A Structural Estimation for the Effects of Uncertainty on Capital Accumulation with Heterogeneous Firms*

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Abstract

This paper develops a structural framework to estimate the effects of uncertainty on investment behaviour and capital accumulation at the firm level. Our model allows uncertainty to affect capital accumulation through three possible channels that have been highlighted in the literature: the Hartman-Abel-Caballero effect; different forms of capital adjustment costs; and a risk premium component in the discount rate. We discuss identification of these three distinct effects, and allow for unobserved heterogeneity in both firm size and growth. Parameters are estimated using simulated method of moments, matching empirical data for UK manufacturing firms. The estimated model indicates that higher uncertainty reduces both firm size and capital intensity in the long run, primarily through the discount rate effect.

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1 Introduction

The central question that this paper aims to answer is what are the effects of uncertainty on a firm's investment behaviour and the resulting capital accumulation. The relationship between uncertainty and investment has interested economists for a long time. The literature has suggested different channels through which uncertainty could affect investment behaviour and capital accumulation.

One channel is through the curvature of the marginal revenue product of capital in the stochastic variable that characterises uncertainty. In the special case of perfect competition and constant returns to scale production technology, as first established in Hartman (1972) and Abel (1983, 1984, 1985), the marginal revenue product of capital is convex in the stochastic price, so that a mean-preserving spread in the price increases the expected desired capital stock due to the Jensen's inequality effect. This relationship is generalised in Caballero (1991) for the case of imperfect competition. In the literature, the effect of uncertainty through this channel is known as the Hartman-Abel-Caballero effect (HAC effect, hereafter).

A second channel emphasizes the option value of investment in the presence of irreversibility. If investment is irreversible and can be postponed, waiting for new information to arrive before committing resources becomes a valuable call option. Since investing extinguishes this option, and since the option value increases with uncertainty, irreversibility implies a negative effect of uncertainty on the incentives to invest. This insight is first formalized in Bertola (1988) and Pindyck (1988), and systematically investigated in Dixit and Pindyck (1994).

A third channel considers the possibility of a risk premium component in the firm's required rate of return, discount rate or cost of capital. Suppose the firm is owned by a representative consumer. In a consumption-CAPM framework, if the consumer is fully-diversified, as Craine (1989) emphasizes, only the component of firm-level uncertainty that is positively correlated with aggregate risk would lower investment. If the consumer is not fully-diversified, either because of incomplete markets, as analysed in Angeletos and Calvet (2006), or as the result of an optimal incentive scheme due to agency conflict, as modelled in Himmelberg, Hubbard and Love (2002), idiosyncratic risks would also affect the required rate of return, reducing investment at a higher level of uncertainty.

Given the importance of this research topic and the rich implications from different theoretical approaches, it is not surprising that much empirical work has been done aimed at signing the effects of uncertainty and sorting the relative importance of these various channels. For example, Leahy and Whited (1996) study the relationship between investment rates and uncertainty by performing various sample splits in order to test comparative static implications of the theories outlined above. The main findings of the paper, as they conclude, appear to be at variance with the HAC effect and the discount rate effect, leaving irreversibility as the most likely explanation of the uncertainty-investment relationship. The significant role of irreversibility has also been found in Bond, Bloom and Van Reenen (2007), both numerically for a model with a rich mix of adjustment costs, and also empirically for a panel of UK manufacturing firms. More recently, in a structural framework, Bloom (2007) finds the effect of irreversibility on investment dominates the response of investment to any moderate change in the discount rate after a large uncertainty shock. In short, compared with the importance of irreversibility, there has been little empirical evidence for the HAC effect and the discount rate effect.

Instead of the uncertainty-investment relationship, the focus of this paper is the effects of uncertainty on capital accumulation. This is motivated for three reasons. First, although the impact of uncertainty on investment dynamics has extremely important business cycle implications, in the long run it is the level of capital stock and capital intensity that determines economic growth and development. Second, the findings from existing empirical work reflect the difficulty of identifying the HAC effect if we only consider the relationship between investment dynamics and uncertainty, while in this paper we illustrate the possibility to identify the HAC effect by studying uncertainty-capital stock relationship. Third, the discount rate essentially determines the Jorgensonian user cost of capital, and the relative price of capital to other inputs. In the short run, investment rates vary with this user cost of capital, but are mainly constrained by the capital adjustment costs. Given that the observed capital stock data is aggregated over periods of both positive and zero investment, and given that the observed capital intensity depends on the relative price, the study of uncertainty-capital stock and uncertainty-capital intensity relationship provides the possibility to identify the discount rate effect.

In this paper, we specify an investment model under uncertainty, which features all three possible channels highlighted in the theoretical literature. Under the specification of this model, with an increase in the level of uncertainty, the discount rate effect could increase, decrease or leave unchanged both the expected capital stock and the expected capital intensity, depending on how uncertainty affects the risk premium; the HAC effect could increase, decrease or leave unchanged the expected capital stock, depending on the source of the uncertainty and the demand elasticity; all three forms of capital adjustment costs could affect both the expected capital stock and the expected capital intensity, depending on the form of the adjustment costs.

This implies that the necessary condition for identification of all three channels is to allow for some variation in the level of uncertainty. Our model allows for variation across firms in the level of uncertainty. In addition, we also allow for heterogeneity in the trend growth rate of the stochastic process in order to get robust estimates for adjustment costs, and allow for heterogeneity in the level of the stochastic process in order to control for other unobserved factors that may lead to permanent differences in firm size. Our specification also allows for the possibility of both permanent and transitory measurement errors in investment rates and sales in the firm-level data.

With this empirical strategy, estimating the effects through each channel separately is transformed into estimating a set of structural parameters of the model. Using a simulated minimum distance estimator, these parameters are then estimated by matching simulated model moments with empirical data moments from a panel of UK manufacturing firms in Datastream. Finally, counterfactual simulations are implemented to estimate the sign and sort the magnitude for each channel based on the estimated model parameters.

Our estimated investment model finds significant empirical evidence for both the HAC effect and the discount rate effect, together with a combination of both convex and non-convex capital adjustment costs. Counterfactual simulations suggest that a permanent lower level of uncertainty would increase both average capital stock levels and aggregate capital intensity. These outcomes are the net effect of a small, negative capital adjustment costs effect, a moderate, positive HAC effect and a large, negative risk-adjusted discount rate effect.

To the best of our knowledge, this is the first paper that studies and finds the empirical importance of the HAC effect and the discount rate effect in a structural framework; and also the first paper that explicitly allows for unobserved heterogeneity across firms in the investment literature using structural estimation.

The rest of the paper is organised as follows. Section 2 outlines the investment model under uncertainty that we estimate. Section 3 investigates how uncertainty would affect the expected capital stock and expected capital intensity through three possible channels, which provides the theoretical basis for our identification strategy discussed in Section 4. Section 5 reports the empirical results. Section 6 illustrates the counterfactual simulations. And Section 7 concludes.

2 An Investment Model under Uncertainty

This section sets up a standard model of investment for a firm operating under uncertainty. The functional forms are chosen following three principles: first, they are widely adopted in the literature; second, they are tractable enough to derive closed-form solution in special cases; and finally, the feasibility for identification.

