can be solved more efficiently while maintaining good accuracy.

It should also be noted that, the larger the size of EM problem, the more effective the wavelet matrix transform method. However, there are some limitations to this approach. It has been observed that this method is sensitive to the number of singularity in the problem. The more the number of singularity, the less effective the method. Therefore, this application is more suitable for large and smooth problems.

After examining the accuracy of the new NEC-WTM, we next proceed to another numerical example designed mainly to examine the relation between the compression rate and the accuracy of both the current distribution and radiation pattern.

This example involves an elliptical scatterer. The geometry of the considered structure is shown in Fig. 3(a). The figure shows an elliptical scatterer excited by an incident plane wave coming down from the z-direction in the x-y plane. The frequency used in the simulation is 3 GHz.

The sparsity pattern of the $[Z']$ matrix with a threshold value $\tau = 10^{-6}$ is shown in Fig. 3(b). For this example,

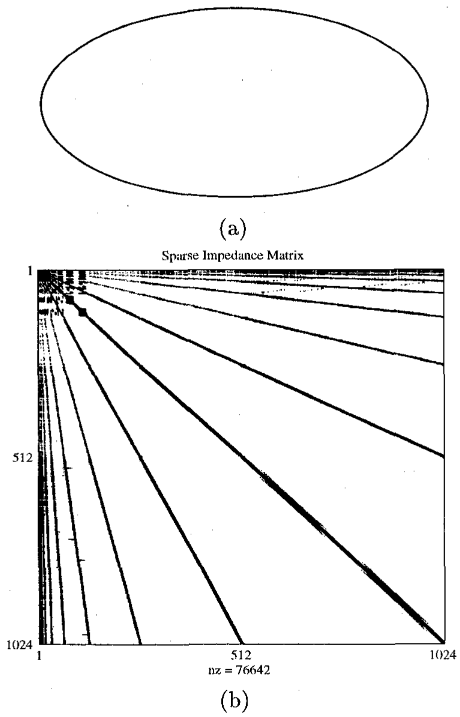

Fig. 3. (a) The geometry of the test example 2; (b) the sparsity pattern of the matrix $[Z']$ after the wavelet transform and a threshold value of $\tau = 10^{-6}$.

only 76642 elements are left out of the total 1048576 elements. This is 7.31% of the original full 1024 by 1024 matrix.

A comparison was done for three different threshold values ($\tau = 10^{-5}$, $\tau = 10^{-6}$, $\tau = 10^{-7}$). The compression rates of these cases are 3.37%, 7.31% and 12.45%, respectively.

Fig. 4 compares the full matrix solutions with the sparse matrix solutions for different compression rates. With a compression rate of 3.37%, the current intensity deviates from the true value by quite a bit. As the compression rate decreases to 7.31%, it was noted that the result tends to be closer to the true value with many points oscillating around the actual solution. At a compression rate of 12.45%, there is minimal error between the actual value and that obtained from the sparse matrix. This again justifies that the NEC-WTM can produce results of high accuracy with a high compression rate.

Fig. 5 and Fig. 6 show the normalized scattering power pattern in both the x-z plane and y-z plane, respectively. A review of Fig. 4(a), Fig. 5(a) and Fig. 6(a) shows that, for a compression rate of 3.37%, although the results obtained for the current intensity might not be very accurate, the scattering power pattern obtained still remains at quite an acceptable range from the actual solution. From Figs. 5 and 6, it can be seen that with a compression rate of 7.31%, the accuracy of the scattering power pattern is sufficiently accurate. This shows that for those who are only interested in the radiation pattern, a much better compression rate can be used and yet results of very high accuracy can be maintained. The use of a Z matrix of higher compression rate will then translate into less computation time and memory space required.

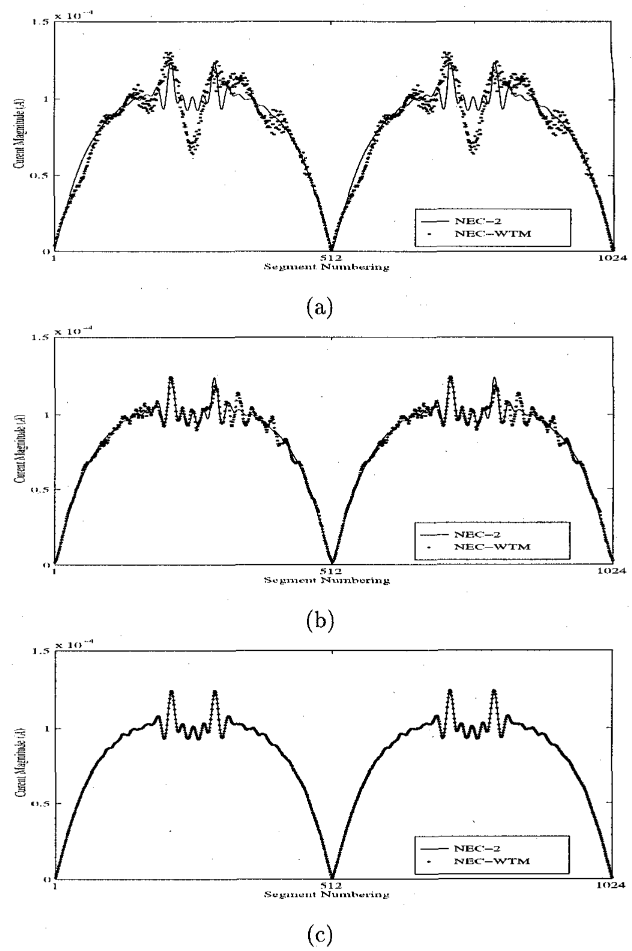

Fig. 4. Comparison of current intensity plots with compression rates of (a) 3.37%, (b) 7.31%, and (c) 12.45% with the full matrix, respectively.

