TABLE 2
MATLAB Code *(continued)*

```
dtau=ceil(T*m)*dt/N;
tau=round([0:1:N]*dtau/dt)*dt;
f=[0:1:K]*df;
f=[-fliplr(f) f];
mat1=spdiags(u',0,m+ceil(T*m),m);
u_padded=[zeros(1,ceil(T*m)),u,zeros(1,ceil(T*m))];
cidx=[1:m+ceil(T*m)];
ridx=round(tau/dt)';
index = cidx(ones(N+1,1),:) + ridx(:,ones(1,m+ceil(T*m)));
mat2 = sparse(u_padded(index));
uu_pos=mat2*mat1;
clear mat2 mat1
e=exp(-j*2*pi*f'*t);
a_pos=abs(e*uu_pos');
a_pos=a_pos/max(max(a_pos));
a=[flipud(conj(a_pos(1:K+1,:))) fliplr(a_pos(K+2:2*K+2,:))];
delay=[-fliplr(tau) tau];
freq=f(K+2:2*K+2)*ceil(max(t));
delay=[delay(1:N) delay(N+2:2*N)];
a=a(:,[1:N,N+2:2*N]);
[amf amt]=size(a);
cm=zeros(64,3);
cm(:,3)=ones(64,1);

figure(2), clf, hold off % Ambiguity function
mesh(delay, [0 freq], [zeros(1,amt);a])
hold on
surface(delay, [0 0], [zeros(1,amt);a(1,:)])
colormap(cm)
view(-40,50)
axis([-inf inf -inf inf 0 1])
xlabel(' {\it\tau}/{\itt_b}','Fontsize',12);
ylabel(' {\it\nu}*{\itMt_b}','Fontsize',12);
zlabel(' |{\it\chi}({\it\tau},{\it\nu})| ','Fontsize',12);
hold off
```

## Doppler Compensation For Binary Phase-Coded Waveforms

**Codes used in phase-coded pulse compression waveforms suffer from Doppler mismatch. Based on an analysis of the effects of Doppler mismatch on binary phase-coded waveforms, a class of codes is proposed (hereafter referred to as the component codes), its derivation and processing is described, and the performance results are analyzed. This technique solves the problem of the Doppler mismatch of binary phase-coded waveforms at the expense of about 3 dB loss in signal-to-noise ratio (SNR). This technique may be used in tracking applications, such as accurately and rapidly measuring and tracking targets of unknown speeds. Simulation results demonstrate the validity of this technique.**

Manuscript received January 25, 2001; revised November 30, 2001 and April 13, 2002; released for publication May 25, 2002.

IEEE Log No. T-AES/38/3/06451.

Refereeing of this contribution was handled by E. S. Chornoboy.



### I. INTRODUCTION

Because of limitations in radar transmitter peak power, there is a conflict between long-range detection and range resolution [1, 2]. Pulse compression signals are an effective solution to the power/bandwidth problem, and among the phase-coded signals, binary phase-coded signals are of considerable interest due to their simple realization.

Matched filtering and correlation processing have been used to realize low-loss pulse compression, but Doppler causes mismatch in processing [1–4]. Doppler mismatch causes losses and time spreading of the peak response of the matched filtering and makes the sidelobe levels larger, thus affecting range resolution and decreasing the probability of detection. Doppler mismatch also affects the performance [5–8] of the subsequent sidelobe suppression filters used to reduce the time sidelobe and decreases the PSL (peak sidelobe level) [1, 3, 8]. Hence, it is important to eliminate or reduce Doppler mismatch as much as possible.

In phase-coded waveform applications, there are many ways for Doppler compensation, such as using multiple Doppler channels or employing a bank of digital filters [1–4]. The conventional methods compensate Doppler mismatch "point to point," as illustrated in Fig. 1 for one Doppler channel or one

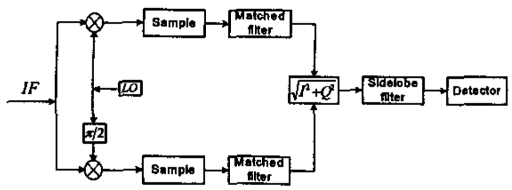


Fig. 1. Quadrature model of matched filter.

digital filter. If the Doppler frequency is constant and known *a priori*, Doppler mismatch is exact and there is no loss in signal-to-noise ratio (SNR). However, in many radar applications, echo Doppler frequency is usually known to be within a range of frequencies. When the Doppler frequency lies midway between adjacent Doppler channels or adjacent digital filters, loss in SNR could reach 3 dB [9]. Employing more Doppler channels or more digital filters could partially solve the above SNR loss problem but could result in higher detection false alarm probability [9], and heavier computational load.