2.1 Production and Demand

Assumption 1 ***Timing:*** *Time is discrete and horizon is infinite. By paying capital adjustment costs, new investment I_t contributes to productive capital $\widehat{K}_t$ immediately in period t, which depreciates at the end of each period.*[1] *The capital accumulation formula is therefore*

$$K_{t+1} = (1-\delta)(K_t + I_t) \equiv (1-\delta)\widehat{K}_t \tag{1}$$

where δ is the constant depreciation rate.

Assumption 2 ***Production:*** *The firm uses capital $\widehat{K}_t$ and a variable input L_t to produce output Q_t, according to a constant returns to scale Cobb-Douglas technology*

$$Q_t = A_t L_t^{1-\beta} \widehat{K}_t^{\beta} \tag{2}$$

where A_t represents the randomness in productivity and β corresponds to the coefficient on productive capital in the production function.

[1] Compared with alternative lagged timing assumption, such as $K_{t+1} = (1-\delta)K_t + I_t$, Assumption 1 does not affect the qualitative implication of our model, but allows for a closed-form solution to the investment problem in the frictionless case, which provides a convenient benchmark for studying the effets of captial adjustment costs.

Assumption 3 ***Demand:*** *The firm faces isoelastic, downward-sloping, stochastic demand schedules of the form*

$$Q_t = X_t P_t^{-\varepsilon} \tag{3}$$

where P_t is price and $-\varepsilon < -1$ is the demand elasticity with respect to price. X_t represents the randomness in demand and can be interpreted as changes in the quantity demanded for any given price.[2]

Definition 1 ***Operating Profit*** $\pi(X_t, A_t, \widehat{K}_t)$ *is the maximized short-run profit for given capital stock and factor price by choosing optimal variable inputs.*

Denote sales as $Y_t = P_t Q_t$. Suppose the price for variable input is a constant w.[3] Lemma 1 summarises the relationship between the operating profit, variable inputs and sales.

Lemma 1 *Properties from short-run profit maximization*

$$\pi_t = const0 \cdot X_t^{\gamma} \left(A_t^{\gamma}\right)^{\varepsilon-1} \widehat{K}_t^{1-\gamma} \tag{4}$$

$$L_t = \frac{\gamma\varepsilon - 1}{w} \cdot \pi_t \tag{5}$$

$$Y_t = \gamma\varepsilon \cdot \pi_t \tag{6}$$

where

$$0 < \frac{1}{\varepsilon} < \gamma = \frac{1}{1 + \beta(\varepsilon - 1)} < 1 \tag{7}$$

and

$$const0 = \left(\frac{\gamma\varepsilon - 1}{w}\right)^{\gamma\varepsilon - 1} (\gamma\varepsilon)^{-\gamma\varepsilon} \tag{8}$$

Proof: See Appendix 1.1.

[2] This is called "horizontal demand shocks" in Abel and Eberly (1999). Alternatively, if we model "vertical demand shocks", such as $P_t = X_t Q_t^{-1/\varepsilon}$ in Caballero (1991), the operating profit can be derived as $\pi(X_t, A_t, \widehat{K}_t) = const0 \cdot (X_t^{\gamma})^{\varepsilon} \left(A_t^{\gamma}\right)^{\varepsilon-1} \widehat{K}_t^{1-\gamma}$. As it will become clear in Section 3.2, this specification does not allow us to estimate the relative importance of the HAC effect. On the other hand, both horizontal and vertical demand shocks could be justified to model demand uncertainty faced by a monopoly (Klemperer and Meyer, 1986).

[3] As it will become clear in section 2.2 and 3.2, if w is also stochastic, $\pi(X_t, A_t, \widehat{K}_t) = const0' \cdot X_t^{\gamma} \left(A_t^{\gamma}\right)^{\varepsilon-1} \left(w_t^{\gamma}\right)^{(1-\beta)(1-\varepsilon)} \widehat{K}_t^{1-\gamma}$. Assuming w_t has the same structure as X_t and A_t in its law of motion, it can also be incorporated into P_t with $\sigma^2 = \sigma_x^2 + (\varepsilon - 1)^2 \left((1-\beta)^2 \sigma_w^2 + \sigma_a^2\right)$. This implies uncertainty in factor prices will also lead to a positive Hartman-Abel-Caballero effect and its magnitude depends on the share of variable inputs in the production function, consistent with the insight in Lee and Shin (2000). However, given we cannot identify σ_w^2 and σ_a^2 separately within this model and given they both lead to a positive Hartman-Abel-Caballero effect, we simplify this issue by assuming non-stochastic factor prices.

2.2 Stochastic Processes

The demand shift parameter X_t and the level of productivity A_t are the two possible sources of uncertainty in this model.

Assumption 4 ***Demand Stochastic:*** *The law of motion for X_t is*

$$\begin{aligned} x_t &= \log X_t \\ x_t &= c_x + \mu_x t + \zeta_t^x \\ \zeta_t^x &= \rho_x \zeta_{t-1}^a + e_t^x = \zeta_0^x + \sum_{s=0}^{t-1} \rho_x^s e_{t-s}^x \end{aligned} \tag{9}$$

where $0 < \rho_x < 1$ and $e_t^x \overset{i.i.d}{\sim} N(0, \sigma_x^2)$.

Assumption 5 ***Productivity Stochastic:*** *The law of motion for A_t is*

$$\begin{aligned} a_t &= \log A_t \\ a_t &= c_a + \mu_a t + \zeta_t^a \\ \zeta_t^a &= \rho_a \zeta_{t-1}^a + e_t^a = \zeta_0^a + \sum_{s=0}^{t-1} \rho_a^s e_{t-s}^a \end{aligned} \tag{10}$$

where $0 < \rho_a < 1$ and $e_t^a \overset{i.i.d}{\sim} N(0, \sigma_a^2)$.

(9) and (10) imply that demand shocks e_t^x and productivity shocks e_t^a have effects that are persistent but not permanent, decaying at the rate $0 < \rho_x < 1$ and $0 < \rho_a < 1$, and on average demand and productivity grow at the trend rates μ_x and μ_a, respectively.

Firms making decisions in period t know X_t and A_t, but are uncertain about future levels of demand and productivity, which depend on future realizations of the demand and productivity shocks. Hence the variance of these shocks, i.e. σ_x^2 and σ_a^2, measure the level of uncertainty from demand and productivity faced by the firm in our model.

Furthermore, as (4) indicates, it is $X_t^\gamma (A_t^\gamma)^{\varepsilon-1}$ that jointly determines the marginal revenue product of capital hence the investment decision.

