SUPPLEMENTARY MATERIALS FOR FACT: FEATURE ADAPTIVE CONTINUAL-LEARNING TRACKER FOR MULTIPLE OBJECT TRACKING

A PREPRINT

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1 Proof of Theorem 3.1

According to Eq. (7), we could derive the feature autocorrelation unit for frame k as:

$$\begin{aligned}\mathbf{R}^{(k)} &= \left(\gamma\mathbf{I} + \sum_{i=0}^{k}\mathbf{X}_i^{(\mathrm{et})T}\mathbf{X}_i^{(\mathrm{et})}\right)^{-1} \\ &= \left(\left(\mathbf{R}^{(k-1)}\right)^{-1} + \mathbf{X}_k^{(\mathrm{et})T}\mathbf{X}_k^{(\mathrm{et})}\right)^{-1},\end{aligned} \tag{a}$$

According to the Woodbury matrix identity, for any invertible square matrices A and C, we have

$$(\boldsymbol{A}+\boldsymbol{U}\boldsymbol{V}\boldsymbol{C})^{-1} = \boldsymbol{A}^{-1} - \boldsymbol{A}^{-1}\boldsymbol{U}\left(\boldsymbol{C}^{-1}+\boldsymbol{V}\boldsymbol{A}^{-1}\boldsymbol{U}\right)^{-1}\boldsymbol{V}\boldsymbol{A}^{-1}. \tag{b}$$

Let $\boldsymbol{A} = \left(\mathbf{R}^{(k-1)}\right)^{-1}$, $\boldsymbol{U} = \mathbf{X}_k^{(\mathrm{et})T}$, $\boldsymbol{V} = \mathbf{X}_k^{(\mathrm{et})}$, and $\boldsymbol{C} = \mathbf{I}$. According to Eq. (a), (b), we have

$$\mathbf{R}^{(k)} = \mathbf{R}^{(k-1)} - \mathbf{R}^{(k-1)}\mathbf{X}_k^{(\mathrm{et})T}\left(\mathbf{I}+\mathbf{X}_k^{(\mathrm{et})}\mathbf{R}^{(k-1)}\mathbf{X}_k^{(\mathrm{et})T}\right)^{-1}\mathbf{X}_k^{(\mathrm{et})}\mathbf{R}^{(k-1)}. \tag{c}$$

To this, we complete the proof of feature autocorrelation unit (Eq. (11)) using $\mathbf{R}^{(k-1)}$ and $\mathbf{X}_k^{(et)}$ in a recursive manner. According to Eq. (4), (5), (6), (7), we have

$$\begin{aligned}\hat{\mathbf{W}}_{\mathrm{FCN}}^{(k)} &= \left(\gamma\mathbf{I} + \sum_{i=0}^{k}\mathbf{X}_i^{(\mathrm{et})T}\mathbf{X}_i^{(\mathrm{et})}\right)^{-1}\left[\mathbf{Q}^{(k-1)}+\Delta\mathbf{Q}^{(k)} \quad \boldsymbol{X}_k^{(\mathrm{et})T}\hat{\boldsymbol{Y}}_k^{(k)}\right] \\ &= \boldsymbol{R}^{(k)}\left[\mathbf{Q}^{(k-1)}+\Delta\mathbf{Q}^{(k)} \quad \boldsymbol{X}_k^{(\mathrm{et})T}\hat{\boldsymbol{Y}}_k^{(k)}\right] \\ &= \left[\boldsymbol{R}^{(k)}\mathbf{Q}^{(k-1)}+\boldsymbol{R}^{(k)}\Delta\mathbf{Q}^{(k)} \quad \boldsymbol{R}^{k}\boldsymbol{X}_k^{(\mathrm{et})T}\hat{\boldsymbol{Y}}_k^{(k)}\right],\end{aligned} \tag{d}$$

Since $\boldsymbol{R}^{(k)}\Delta\mathbf{Q}^{(k)}$ and $\boldsymbol{R}^{(k)}\boldsymbol{X}_k^{(\mathrm{et})T}\hat{\boldsymbol{Y}}_k^{(k)}$ are from the latest frame data which we have already known, we intend to derive the $\boldsymbol{R}^{(k)}\mathbf{Q}^{(k-1)}$ from the $\hat{\mathbf{W}}_{\mathrm{FCN}}^{(k-1)}$. We have

$$\begin{aligned}\boldsymbol{R}^{(k)}\mathbf{Q}^{(k-1)} &= \boldsymbol{R}^{(k-1)}\mathbf{Q}^{(k-1)} - \boldsymbol{R}^{(k-1)}\boldsymbol{X}_k^{(\mathrm{et})T}\left(\boldsymbol{I}+\boldsymbol{X}_k^{(\mathrm{et})}\boldsymbol{R}^{(k-1)}\boldsymbol{X}_k^{(\mathrm{et})T}\right)^{-1}\boldsymbol{X}_k^{(\mathrm{et})}\boldsymbol{R}^{(k)}\mathbf{Q}^{(k-1)} \\ &= \hat{\mathbf{W}}_{\mathrm{FCN}}^{(k-1)} - \boldsymbol{R}^{(k-1)}\boldsymbol{X}_k^{(\mathrm{et})T}\left(\boldsymbol{I}+\boldsymbol{X}_k^{(\mathrm{et})}\boldsymbol{R}^{(k-1)}\boldsymbol{X}_k^{(\mathrm{et})T}\right)^{-1}\boldsymbol{X}_k^{(\mathrm{et})}\hat{\mathbf{W}}_{\mathrm{FCN}}^{(k-1)}\end{aligned} \tag{e}$$