We propose a new method for compensating Doppler mismatch without using multiple Doppler channels or prior knowledge of the Doppler frequency. The paper is an extension of [11, 12] in which a class of codes named component codes and its associated processing were proposed. Using a component code, the modulus of the matched filtering response is independent of the Doppler frequency, hence effectively solving the Doppler mismatch problem. Possible applications of the new method include accurately measuring and tracking of high-velocity objects with unknown speeds, such as satellites and missiles. Although the idea of processing the component codes is the same as the differential phase shift keying (DPSK) technique widely used in communications [10], the objectives are not the same. While DPSK is used in communications for processing any given binary signal, the component codes discussed here are those that produce the matched-filter response for target detection.

This paper is organized as follows. Analysis of the Doppler mismatch and its compensation are presented in Section II. Performance analysis and simulations are presented in Section III. Discussion and conclusion are given in Section IV.

## II. DOPPLER MISMATCH AND ITS COMPENSATION

Fig. 1 shows a quadrature detection model of a matched filter [1]. Let the transmitted RF signal for a binary phase-code $\{b_n\}$ be

$$S_0(t) = \sum_{n=0}^{N} b_n \mathrm{Re}ct[(t-n\tau)/\tau]\cdot\exp(j2\pi f_0 t) \qquad (1)$$

where

$$\mathrm{Re}ct[t] = \begin{cases} 1 & 0 < t \le 1 \\ 0 & \text{other} \end{cases},$$

$f_0$ is the carrier frequency and $\tau$ is the duration of each subpulse.

The return (assuming IF at zero) is then

$$S(t) = \sum_{n=0}^{N} g b_n \mathrm{Re}ct[(t-n\tau)/\tau]\cdot\exp(j2\pi f_d t + j\theta) \qquad (2)$$

where $f_d$ is the Doppler frequency, $\theta$ is the initial phase of the return, and $g$ is the amplitude of the signal.

Let $h_1(t)$ represent the matched filter for the binary phase-coded signal $\sum_{n=0}^{N} b_n \mathrm{Re}ct[(t-n\tau)/\tau]$. Then the matched-filter response for the return is given by

$$R(t) = S(t)\otimes h_1(t) \qquad (3)$$

where $\otimes$ denotes convolution. Simple derivation shows that the modulus of $R(t)$ is modulated by the Doppler frequency $f_d$ for a general binary phase-code $\{b_n\}$ [5, 11, 12]. In the following, it is shown that the magnitude of $R(t)$ is independent of $f_d$ if $\{b_n\}$ is a component code [11, 12].

Given a desired binary phase-code $\{a_n\}$ ($n = 1,\ldots,N$), its component code $\{b_n\}$ ($n = 0,\ldots,N$) is defined as that code such that $a_n = b_n \cdot b_{n-1}$. Note, for a given code $\{a_n\}$ the length of a component code is necessarily one bit longer and there exist but two associated component codes, the two being complementary to one another.

If the binary phase-code in (1) is a component code, let

$$X_\tau(t) = S(t)\cdot S^*(t-\tau) \qquad (4)$$

where ( )* denotes complex conjugate. Then

$$X_\tau(t) = \sum_{n=1}^{N} g^2 a_n \mathrm{Re}ct[(t-\tau-n\tau)/\tau]\exp(j2\pi f_d\tau). \qquad (5)$$

Let $h(t)$ represent the matched filter for the desirable signal $p(t) = \sum_{n=1}^{N} a_n \mathrm{Re}ct[(t-n\tau)/\tau]$ and let

$$A(t) = p(t)\otimes h(t). \qquad (6)$$

The response of matched filter $h(t)$ to $X_\tau(t)$ is then given by

$$\begin{aligned} R_\tau(t) &= X_\tau(t)\otimes h(t) \\ &= \left\{\sum_{n=1}^{N} g^2 a_n \mathrm{Re}ct[(t-\tau-n\tau)/\tau]\exp(j2\pi f_d\tau)\right\}\otimes h(t) \\ &= g^2\{p(t-\tau)\exp(j2\pi f_d\tau)\}\otimes h(t) \\ &= g^2 A(t-\tau)\cdot\exp(j2\pi f_d\tau). \end{aligned} \qquad (7)$$