Lemma 2 *By imposing $\rho_x = \rho_a = \rho$, and assuming that e_t^x and e_t^a are independent, X_t and A_t can be combined into one single random variable, i.e.*

$$Z_t = X_t (A_t)^{\varepsilon-1} \tag{11}$$

The law of motion for Z_t is given by

$$\begin{aligned} z_t &= \log Z_t \\ z_t &= c + \mu t + \zeta_t \qquad (12) \\ \zeta_t &= \rho \zeta_{t-1} + e_t = \zeta_0 + \sum_{s=0}^{t-1} \rho^s e_{t-s} \end{aligned}$$

where $0 < \rho < 1$ and $e_t \overset{i.i.d}{\sim} N\left(0, \sigma^2\right)$. In particular,

$$\begin{aligned} \zeta_0 &= \zeta_0^x + (\varepsilon - 1)\,\zeta_0^a \\ c &= c_x + (\varepsilon - 1)\,c_a \\ \mu &= \mu_x + (\varepsilon - 1)\,\mu_a \\ \sigma^2 &= \sigma_x^2 + (\varepsilon - 1)^2\,\sigma_a^2 \qquad (13) \end{aligned}$$

Proof: See Appendix 1.2.

With this reparameterization, the operating profit can be written as

$$\pi(Z_t, \widehat{K}_t) = const0 \cdot Z_t^{\gamma} \widehat{K}_t^{1-\gamma} \qquad (14)$$

where Z_t incorporates stochastic from both demand and productivity, which is called "profitability" in Cooper and Haltiwanger (2006), or "business condition" in Bloom (2007). If $\sigma_a^2 = 0$, it is equivalent to a model where all the uncertainty is from demand; if $\sigma_x^2 = 0$, it is equivalent to a model where all the uncertainty is from productivity. When uncertainty comes from both demand and productivity, σ^2 is a measure of the overall uncertainty faced by the firm.

Assumption 6 ***Constant Proportion of Demand Uncertainty:*** *Among the overall uncertainty, there is a constant proportion τ of uncertainty coming from demand, i.e.*

$$\sigma_x^2 = \tau \sigma^2 \qquad (15)$$

Since $\sigma^2 = \sigma_x^2 + (\varepsilon - 1)^2\,\sigma_a^2$, this assumption also implies a constant proportion of uncertainty from productivity, i.e. $\sigma_a^2 = \frac{(1-\tau)}{(\varepsilon-1)^2}\sigma^2$. Now $\tau = 1$ is equivalent to a model where all the uncertainty is from demand; and $\tau = 0$ is equivalent to a model where all the uncertainty is from productivity.

Remark 1 *The operating profit π_t and therefore the optimal variable inputs L_t and the sales Y_t are all linear homogenous in $(Z_t, \widehat{K}_t)$.*

2.3 Adjustment Cost Function

Besides the demand conditions and the level of productivity, the firm's investment behaviour also depends on capital adjustment costs. The investment literature of the last four decades has focused on three forms of cost in capital adjustment.

2.3.1 Quadratic Adjustment Costs

Quadratic adjustment costs reflect those costs that increase convexly in the level of investment or disinvestment. We consider a specification that includes three features. First, the costs are quadratic in investment rate, to reflect higher costs due to more rapid changes and to allow for analytical tractability. Second, the costs attain their minimum value of zero at zero investment, so that the firm can avoid these costs by setting investment equal to zero. Third, the level of these costs is proportional to capital stock, so that a given investment rate imposes costs that increase with the size of the firm, and do not become irrelevant as the firm grows larger.

Assumption 7 ***Quadratic Adjustment Costs:*** *The functional form of quadratic adjustment costs is*

$$G(K; I_t) = \frac{b_q}{2}\left(\frac{I_t}{K_t}\right)^2 K_t$$

where b_q measures the magnitude of quadratic adjustment costs.

2.3.2 Partial Irreversibility

Partial irreversibility allows a gap between the purchase price of capital p^I and the sale price of capital p^S, as a result of capital specificity, or more generally, the adverse selection in the market for used capital goods. We normalise the purchase price p^I to one and denote $b_i = 1 - p^S \geqslant 0$, so that the parameter b_i can be interpreted as the difference between the purchase price and the sale price expressed as a percentage of the purchase price. For example, $p^S = 0.8$ gives $b_i = 0.2$, indicating that the sale price is 20% lower than the purchase price. Letting p^S approach zero or letting b_i approach one ensures that the firm never chooses to sell any capital, and mimics investment behaviour under a complete irreversibility constraint.

Assumption 8 ***Partial Irreversibility:*** *The functional form of partial irreversibility is*

$$G(I_t) = -b_i I_t 1_{[I_t<0]}$$

where $1_{[I_t<0]}$ *is an indicator equal to one if investment is strictly negative.*

2.3.3 Fixed Adjustment Costs

Fixed adjustment costs reflect those costs that are independent of the level of investment or disinvestment and are paid at each point of time if any investment or disinvestment is undertaken. We model the level of these costs to be proportional to the operating profit, so that first, these costs can be rationalized as output loss due to the interruption in production during periods of large investment or disinvestment; second, these costs again do not become irrelevant as the firm grows larger; third, they can be avoided by choosing zero investment.

Assumption 9 ***Fixed Adjustment Costs:*** *The functional form of fixed adjustment costs is*

$$G(Z_t, K_t; I_t) = b_f 1_{[I_t \neq 0]} \pi_t$$

where $1_{[I_t \neq 0]}$ *is an indicator equal to one if investment is non-zero. The parameter* b_f *is interpreted as the fraction of operating profit loss due to any non-zero investment.*

Our model allows for these three forms of adjustment costs, specifying the adjustment cost function to be

$$G(Z_t, K_t; I_t) = \frac{b_q}{2} \left(\frac{I_t}{K_t} \right)^2 K_t - b_i I_t 1_{[I_t<0]} + b_f 1_{[I_t \neq 0]} \pi_t \tag{16}$$

Remark 2 *The adjustment cost function* $G(Z_t, K_t; I_t)$ *is linear homogenous in* $(Z_t, K_t; I_t)$*.*

2.4 Investment Decisions

Denote $\Pi(Z_t, K_t; I_t)$ as the net revenue of the firm in each period t. That is

$$\Pi(Z_t, K_t; I_t) = \pi(Z_t, K_t; I_t) - G(Z_t, K_t; I_t) - I_t \tag{17}$$

Assumption 10 *The firm is owned by a representative consumer who values future net revenue with a discount rate adjusted with the level of uncertainty in the form of*

$$r = \overline{r} + \theta\sigma \tag{18}$$

where $\overline{r}$ *is a risk-free interest rate;* θ *is a parameter which could be positive, negative or zero.*

In each period investment is chosen to maximize the present value of current and expected future net revenues, where expectations are taken over the distribution of future demand/productivity shocks. According to the Principle of Optimality (Theorem 9.2, Stokey and Lucas, 1989), this investment decision can be represented as the solution to a dynamic optimization problem defined by the stochastic Bellman equation

$$V(Z_t, K_t) = \max_{I_t}\{\Pi(Z_t, K_t; I_t) + \frac{1}{1+r}E_t\left[V(Z_{t+1}, K_{t+1})\right]\} \tag{19}$$

together with the law of motion (1) and (12) for K_t and Z_t. Here $V(Z_t, K_t)$ is the value of the firm in period t; $E_t\left[V(Z_{t+1}, K_{t+1})\right]$ is the expected value of the firm in period $t+1$ conditional on information available in period t.