Let $\boldsymbol{Z} = \boldsymbol{R}^{(k-1)}\boldsymbol{X}_k^{(\mathrm{et})T}\left(\boldsymbol{I} + \boldsymbol{X}_k^{(\mathrm{et})}\boldsymbol{R}^{(k-1)}\boldsymbol{X}_k^{(\mathrm{et})T}\right)^{-1}$, $\boldsymbol{S} = \left(\boldsymbol{I} + \boldsymbol{X}_k^{(\mathrm{et})}\boldsymbol{R}^{(k-1)}\boldsymbol{X}_k^{(\mathrm{et})T}\right)^{-1}$. Since,

$$\begin{aligned}\boldsymbol{I} &= \boldsymbol{S}\boldsymbol{S}^{-1} \\ &= \boldsymbol{S}\left(\boldsymbol{I} + \boldsymbol{X}_k^{(\mathrm{et})}\boldsymbol{R}^{(k-1)}\boldsymbol{X}_k^{(\mathrm{et})T}\right)\end{aligned}$$

We have $\boldsymbol{S} = \boldsymbol{I} - \boldsymbol{S}\boldsymbol{X}_k^{(\mathrm{et})}\boldsymbol{R}^{(k-1)}\boldsymbol{X}_k^{(\mathrm{et})T}$. Thus

$$\begin{aligned}\boldsymbol{Z} &= \boldsymbol{R}^{(k-1)}\boldsymbol{X}_k^{(\mathrm{et})T}\left(\boldsymbol{I} + \boldsymbol{X}_k^{(\mathrm{et})}\boldsymbol{R}^{(k-1)}\boldsymbol{X}_k^{(\mathrm{et})T}\right)^{-1} \\ &= \boldsymbol{R}^{(k-1)}\boldsymbol{X}_k^{(\mathrm{et})T}\boldsymbol{S} \\ &= \boldsymbol{R}^{(k-1)}\boldsymbol{X}_k^{(\mathrm{et})T}\left(\boldsymbol{I} - \boldsymbol{S}\boldsymbol{X}_k^{(\mathrm{et})}\boldsymbol{R}^{(k-1)}\boldsymbol{X}_k^{(\mathrm{et})T}\right) \\ &= \left(\mathbf{R}^{(k-1)} - \mathbf{R}^{(k-1)}\mathbf{X}_k^{(\mathrm{et})T}\boldsymbol{S}\mathbf{X}_k^{(\mathrm{et})}\mathbf{R}^{(k-1)}\right)\mathbf{X}_k^{(\mathrm{et})T} \\ &= \boldsymbol{R}^{(k)}\mathbf{X}_k^{(\mathrm{et})T} \end{aligned} \tag{f}$$

It allows the Eq. (e) to be simplified to

$$\begin{aligned}\boldsymbol{R}^{(k)}\mathbf{Q}^{(k-1)} &= \hat{\mathbf{W}}_{\text{FCN}}^{(k-1)} - \boldsymbol{Z}\boldsymbol{X}_k^{(\mathrm{et})}\hat{\mathbf{W}}_{\text{FCN}}^{(k-1)} \\ &= \hat{\mathbf{W}}_{\text{FCN}}^{(k-1)} - \boldsymbol{R}^{(k)}\boldsymbol{X}_k^{(\mathrm{et})T}\boldsymbol{X}_k^{(\mathrm{et})}\hat{\mathbf{W}}_{\text{FCN}}^{(k-1)}. \end{aligned} \tag{g}$$

By putting Eq. (g), (9) into Eq. (d), we get

$$\begin{aligned}\hat{\mathbf{W}}_{FCN}^{(k)} &= \left[\boldsymbol{R}^{(k)}\mathbf{Q}^{(k-1)} + \boldsymbol{R}^{(k)}\Delta\mathbf{Q}^{(k)} \quad \boldsymbol{R}^{(k)}\boldsymbol{X}_k^{(\mathrm{et})T}\hat{\boldsymbol{Y}}_k^{(k)}\right] \\ &= \left[\hat{\mathbf{W}}_{\text{FCN}}^{(k)} - \boldsymbol{R}^{(k)}\boldsymbol{X}_k^{(\mathrm{et})T}\boldsymbol{X}_k^{(\mathrm{et})}\hat{\mathbf{W}}_{\text{FCN}}^{(k)} + \boldsymbol{R}^{(k)}\Delta\mathbf{Q}^{(k)} \quad \boldsymbol{R}^{(k)}\boldsymbol{X}_k^{(\mathrm{et})T}\hat{\boldsymbol{Y}}_k^{(k)}\right] \\ &= \left[\boldsymbol{V}_k\hat{\mathbf{W}}_{\text{FCN}}^{(\text{k-1})} + \mathbf{R}^{(\text{k})}\triangle\mathbf{Q}^{(\text{k})} \quad \mathbf{R}^{(\text{k})}\boldsymbol{X}_k^{(\mathrm{et})\mathrm{T}}\hat{\boldsymbol{Y}}_k^{(\text{k})}\right]. \end{aligned} \tag{h}$$

To this, we complete the proof for FCN layer weight (Eq. (8)) at frame k.