Note now that the modulus of the matched-filter output is not modulated by the Doppler, and furthermore that it equals the modulus of the

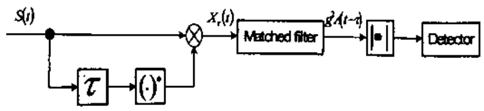


Fig. 2. Block diagram for processing component-coded waveform.

desired-signal autocorrelation function with delay $\tau$ and a factor of $g^2$. The delay $\tau$ must be considered in measuring the distance from a target to the radar, but note also that $\tau$ is known and fixed. The processing of component-coded waveform can be implemented as in Fig. 2.

## III. PERFORMANCE ANALYSIS AND SIMULATION

Compared with conventional Doppler compensation methods, the main difference of the proposed method is the introduction of the component-code processing prior to matched filtering (from $S(t)$ to $X_\tau(t)$ in Fig. 2). We now analyze the SNR loss incurred in this part making use of results presented in [13].

Suppose that the $n$th subpulse echo from a target is

$$z_n = gb_n \exp(j2\pi f_d n\tau) + u_n \tag{8}$$

where $u_n$ is a complex Gaussian process, whose real and imaginary parts are independent zero-mean Gaussian variables, each of whose variances is $\sigma^2$. For the purpose of analyzing SNR loss, we can ignore $b_n$ since $|b_n| = 1$. The corresponding $n$th subpulse of $X_\tau(t)$ is given by

$$X_n = z_n z_{n-1}^* \tag{9}$$

which is a special case of the signal analyzed in [13]

$$Z'_k = \sum_{m=k+1}^{M} z_m z_{m-k}^* \tag{10}$$

when $k = 1$ and $M = 2$. Applying the results of [13] (see [13, equation (6)–(14)]) and simplifying, it is easy to show that the SNR in $X_n$ is

$$\mathrm{SNR}_{X_n} = \frac{g^4}{4g^2\sigma^2 + 4\sigma^4} = \frac{\mathrm{SNR}_{z_n}}{2 + \dfrac{1}{\mathrm{SNR}_{z_n}}} \tag{11}$$

where $\mathrm{SNR}_{z_n} = g^2/2\sigma^2$ is the SNR of echo before the component-code processing. Fig. 3 plots the SNR loss as a function of $\mathrm{SNR}_{z_n}$. The SNR loss is at least 3 dB as seen for high $\mathrm{SNR}_{z_n}$ and becomes progressively worse as $\mathrm{SNR}_{z_n}$ decreases.

Despite the SNR loss incurred, the component-code processing has the advantage of making the PSL of the matched-filter output independent of Doppler frequency, as shown in the following simulation example.

The computer simulation is based on the maximal-length sequence $\{a_n\}$ of length 127 bit

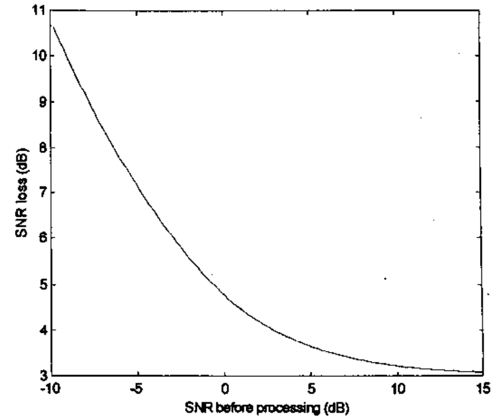


Fig. 3. SNR loss in component-code processing.