2.4.1 Frictionless Case

Lemma 3 ***Investment Policy in the Frictionless Case:*** *in the absence of any capital adjustment cost, the Euler equation for this optimization problem is*

$$const0 \cdot (1-\gamma) \cdot \left[\frac{Z_t}{\widehat{K}_t}\right]^{\gamma} = 1 - \frac{1-\delta}{1+r} \tag{20}$$

Hence the optimal investment rate can be derived as

$$\frac{I_t^*}{K_t} = const1 \cdot \frac{Z_t}{K_t} - 1 \tag{21}$$

Or equivalently expressed in levels, the optimal productive capital stock is

$$\widehat{K}_t^* = I_t^* + K_t = const1 \cdot Z_t \tag{22}$$

where

$$const1 = \left[const0 \cdot (1-\gamma) \,/\left(1 - \frac{1-\delta}{1+r}\right)\right]^{\frac{1}{\gamma}} \tag{23}$$

Proof: See Appendix 1.3.

The right hand side of equation (20) is simply the marginal revenue product of capital, while the left side is known as the Jorgensonian user cost of capital. Hence in spite of the uncertainty about future demand/productivity, this intertemporal optimality condition is equivalent to the first order condition in a static decision problem of the neoclassical producer theory. This is solely the result of the firm being able to adjust its capital stock instantaneously and costlessly in this case.

Equation (21) and (22) imply that without any friction, the optimal investment rate is a linear function of demand/productivity relative to inherited capital stock to meet the imbalance between the productive capital stock and the level of demand/productivity in each period, where the slope term $const1$ reflects production technology (β), demand elasticity (ε), factor price (w), and the Jorgensonian user cost of capital.

2.4.2 Friction Cases

In the presence of capital adjustment costs, uncertainty about future demand/productivity affects current investment since future adjustment of capital stock incurs costs. Optimal investment then needs to take into account the intertemporal linkage between current investment and future returns to capital and becomes indeed an interesting dynamic problem. However, with capital adjustment costs that we consider in equation (16), there is in general no closed-form solution. Appendix 2.1 explains how we solve the dynamic programming (19) numerically.

Figures 1-3 present the investment decision rules derived from the numerical solutions. We plot the optimal investment rate (I_t/K_t) against ($const1 \cdot Z_t/K_t - 1$), that is the scaled demand/productivity, where the 45^o line for the frictionless case (21) is plotted as a benchmark. With this scaling, both in the absence and presence of adjustment costs, a value of zero on the horizontal axis would always be associated with zero investment on the vertical axis. In the absence of any adjustment costs, investment occurs at all levels of scaled demand/productivity beyond zero while disinvestment occurs below zero. In the presence of adjustment costs, we show these decision rules separately for three special cases of the model.[4]

Figure 1 illustrates the optimal investment policy with quadratic adjustment costs only. Investment and disinvestment still occur at all levels of scaled demand/productivity beyond and below zero. However, with quadratic adjustment costs, the increasing marginal adjustment costs penalize high rates of investment or disinvestment, capital stock adjusts to new information about demand/productivity through a series of small and continuous adjustments. Hence, compared with the 45^o line, the investment policy in this case is also smooth but much dampened.

Figure 2 illustrates a region of inaction in the investment policy determined by

[4] These figures impose common parameters: $\beta = 0.10$, $\varepsilon = 6.00$, $w = 0.50$, $r = 0.065$, $\delta = 0.02$, $\rho = 0.90$, $\mu = 0.02$, and $\sigma = 0.10$.

two critical values with partial irreversibility. With partial irreversibility, there is no positive investment unless scaled demand/productivity reaches a right critical level that is larger than zero; and for further higher levels of demand/productivity the investment rate continues to be lower than what would be chosen in the frictionless case. Similarly, no disinvestment occurs unless scaled demand/productivity falls to a left critical level that is smaller than zero; and for further lower levels of demand/productivity the rate of disinvestment that occurs is much lower than what would be chosen in the frictionless case. In the extreme case of complete irreversibility, no disinvestment would ever happen, no matter how low is the level of demand/productivity relative to the inherited capital stock.

Figure 3 illustrates both a region of inaction and investment bursts as a result of corner-solution in the investment policy with fixed adjustment costs. Similar to partial irreversibility, investment or disinvestment occurs only when scaled demand/productivity exceeds the right and left critical values that are larger and smaller respectively than zero. Outside this region of inaction, the optimal investment decisions are quite different from those under partial irreversibility. Small adjustments to the capital stock do not generate benefits that are sufficiently high to warrant paying a fixed cost to implement them. Therefore capital stock adjusts to new information about demand/productivity through infrequent but large adjustments. When the scaled demand/productivity exceeds the right or left critical value, optimal investment jumps discontinuously to an investment policy, in which positive investment rate is higher than those in the frictionless case and negative investment rate is lower than those in the frictionless case.

3 The Effects of Uncertainty

This section analyses the effects of uncertainty on two interesting quantities. Given σ is the measure of overall uncertainty in this model, we define these quantities as explicit functions of σ.

Definition 2 ***Expected Capital Stock*** $E\left[\widehat{K}_t(\sigma)\right]$ *is the mathematical expectation for the optimal productive capital stock in period* t*.*

Definition 3 ***Expected Capital Intensity*** $E\left[\widehat{K}_t(\sigma)/Y_t(\sigma)\right]$ *is the mathematical expectation for the ratio of optimal productive capital stock to sales in period* t. [5]

Lemma 4 *In the absence of any capital adjustment cost,*

$$\begin{aligned} E\left[\widehat{K}_t^*(\sigma)\right] &= const1 \cdot E[Z_t] \\ E\left[\widehat{K}_t^*(\sigma)/Y_t(\sigma)\right] &= const2 \end{aligned}$$

where

$$const2 = \beta\left(1 - \frac{1}{\varepsilon}\right)/\left(1 - \frac{1-\delta}{1+r}\right) \tag{24}$$

Proof: See Appendix 1.4.

Lemma 4 implies that in the frictionless case, uncertainty would affect the expected capital stock only if $const1$ or $E[Z_t]$ depends on σ; and would affect the expected capital intensity only if $const2$ depends on σ. In the friction cases, the effects of uncertainty on these quantities also depend on different forms of capital adjustment costs. Therefore, our model provides a structural framework, which allows for uncertainty to affect capital accumulation through three possible channels: the risk-adjusted discount rate effect (through $const1$ and $const2$); the HAC effect (through $E[Z_t]$); and the capital adjustment costs. We examine these three channels separately one by one.

3.1 Uncertainty and the Discount Rate Effect

In order to abstract from any effects of uncertainty through the HAC effect and capital adjustment costs, we impose $E[Z_t]$ to be invariant to σ and $G(Z_t, K_t; I_t) = 0$ in this subsection.

According to (23) and (24), both $const1$ and $const2$ reflect production technology (β), demand elasticity (ε), depreciation rate (δ) and the discount rate (r).

Lemma 5 *All else being equal,* $\partial const1/\partial\beta > 0$, $\partial const2/\partial\beta > 0$, $\partial const1/\partial\varepsilon > 0$, $\partial const2/\partial\varepsilon > 0$, $\partial const1/\partial\delta < 0$, $\partial const2/\partial\delta < 0$, $\partial const1/\partial r < 0$, *and* $\partial const2/\partial r < 0$.