Table I shows a comparison between the computational time required to solve the sparse matrix using a sparse solver as compared to the original LU factorization method. This was done on the matrix shown in this example. As seen from the table, when 3.37% of the elements remain, the time required to solve this 1024 by 1024 matrix is approximately 29 times faster then that of the original time required. This shows a considerable reduction in CPU time required for the NEC-WTM. It was also observed that, the higher the compression rate, the more efficient is the sparse solver.

TABLE I
SPARSITY AND CPU TIME

Sparsity	CPU Time (sec)	No. Times Faster
100%	374.7	1.00
12.5%	73.62	5.09
7.31%	68.78	5.45
3.37%	12.98	28.86

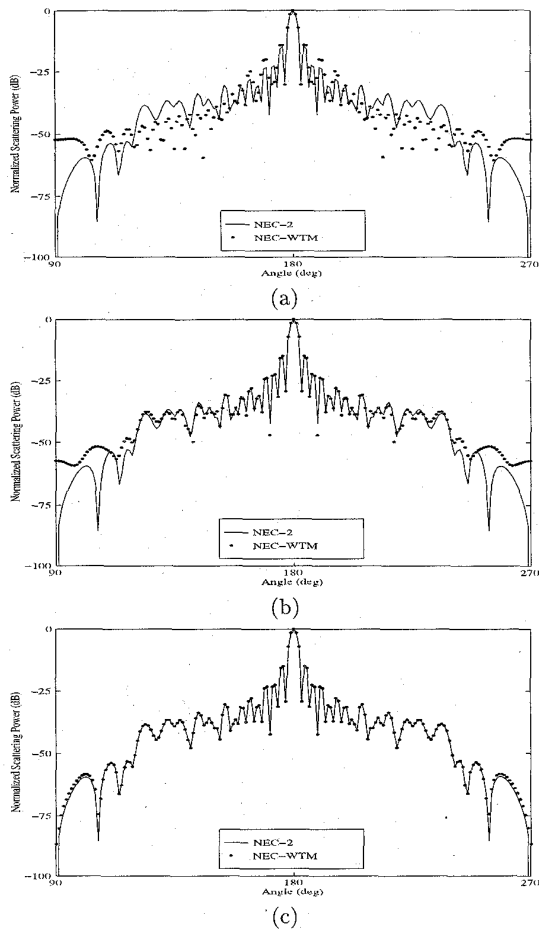

Fig. 5. Comparison of scattering power patterns with compression rates of (a) 3.37%, (b) 7.31%, and (c) 12.45% with full matrix for x-z plane, respectively.

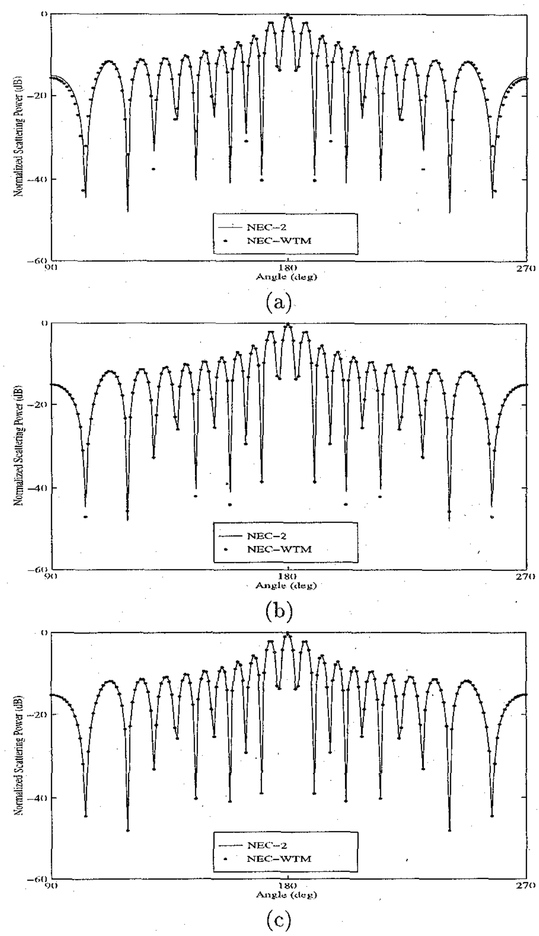

Fig. 6. Comparison of scattering power patterns with compression rates of (a) 3.37%, (b) 7.31%, and (c) 12.45% with ull matrix for y-z plane, respectively.

In this second example, the effect of threshold value on the accuracy of the results has been examined. The results of both the current intensity and radiation pattern were observed for various threshold. From this, it can be concluded that, depending on the required results, a suitable threshold should be chosen so as to optimize both the computational time and the memory required. The large improvement in computational time required to solve the sparse matrix of different threshold has also been examined.

IV. Conclusions

In this paper, acceleration of NEC computation by using the wavelet transform method is proposed and studied. By using the wavelet matrix transform method, instead of solving a full matrix equation in the original NEC a sparse matrix can be obtained and solved efficiently, by sparse solvers.

It is shown that with the NEC-WTM approach, one can obtain high accuracy approximate solutions with a very sparse matrix equation. The effectiveness and accuracy of the method are shown by numerical examples. Some of the limitations to this method was also examined.

It has also been shown that, if only the radiation pattern are of interest, a higher compression rate can be used. This would further reduce the computational time and memory required.

Acknowledgment

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