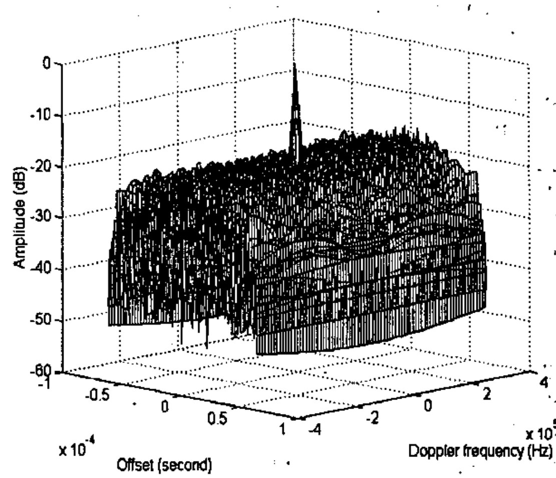


Fig. 4. Ambiguity function for direct matched filter.

produced by a 7-stage shift register. The feedback function of the shift register is $F = X_1 \oplus X_3 \oplus X_7$ with initial state (1,0,1,1,1,0,0) [1, 14]. The duration of each subpulse is set as $\tau = 0.5\ \mu$s. The ambiguity function for the direct matched filtering and the method proposed here are shown in Figs. 4 and 5, respectively. From the ambiguity functions, we can note that with preprocessing the PSL of matched-filter output is independent of Doppler frequency. The average PSL is plotted as a function of Doppler frequency in Figs. 6–8 for input SNRs of 5 dB, 0 dB, and $-8.5$ dB, respectively, where average is taken with respect to 1000 Monte Carlo runs. The solid curve corresponds to direct matched filtering while the dotted curve shows the proposed method. As can be seen from Figs. 6–8, without Doppler compensation, the PSL increases drastically when the Doppler frequency increases. Using the proposed method, Doppler frequency obviously has no effect on the PSL. However, note that at low SNR the PSL using the proposed method increases significantly due to the SNR loss incurred by component-code processing.

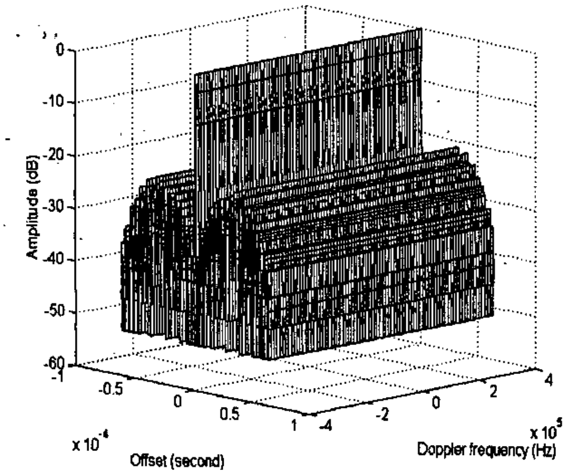


Fig. 5. Ambiguity function for proposed method.

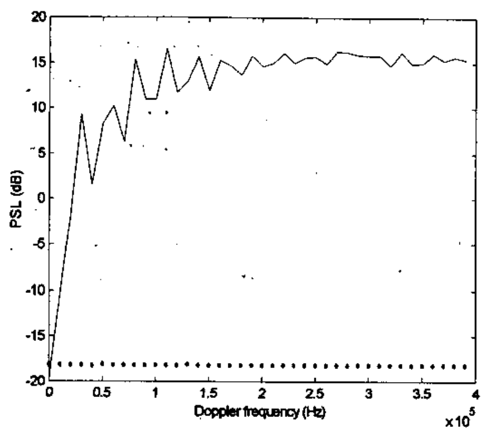


Fig. 6. PSL versus Doppler frequency for SNR = 5 dB.

## IV. DISCUSSION AND CONCLUSION

In this paper we have presented a component-code processing method for Doppler compensation with binary phase-coded waveforms. The main advantage is that Doppler compensation can be achieved using a single matched-filter channel instead of multiple Doppler channels or bank of digital filters. However, the proposed method has several limitations, which are to be discussed next along with some potential solutions.

First, as analyzed in Section III, the SNR loss incurred by processing the component code is more severe when the original SNR (before component-code processing) is low. Hence, we suggest that for practical radar systems the proposed method is used only as a supplement to existing Doppler compensation methods, being implemented only when the original SNR is relatively high.

Second, the proposed method does not provide Doppler resolution, which could be obtained using existing Doppler estimation methods in a radar system.