[5] An alternative measure for capital intensity is the capital-labour ratio. By Lemma 1, $E\left[\widehat{K}_t(\sigma)/L_t(\sigma)\right] = \frac{\gamma\varepsilon w}{\gamma\varepsilon - 1} \cdot E\left[\widehat{K}_t(\sigma)/Y_t(\sigma)\right]$. Therefore as long as (γ, ε, w) is uncorrelated with σ, the sign of the effect of uncertainty on capital intensity does not depend on which measure we use.

Proof: Comparative static analysis. Intuitively, an increase in β and ε would both decrease γ so that the operating profit function becomes less concave in capital stock, hence leads to more capital stock and capital intensity. In contrast, an increase in δ and r would both increase the Jorgensonian user cost of capital, hence leads to less capital stock and capital intensity.

Therefore uncertainty would affect the expected capital stock and capital intensity if any of these four parameters varies with the level of uncertainty. Under Assumption 10, for the discount rate $r = \overline{r} + \theta\sigma$, if the demand/productivity shocks are systematic, θ would be greater than, less than or equal to 0, depending on whether the marginal utility of the owner is negatively correlated, positively correlated or uncorrelated with the marginal revenue product of capital. If the demand/productivity shocks are idiosyncratic and the owner is fully-diversified, θ would be 0. If the demand/productivity shocks are idiosyncratic, but the owner is not fully-diversified and a large proportion of his consumption comes from the revenue of the firm, θ would be greater than 0, as rationalized in Angeletos and Calvet (2006), or Himmelberg, Hubbard and Love (2002).

Proposition 1 *When $E[Z_t]$ is invariant to σ and $G(Z_t, K_t; I_t) = 0$,* $\partial E\left[\widehat{K}_t^*(\sigma)\right]/\partial\sigma \begin{cases} <0 & \text{if } \theta>0 \\ \geqslant 0 & \text{if } \theta \leqslant 0 \end{cases}$ *and* $\partial E\left[\widehat{K}_t^*(\sigma)/Y_t(\sigma)\right]/\partial\sigma \begin{cases} <0 & \text{if } \theta>0 \\ \geqslant 0 & \text{if } \theta \leqslant 0 \end{cases}$, *i.e. the effects of uncertainty on the expected capital stock and expected capital intensity depend on the sign of θ.*

Proof: By Assumption 10, Lemma 5 and applying the chain rule in partial differentiation.

3.2 Uncertainty and the HAC Effect

In order to abstract from any effects of uncertainty through discount rate effect and capital adjustment costs, we impose $\theta = 0$ and $G(Z_t, K_t; I_t) = 0$ in this subsection.

To study the effects of uncertainty through the HAC effect, it is standard to apply **a mean-preserving spread** for the underlying stochastic process. Equation (9) and (10) imply that keeping $\mu_x$and μ_a constant while increasing σ_x^2 and σ_a^2, is a mean-preserving spread for x_t and a_t respectively. Since the demand shift parameter X_t and the level of productivity A_t are the two stochastic variables that expectation is taken over, we would like to focus on increases in uncertainty that preserve the mean of X_t and A_t. This is easily achieved by Lemma 6.

Lemma 6 *By setting* $c_x = -0.5\sigma_x^2/\left(1-\rho^2\right)$ *and* $c_a = -0.5\sigma_a^2/\left(1-\rho^2\right)$, $E\left[X_t\right] = \exp\left(\zeta_0^x + \mu_x t\right)$ *and* $E\left[A_t\right] = \exp\left(\zeta_0^a + \mu_a t\right)$, *i.e. keeping* μ_x*and* μ_a *constant while increasing* σ_x^2 *and* σ_a^2, *is a mean-preserving spread for* X_t *and* A_t, *respectively.*

Proof: See Appendix 1.5.

Recall the operating profit is $\pi(Z_t, \widehat{K}_t) = const0 \cdot Z_t^{\gamma} \widehat{K}_t^{1-\gamma}$, where $Z_t = X_t \left(A_t\right)^{\varepsilon-1}$. Since $\mu = \mu_x + \left(\varepsilon-1\right)\mu_a$ and $\sigma^2 = \sigma_x^2 + \left(\varepsilon-1\right)^2 \sigma_a^2$, keeping $\mu_x$and μ_a constant while increasing σ_x^2 and σ_a^2 also implies keeping μ constant while increasing σ^2. However, this is in general not a mean-preserving spread for Z_t.

Lemma 7 *Keeping* μ *constant while increasing* σ^2 *is in general not a mean-preserving spread for* Z_t. *In particular,*

$$E\left[Z_t\right] = \exp\left[\zeta_0 + \mu t + \frac{\left(\varepsilon-2\right)\left(1-\tau\right)}{2\left(\varepsilon-1\right)\left(1-\rho^2\right)}\sigma^2\right] \tag{25}$$

Proof: See Appendix 1.6.

Lemma 7 implies that the effect of keeping μ constant while increasing σ^2 on $E\left[Z_t\right]$ includes three cases. First, either when all uncertainty is from demand so that $\sigma_x^2 = \sigma^2$ or equivalently $\tau = 1$, or when the demand elasticity $\varepsilon = 2$, then $\partial E\left[Z_t\right]/\partial\sigma = 0$. Second, if there is any uncertainty from productivity so that $\sigma_a^2 > 0$ or equivalently $\tau < 1$, and the demand elasticity $\varepsilon > 2$, then $\partial E\left[Z_t\right]/\partial\sigma > 0$. Finally, if there is any uncertainty from productivity so that $\sigma_a^2 > 0$ or equivalently $\tau < 1$, and the demand elasticity $1 < \varepsilon < 2$, then $\partial E\left[Z_t\right]/\partial\sigma < 0$.

Proposition 2 *When* $\theta = 0$ *and* $G(Z_t, K_t; I_t) = 0$,

$$\partial E\left[\widehat{K}_t^*\left(\sigma\right)\right]/\partial\sigma \begin{cases} = 0 & \textit{if } \tau = 1 \textit{ or } \varepsilon = 2 \\ > 0 & \textit{if } \tau < 1 \textit{ and } \varepsilon > 2 \\ < 0 & \textit{if } \tau < 1 \textit{ and } 1 < \varepsilon < 2 \end{cases} \quad \textit{and } \partial E\left[\widehat{K}_t^*\left(\sigma\right)/Y_t\left(\sigma\right)\right]/\partial\sigma = 0,$$

i.e. the effects of uncertainty on the expected capital stock depend on the value of τ *and* ε, *but uncertainty has no effect on the expected capital intensity through the HAC effect.*

Proof: It is straightforward to derive the results for $\partial E\left[\widehat{K}_t^*\left(\sigma\right)\right]/\partial\sigma$ by Lemma 7 and $\partial E\left[\widehat{K}_t^*\left(\sigma\right)/Y_t\left(\sigma\right)\right]/\partial\sigma = 0$ by Lemma 4.

