Holographic Sensing (on Images)

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Introduction

Hologram to Data Representation

Holographic Sensing on Images

Simulation and Software
Thank you for the invitation to share some nice mathematics with you today.

- I did my undergraduate studies (2000–05: Mathematics and Philosophy) at Ateneo de Manila (AdMU).
- Then M.Sc. Math. (2005–07) also at AdMU with 3 graduate courses completed at UP Diliman.
- I love the beautiful campuses of both universities;
- And continue to cherish the friendship developed there.

Webpage: you can find all papers and software codes at https://personal.ntu.edu.sg/fredezerman/
Current Research Projects

Main fund: NTU Research Grant No. 04INS000047C230GRT01.

- **Holographic Data Representation.** Also funded by Singapore National Research Foundation (NRF) and Israel Research Foundation (ISF).

- **Cryptographic Sequences:** with Zuling Chang (Zhengzhou Univ.), Adamas Fahreza (NTU), Janusz Szmidt (Military Commun. Div., Poland) and Steven Wang (Carleton Univ., Canada). Also funded by China NRF.

- **Algebraic Coding Theory:** e.g. quantum error-control codes, MDS matrices for cryptography, spectral analysis. Mostly with Ling San and Buket Özkaya (NTU) and Markus Grassl (ICTQT-Gdansk, Poland).
Just-for-fun Projects

- Code-based cryptography.
- Lattice-based cryptography: with Indah Emilia, Uha Isnaeni, and their students (UGM, Yogyakarta).
- Complexity of Modified de Bruijn Sequences: PhD thesis supervision of Mustofa (UGM).
- Codes over Modules and Rings: mostly with Patrick Solé (Univ. Paris 8, France), Nasro Didine (USTH, Algeria), Taher Abualrub (Sharjah, UAE).
- Deep Learning on Images: with Risman Adnan and his team at Samsung Research Indonesia.
- Metacirculant Graphs and Quantum Codes: with Pani Seneviratne (Texas A&M Univ.-Commerce).
Collaborators in *Project Holography*

- **Ling San**: Prof. of Math., Provost and Vice President (NTU).
- **Adamas**: B.Sc. Math 2016 NTU; lead programmer.
- **Freddy**: SIAM Fellow (since 2014), Chair Prof. (Technion), Visiting Prof. (NTU).
A Hologram

In physics, a *hologram* is produced by optical sensing process.

![Diagram of a hologram process]

- **Scene:**
- **Output:**

Typically capturing the reflected sensing waves

*A recording of the SCENE.*
A Hologram

What’s amazing about a hologram?

1. A hologram is visual. It can be seen.
2. Reduced dimension: it is typically a 2-dimensional plate, yet produces a representation of the scene, which is 3-dimensional.
3. Shining rays of light on this 2D plate makes the 3D scene appear magically.
Most Magical Properties

The **same entire 3D representation** remains even if we break the holographic plate and shine lights only on a **randomly chosen part**.

- Most likely with less fidelity; e.g. blurry; "weaker", etc.
- Quality of recovery depends only on the ratio of the parts available.
Emulating Hologram for Stochastic Data

Freddy Bruckstein’s lecture with background and motivation: https://www.youtube.com/watch?v=iZQZj5b9Cf8.

1. Optical holography was invented by Denis Gabor circa 1947; Nobel Prize in 1971.
2. Successful applications in, e.g., arts, microscopy sensors, security, and optical data storage.
3. The core idea behind digital watermarking of copyrighted media items esp. images and videos.

What we do: revisit the idea of OPTICAL holographic sensing and representation to formulate a distributed DIGITAL data acquisition process.
Desirable Properties of Data Representation

1. **Fidelity** in recovery. Recovered value should approximate the actual value up to some threshold.
2. **Efficient** enough to be of practical value.
3. Meeting other performance **measures specific to application domain(s)**.
Other Models

• **Successive Refinement** of information from rate-distortion theory. First developed by Equitz and Cover in 1991.

• **Compressed (or Compressive) Sensing**. Pioneered by Donoho, Romberg, and Tao around 2006. Impressive tools, leading to astonishing progress.

• **Multiple Description Coding**, proposed initially by Vivek Goyal in 2001.
Objectives of Holographic Sensing

There are two main objectives in sensing holographically.

1. The sensing process generates and then distributes *multiple descriptions* of information in packets that enable *progressive recovery*.

2. The packets are designed to have as equal importance as possible to guarantee *smoothness* in the quality of the recovered information, *independent* of the order of the packet’s arrival.
Distributed Acquisition of Representation Packets

The main task is to design projection operators to ensure that the objectives are met.
General Framework

General theoretical framework for stochastic data.


Holographic sensing

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ABSTRACT

Holographic representations of data encode information in packets of equal importance that enable progressive recovery. The quality of recovered data improves as more and more packets become available. This progressive recovery of the information is independent of the order in which packets become available. Such representations are ideally suited for distributed storage and for the transmission of data packets over networks with unpredictable delays and or erasures.