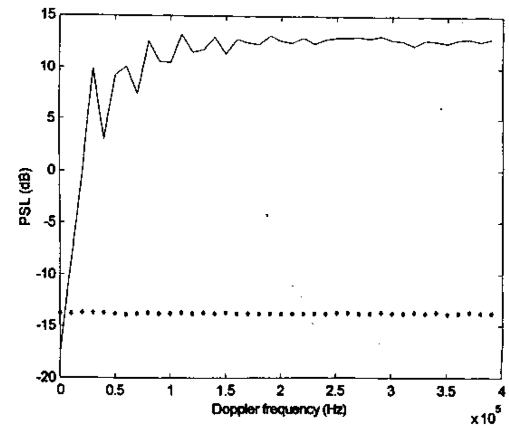


Fig. 7. PSL versus Doppler frequency for SNR = 0 dB.

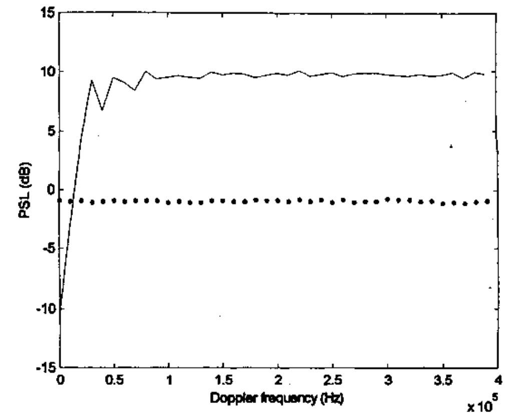


Fig. 8. PSL versus Doppler frequency for SNR = −8.5 dB.

Third, in order to obtain the desired matched-filter output, one has to generate a component code for the desired code and transmits a signal modulated by this component code. This may introduce additional cost for an existing radar system. However, if a given radar system adopts a maximal-length sequence, then the new method can be implemented easily without additional cost due to a special delay and add property of maximal-length sequences [14]. Specially, if $b_n$ $(n = 1,\ldots,N)$ is a maximal-length sequence, $a_n = b_n b_{n-1}$ $(n = 1,\ldots,N)$ is also a maximal-length sequence that is a shifted version of $\{b_n\}$ [14]. Note that in the above, we choose $b_0 = 1$ for computing $a_n$. Hence, for the special but important case of maximal-length sequences, instead of calculating a component code from a desired code, we start with a maximal-length sequence $\{b_n\}$, and the resultant matched-filter output will correspond to $\{a_n\}$ that is just a shifted version of $\{b_n\}$.

Taking into account the above advantage and limitations of the proposed method, we feel that the new method can be used as an effective supplement to existing Doppler compensation methods for radar systems, particularly for those with maximal-length sequences and long binary phase-coded pulses.

ACKNOWLEDGMENT

The authors wish to thank Prof. G. R. Krupholz for helpful comments and pointing out [13].

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## Characterizing Performance of $\alpha$–$\beta$–$\gamma$ Filters

**The $\alpha$–$\beta$–$\gamma$ filter, a sampled data target tracker which can asymptotically track a constant acceleration target, is discussed in detail. The $\alpha$, $\beta$, $\gamma$ parameters are studied to characterize the stability of the filter and its performance viz its transient behavior. A closed-form equation for the mean square response of the system to white noise is derived. In addition, performance measures to gauge the transient response and the steady state tracking error are derived. The closed-form equations for the noise ratio and the transient and steady state error are exploited to optimally select the $\alpha$, $\beta$, $\gamma$ parameters. The resulting solutions are shown to reduce to some results presented in the literature for the $\alpha$–$\beta$ filter.**

## I. INTRODUCTION

Numerous applications such as air-traffic control, missile interception, and antisubmarine warfare require the use of discrete-time data to predict the kinematics of a dynamic object. The use of passive sonobuoys which have limited power capacity constrain us to implement target-trackers which are computationally inexpensive. It is with these considerations in mind, we analyze an $\alpha$–$\beta$–$\gamma$ filter to study its ability to predict the object kinematics in the presence of noisy discrete-time data.

There exists a significant body of literature which addresses the problem of *track-while-scan systems*. Sklansky [1] in his seminal paper analyzed the behavior of an $\alpha$–$\beta$ filter. His analysis of the range of the $\alpha$–$\beta$ smoothing parameters is based on the prediction characteristics, which resulted in a stable filter constraining the parameters to lie within

Manuscript received August 4, 2000; revised September 14, 2001, March 18 and May 9, 2002; released for publication May 9, 2002.

IEEE Log No. T-AES/38/3/06452.

Refereeing of this contribution was handled by P. K. Willett.