The first part of Proposition 2 implies that our model allows for the uncertainty to affect the expected capital stock through the marginal revenue product of capital, and this effect could be positive, negative or zero under our setting, depending on the source of uncertainty and the demand elasticity. The cases studied in the literature that lead to the HAC effect, for example, uncertainty in output price (Hartman, 1972; Abel, 1983), in the price of variable input (Abel, 1985; Lee and Shin, 2000), or in horizontal demand shocks (Caballero, 1991; Pindyck, 1993) can be represented by the case of $\tau < 1$ and $\varepsilon > 2$. Furthermore, within this case, we have $\partial\left(\partial E\left[\widehat{K}_t^*(\sigma)\right]/\partial\sigma\right)/\partial\varepsilon > 0$, which verifies the insight in Caballero (1991) about the role of degree of competition in determining the importance of the HAC effect. In the extreme case of perfection competition, i.e. $\varepsilon = \infty$, the magnitude of the HAC effect is infinitely large and dominates the effects of uncertainty through any other channel, one special case studied in Abel and Eberly (1994).

The second part of Proposition 2 implies that due to the linear homogeneity property of our investment model, the HAC effect would affect all the variables in levels, such as capital stock, investment, variable input, sales and operating profit, in the same proportion; hence it would not affect any variable in ratio, such as investment rate, capital-to-sales ratio, profit-to-sales ratio and sales growth rate. This might explain why in empirical research, such as Leahy and Whited (1996), that only consider the effects of uncertainty on investment rate rather than on capital stock, the HAC effect has not been detected.

3.3 Uncertainty and the Adjustment Cost Effect

In order to abstract from any effects of uncertainty through discount rate effect and the HAC effect, we impose $\theta = 0$ and $\tau = 1$ so that $E\left[\widehat{K}_t^*(\sigma)\right] = const1 \cdot \exp(\zeta_0 + \mu t)$ is invariant to σ in this subsection. Given there is no closed-form solution to the investment model in the presence of capital adjustment costs, we provide intuition and illustrate simulation results as proof to the following results.

Proposition 3 *When $\theta = 0$ and $\tau = 1$,*
$\partial E\left[\widehat{K}_t(\sigma)\right]/\partial\sigma \begin{cases} < 0 & \text{if } b_q > 0 \\ \lesseqgtr 0 & \text{if } b_i > 0 \text{ or } b_f > 0 \end{cases}$ *and*
$\partial E\left[\widehat{K}_t(\sigma)/Y_t(\sigma)\right]/\partial\sigma \lesseqgtr 0$, *if $b_q > 0$, $b_i > 0$ or $b_f > 0$.*

Proof: The first part of Proposition 3 implies that an increase in the level of uncertainty must lower the expected capital stock in the presence of quadratic adjustment costs; but has an ambiguous effect on the expected capital stock in the presence of partial irreversibility or fixed adjustment costs.

For quadratic adjustment costs only, analogy to Abel (1984), for any given inheritated capital stock K_t, if $\gamma = 0$, equation (19) represents a linear-quadratic problem in which certainty-equivalence applies, hence $E[I_t(\sigma)]$ would be invariant to σ. Take this case as a benchmark. The case under our consideration is $\gamma > 0$, so certainty-equivalence fails since (19) is no longer a linear-quadratic problem. Given $\gamma > 0$ implies $\Pi(K_t, Z_t; I_t)$ being concave in I_t, $E[I_t(\sigma)]$ is decreasing in σ due to Jensen's inequality effect. Since $\widehat{K}_t = K_t + I_t$, this implies $E\left[\widehat{K}_t(\sigma)\right]$ is decreasing in σ, or $\partial E\left[\widehat{K}_t(\sigma)\right]/\partial\sigma < 0$ if $b_q > 0$.

For partial irreversibility only, Abel and Eberly (1999) demonstrate that complete irreversibility and uncertainty increase the user cost of capital which tends to reduce the capital stock. Working in the opposite direction is a hangover effect, which arises because irreversibility prevents the firm from selling capital even when the marginal revenue product of capital is low. Neither the user cost effect nor the hangover effect dominates globally, so that irreversibility may increase or decrease the expected capital stock $E\left[\widehat{K}_t(\sigma)\right]$ relative to that under reversibility $E\left[\widehat{K}_t^*\right]$. Furthermore, both the user cost effect and the hangover effect are stronger with higher level of uncertainty, again neither of them dominates globally. Hence the sign of $\partial\left(E\left[\widehat{K}_t(\sigma)\right]/E\left[\widehat{K}_t^*\right]\right)/\partial\sigma$ is ambiguous. Given $E\left[\widehat{K}_t^*\right]$ is invariant to σ, this implies the ambiguity in the sign of $\partial E\left[\widehat{K}_t(\sigma)\right]/\partial\sigma$ if $b_i > 0$.

For fixed adjustment costs only, Cooper, Haltiwanger and Power (1999) provide intuition for the trade-off between the threshold effect and the target effect in the presence of fixed adjustment costs. Under a higher level of uncertainty, the thresholds for investment and disinvestment enlarge, but meanwhile the firm has more incentive to overshoot its investment target to adjust capital stock due to physical depreciation and demand/productivity shocks. This implies an increase in uncertainty will lead to both more frequent investment inaction and larger investment/disinvestment bursts, hence the ambiguity in the sign of $\partial E\left[\widehat{K}_t(\sigma)\right]/\partial\sigma$ if $b_f > 0$.

In Bond, Söderbom and Wu (2007), we replicate the analytical results in Abel and Eberly (1999) for complete irreversibility by numerical simulation, and generalize the analyses for quadratic adjustment costs, partial irreversibility and fixed adjustment costs, which confirms the claim in Proposition 3.

The second part of Proposition 3 implies that an increase in the level of uncertainty has an ambiguous effect on the expected capital intensity in the presence of adjustment costs.

As Lemma 1 indicates, the sales Y_t is linear homogeneous in Z_t and $\widehat{K}_t$. Together with Lemma 3, in the frictionless case, Y_t is always proportional to $\widehat{K}_t^*$ hence $\widehat{K}_t^*/Y_t = const2$ is invariant to σ. In the friction case, when Z_t decreases due to negative shocks, all three forms of capital adjustment costs make $\widehat{K}_t$ decrease less than Z_t, linear homogeneity implies Y_t would decrease more than $\widehat{K}_t$ but less than Z_t. Hence $\widehat{K}_t/Y_t$ must be higher than $\widehat{K}_t^*/Y_t$ conditional on $e_t < 0$. When Z_t increases due to positive shocks, quadratic adjustment costs and partial irreversibility make $\widehat{K}_t$ increase less than Z_t, linear homogeneity implies Y_t would increase more than $\widehat{K}_t$ but less than Z_t. Fixed adjustment costs have ambiguous effect, depending on the relative importance of the threshold effect and the target effect. Hence $\widehat{K}_t/Y_t$ tends to be lower than $\widehat{K}_t^*/Y_t$ conditional on $e_t > 0$. When σ increases, Z_t would decrease or increase both with a larger magnitude, which means $\widehat{K}_t/Y_t$ would be higher or lower than $\widehat{K}_t^*/Y_t$ both with a larger magnitude. Since the expectation is taken over both positive and negative shocks, this implies the ambiguity in the sign of $\partial E\left[\widehat{K}_t(\sigma)/Y_t(\sigma)\right]/\partial\sigma$.