Several methods for holographic representations of signals and images have been proposed over the years and multiple description information theory also deals with such representations. Surprisingly, however, these methods had not been considered in the classical framework of optimal least-squares estimation theory, until very recently. We develop a least-squares approach to the design of holographic representation for stochastic data vectors, relying on the framework widely used in modeling signals and images.

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Patch-Based Sensing on Images
Implementation on natural images as bit-allocation problems.

Abstract. Holographic representations of data enable distributed storage with progressive refinement when the stored packets of data are made available in any arbitrary order. In this paper, we propose and test patch-based transform coding holographic sensing of image data. Our proposal is optimized for progressive recovery under random order of retrieval of the stored data. The coding of the image patches relies on the design of distributed projections ensuring best image recovery, in terms of the $\ell_2$ norm, at each retrieval stage. The performance depends only on the number of data packets that have been retrieved thus far. Several possible options to enhance the quality of the recovery while changing the size and number of data packets are discussed and tested. This leads us to examine several interesting bit-allocation and rate-distortion trade-offs, highlighted for a set of natural images with ensemble estimated statistical properties.

Key words. holographic representation, mean squared error estimation, stochastic image data, Wiener filter

AMS subject classifications. 60G35, 68U10, 94A08, 94A12

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A Simplified Comparison on a Sketch

In holographic sensing the order of availability of the packets is irrelevant and recovery must be uniform.
Tools for Holographic Sensing

Tools needed are **undergraduate** mathematics materials.

- **Linear Algebra**: eigenvalues, eigenvectors, eigendecomposition.
- **Basic Statistics**: autocovariance matrix, Singular Value Decomposition (SVD).
- **Multivariate Calculus**: Optimization by Lagrange Multipliers.
- **Algorithms** for bit allocations e.g. Waterfilling.
Examples of Target Image

(a) dragon

(b) flood

(c) merlion

(d) oldlady

Figure: Four images from the data set.
Outlines of Main Steps

1. Compute the autocovariance matrix, where $\Omega$ is an index set.

$$R_{xx} = \frac{1}{|\Omega|} \sum_{\omega \in \Omega} x_{\omega} x_{\omega}^\top.$$  \hspace{1cm} (1)

2. Use singular value decomposition (SVD):

$$R_{xx} \mapsto \Psi \Lambda \Psi^\top.$$ \hspace{1cm} (2)

3. Determine the sensing operators that minimize the **Mean Squared Error (MSE)**.

4. To measure “smoothness in recovery,” compute the expected

$$E(\text{MSE}, \ell) := \frac{1}{\binom{N}{\ell}} \sum_{K=1}^{\binom{N}{\ell}} \left( \frac{1}{|\Omega|} \sum_{\omega=1}^{|\Omega|} (x_{\omega} - \hat{x}_{\omega,K})^2 \right)$$ \hspace{1cm} (3)

with $$(x_{\omega} - \hat{x}_{\omega,K})^2 = (x_{\omega} - \hat{x}_{\omega,K})^\top (x_{\omega} - \hat{x}_{\omega,K}).$$
A Typical Lambda Matrix

Lambda plots for various images when $M = 64$

Figure: The $\Lambda$ profiles when $M = 64$. The vertical axis is labeled in logarithmic scale.
Outline of Technical Details

1. The model and recovery procedure in the most general situation is in Section 2.
   - How to get the Λ matrix from the images in square patches.
   - The channel is assumed to be noisy.
   - The recovered estimate of the data, per patch in Eq. (2.4).
   - How the MSE is defined in the setup.

2. The sensing design is in Section 3.
   - How to design the projection operators.
   - Allocating the packets of representation.
   - MSE Minimization by the Lagrangian in Eqs. (3.2) to (3.4).
   - Practical calibrations: discretization into 3 modes.

3. The rest are implementation and analysis on actual data set of images.
Nice Properties

1. Adjusting well to changes in parameters i.e., versatile in handling dynamic real-time adjustment.
2. Resilient to changes in channel noise.
3. Aggregate Lambda Profile in a very simple model is enough in most setups.
4. Progressive smoothness as can be seen from the statistical analysis.
An Example: Randomized Recovery

Figure: Recovered dragon; randomized procedure with $(M, m, N, \sigma_n^2) = (64, 8, 8, 0.64)$ on Mode 1.
An Example: Higher Noise Level

Figure: Incrementally recovered using the $\Lambda$ of aggregate with $(M, m, N, \sigma_n^2) = (64, 8, 8, 1.00)$. The three modes coincide.
An Example: A Control Image

(a) $\ell = 1$  
(b) $\ell = 4$  
(c) $\ell = 8$  
(d) the original

Figure: Incrementally recovered boy on Mode 2 with $(M, m, N, \sigma_n^2) = (64, 4, 16, 0.64)$. 
Progressive and Smooth

Figure: Incremental (1st row) and randomized (2nd row) recovery on Mode 1 with \((M, m, N, \sigma_n^2) = (64, 4, 8, 0.25)\).
PoC Software

One can try on the software suite (in python):
https://github.com/adamasstokhorst/holographic

1. Play around with the parameters.
2. Simulate and analyze the outcomes.
3. Change the database and target images.
4. You are invited to improve and expand.

MARAMING SALAMAT