4 Empirical Strategy

The analyses in Section 3 illustrate the rich implications about the effects of uncertainty in our investment model: with an increase in the level of uncertainty, a risk-adjusted discount rate effect would increase/decrease/unchange both the expected capital stock and the expected capital intensity, depending on the sign of θ (Proposition 1); the HAC effect would increase/decrease/unchange the expected capital stock, depending on the value of τ and ε (Proposition 2); capital adjustment costs would affect both the expected capital stock and the expected capital intensity, depending on the exact form of the adjustment costs (Proposition 3). This implies the effects of uncertainty on capital accumulation is fundamentally an empirical question.

4.1 Dataset

We use an empirical sample from Bloom, Bond and Van Reenen (2007), which studies the investment dynamics under uncertainty and partial irreversibility. This sample contains firm-level data for an unbalanced panel of 672 publicly traded U.K. manufacturing firms between 1972 and 1991. These company data are taken from the consolidated accounts of manufacturing firms listed on the U.K. stock market and are obtained from the Datastream on-line service. Our identification strategy only requires four key variables: Investment ($I_{j,t}$); Capital stock ($K_{j,t}$); Sales ($Y_{j,t}$); and Operating Profit ($\pi_{j,t}$) where j denotes firm and t denotes year. The data appendix of Bloom, Bond and Van Reenen (2007) explains how these variables are constructed, cleaned and deflated.

4.2 Uncertainty Heterogeneity

In order to identify the discount rate effect and the HAC effect, the necessary condition is to have some variation in the level of uncertainty. In theory, this variation could be modelled either across time or across firms. Since the empirical sample we use in this paper is a short panel, and a main feature in firm-level investment data is the importance of "fixed-effects" (Bond and Van Reenen, 2003), we model this variation as cross-sectional.

Assumption 11 ***Uncertainty Heterogeneity:*** *The measure of overall uncertainty for firm j is σ_j, where $\log \sigma_j \overset{i.i.d}{\sim} N\left(\mu_{l\sigma}, \sigma_{l\sigma}^2\right)$.*

That is each firm j faces a firm-specific measure of uncertainty σ_j, where $\log \sigma_j$ is drawn independently from an identical normal distribution with mean $\mu_{l\sigma}$ and standard deviation $\sigma_{l\sigma}$.

Under this assumption, Proposition 1 predicts that the sign of $cov[K_{j,t}, \sigma_j^2]$ and $cov[K_{j,t}/Y_{j,t}, \sigma_j^2]$ depends on θ, through the discount rate effect; Proposition 2 predicts the sign of $cov[K_{j,t}, \sigma_j^2] > 0$ depends on τ and ε, through the HAC effect, which means we have transformed the problem of identifying the discount rate effect and the HAC effect into estimating $\mu_{l\sigma}$, $\sigma_{l\sigma}^2$, θ, τ and ε.

4.3 Growth Rate Heterogeneity

In order to identify the capital adjustment costs effect, the investment policies illustrated in Section 2 indicate the possibility of identifying different forms of capital adjustment costs from different features in the investment rate. However, as recognized in both Cooper and Haltiwanger (2006) and Bloom (2007), a key challenge in estimating adjustment costs is to distinguish the persistent differences in the stochastic process from the adjustment costs. For example, both differences across firms in the demand/productivity growth rate and high quadratic adjustment costs can lead to persistent differences across firms in the investment rate. Given the important role of quadratic adjustment costs in determining the effects of uncertainty on the expected capital stock, it is important to distinguish between unobserved heterogeneity and state dependence. Therefore, we explicitly model heterogeneity in the growth rate in order to get robust estimates for the adjustment costs

Assumption 12 ***Growth Rate Heterogeneity:*** *The combined growth rate for firm j is μ_j, where $\mu_j \overset{i.i.d}{\sim} N\left(\mu_\mu, \sigma_\mu^2\right)$ and $cov\left(\mu_j, \sigma_j\right) = 0$.*

That is each firm j has a firm-specific combined growth rate μ_j, where μ_j is drawn independently from an identical normal distribution with mean μ_μ and standard deviation σ_μ. With heterogeneities in both σ and μ, we further assume that they are uncorrelated with each other so that the effects of uncertainty can be separated from the effects of growth rate.

Both the level of uncertainty and the growth rate would affect the investment policy. Hence the dynamic programming described in (19) must be solved for each firm j with value σ_j and μ_j, which is unaffordable even for a small sample. Therefore we adopt a standard approach used in the literature, for example, Eckstein and Wolpin (1999), to allow for a finite mixture of types.

Assumption 13 ***A Finite Mixture of Types:*** *There are a finite mixture of types, say $U \times V$ types of firms, each comprising a fixed proportion $1/(U \times V)$ of the population, where the type set is defined as $F = \{(\sigma_u, \mu_v) : u = 1, \cdots, U;\ v = 1, \cdots, V\}$.*

Appendix 2.2 explains how we solve the dynamic programming and Appendix 2.3 explains how we simulate the data under this assumption.

4.4 Relating $Z_{j,t}$ to Observable Variables

We have shown how optimal investment would response to the scaled demand/productivity $(const1 \cdot Z_{j,t}/\widehat{K}_{j,t} - 1)$ with different forms of capital adjustment costs. We have also allowed for two dimension heterogeneities in the demand/productivity $(Z_{j,t})$. Given the stochastic process is known to the firm but is in general not observable to econometrician, we construct following two proxies.

Denote $yk_{j,t} = \log(Y_{j,t}/K_{j,t})$, i.e. the log of sales-to-capital ratio for firm j in period t. In the absence of capital adjustment costs,

$$\log(Y_{j,t}/K_{j,t}) = \log\left(const0 \cdot Z_{j,t}^{\gamma}\widehat{K}_{j,t}^{1-\gamma}/\widehat{K}_{j,t}\right) = \log const0 + \gamma \log\left(Z_{j,t}/\widehat{K}_{j,t}\right)$$

which is a monotonic increasing transformation of $(const1 \cdot Z_{j,t}/\widehat{K}_{j,t} - 1)$. Since in the presence of capital adjustment costs, $Z_{j,t}$ is also a non-decreasing function of $Z_{j,t}$, we use $yk_{j,t}$ as the proxy for the scaled demand/productivity $(const1 \cdot Z_{j,t}/\widehat{K}_{j,t} - 1)$.

Denote $dy_{j,t} = \log(Y_{j,t}) - \log(Y_{j,t-1})$, i.e. the sales growth rate for firm j in period t. In the absence of capital adjustment costs,

$$\begin{aligned} \log(Y_{j,t}) - \log(Y_{j,t-1}) &= \log(Z_{j,t}) - \log(Z_{j,t-1}) = \mu_j + \zeta_{j,t} - \zeta_{j,t-1} \\ \zeta_{j,t} &= \rho\zeta_{j,t-1} + e_{j,t} \end{aligned}$$

where $0 < \rho < 1$ and $e_{j,t} \overset{i.i.d}{\sim} N\left(0,\, \sigma_j^2\right)$. Then

$$\begin{aligned} Edy_j &= mean_t\,(dy_{j,t}) = \mu_j \\ SDdy_j &= sd_t\,(dy_{j,t}) \simeq \sigma_j \end{aligned}$$

That is the within-group mean of the sales growth rate for firm j is equal to μ_j; and the within-group standard deviation of the sales growth rate for firm j is approximately (exactly iff $\rho = 1$) equal to σ_j. Since in the presence of capital adjustment costs, $Y_{j,t}$ is also a non-decreasing function of $Z_{j,t}$, we use Edy_j and $SDdy_j$ as the proxies for the growth rate and level of uncertainty for firm j.

4.5 Intercept Heterogeneity

In addition to the discount rate effect, the HAC effect and the capital adjustment costs effects that we have explicitly modelled, Lemma 4 indicates that the expected capital stock also depend on production technology (β), demand elasticity (ε), depreciation rate (δ), relative price of variable input (w), the time period a firm has operated (t), the unit in measuring capital stock (£ or £1000), and finally the intercept in the stochastic process (ζ_0). Any differences in these factors across firms will lead to permanent differences in the expected capital stock across firms. Our empirical strategy is to impose common value for β, ε and δ at their sample average, choose arbitrary value for w, t, and the unit of measurement, while model and estimate the distribution of ζ_0.

Assumption 14 ***Intercept Heterogeneity:*** *The intercept in the stochastic process for firm j is ζ_{0j} , where $\zeta_{0j} \overset{i.i.d}{\sim} N\left(\mu_{\zeta_0}, \sigma_{\zeta_0}\right)$ and cov $\left(\zeta_{0j}, \sigma_j\right) = 0$, cov $\left(\zeta_{0j}, \mu_j\right) = 0$.*

That is each firm j has a firm-specific intercept ζ_{0j} in the stochastic process, where ζ_{0j} is drawn independently from an identical normal distribution with mean μ_{ζ_0} and standard deviation σ_{ζ_0}. With heterogeneities in σ, μ and ζ_0, we further assume that they are uncorrelated with each other so that the factors that lead to permanent differences in the expected capital stock are uncorrelated with the level of uncertainty and the growth rate of the firms.

This technical devise is based on the important property summarized by the following lemma.

Lemma 8 *Denote $\Gamma_j = \Gamma(\beta_j, \varepsilon_j, \delta_j, w_j, t_j)$. If cov $(\Gamma_j, \sigma_j) = 0$, the effect of imposing common value for $(\beta, \varepsilon, \delta, w, t)$ on the dispersion of the expected capital stock can be accounted for by adjusting σ_{ζ_0}; the effect of choosing arbitrary value for (w, t) and the unit of measurement on the level of the expected capital stock can be accounted for by adjusting μ_{ζ_0}.*

Proof: See Appendix 1.7.

Different from the level of uncertainty and the growth rate, the value of ζ_0 doesn't affect the investment policy due to the linear homogeneity property of the investment model. Hence there could be "infinite" type for the intercept in the stochastic process. Appendix 2.3 explains how we normalize the dynamic programming and simulate the data under this assumption.

4.6 Measurement Errors

Given the important role of investment rate and sales in our identification strategy, we allow for a rich structure of measurement errors in our empirical specification. This is motivated by two reasons. First, measurement error is a common feature in firm-level recorded data. Second and more fundamentally, allowing for permanent components of measurement errors in the investment rate and sales is a computationally tractable way, to control for persistent differences between firms in investment rate and sales, which might not have been fully controlled for through modelling heterogeneities in the stochastic process.

Assumption 15 ***Measurement Errors in Investment Rate:*** *Denote investment rate* $i_{j,t} = I_{j,t}/K_{j,t}$. *Suppose* $i_{j,t} = i^*_{j,t}\exp(e^I_{j,t})$, *where* $e^I_{j,t} = e^{IT}_{j,t} + e^{IP}_j$, *and* $e^{IP}_j \overset{i.i.d}{\sim} N(0, \sigma^2_{IP})$, $e^{IT}_{j,t} \overset{i.i.d}{\sim} N(0, \sigma^2_{IT})$.

That is there is a standard multiplicative structure for measurement error in the investment rate, where $i_{j,t}$ denotes the observed investment rate, $i^*_{j,t}$ denotes the true underlying investment rate which is not measured accurately in the data, and the measurement error $e^I_{j,t}$ has both transitory and permanent components with mean zero and standard deviation σ_{IT} and σ_{IP}, respectively. This specification has the property that the sign of recorded investment rate is not affected by measurement error, and treats observations with zero investment in the data as true zeros.

Assumption 16 ***Measurement Errors in Sales:*** *Suppose* $Y_{j,t} = Y^*_{j,t}\exp(e^Y_{j,t})$, *where* $e^Y_{j,t} = e^{YT}_{j,t} + e^{YP}_j$, *and* $e^{YP}_j \overset{i.i.d}{\sim} N(0, \sigma^2_{YP})$, $e^{YT}_{j,t} \overset{i.i.d}{\sim} N(0, \sigma^2_{YT})$.

That is there is a standard multiplicative structure for measurement error in sales, where $Y_{j,t}$ denotes the observed level of sales, $Y^*_{j,t}$ denotes the true underlying level of sales which is not measured accurately in the data, and the measurement error $e^Y_{j,t}$ has both transitory and permanent components with mean zero and standard deviation σ_{YT} and σ_{YP}, respectively.

Appendix 2.3 explains how we simulate the data under these two assumptions.

4.7 Aggregation at the Firm-Level

Another feature for firm-level accounting data is that these data might be consolidated across several plants within the firm. Table 4 compares the investment rate data from a sample of the Longitudinal Research Database (plant-level) in Cooper and Haltiwanger (2006) and from a sample of the Compustat Dataset (firm-level) in Bloom (2007), in which investment rate is featured by spikes and zeros at the plant-level but by smooth and continuous serials at the firm-level. Without accounting for possible aggregation in the firm-level data could lead to an overestimate for the quadratic adjustment costs and an underestimate for the non-convex adjustment costs. Given the Datastream is a firm-level dataset, it is important for us to consider the possible aggregation at the firm-level.

Assumption 17 ***Aggregation:*** *Each firm is made of m plants, where $m \geq 1$. For plant i of firm j in period t, the law of motion for $Z_{i,j,t}$ is given by*

$$\begin{aligned} z_{i,j,t} &= \log Z_{i,j,t} \\ z_{i,j,t} &= c_j + \mu_j t + \zeta_{i,j,t} \\ \zeta_{i,j,t} &= \rho\zeta_{i,j,t-1} + e_{i,j,t} = \zeta_{0j} + \sum_{s=0}^{t-1} \rho^s e_{i,j,t-s} \end{aligned} \tag{26}$$

where $0 < \rho < 1$, $e_{i,j,t} \overset{i.i.d}{\sim} N\left(0,\, \sigma_j^2\right)$, $c_j = -\left[\tau + \frac{(1-\tau)}{(\varepsilon-1)}\right] \frac{\sigma_j^2}{2(1-\rho^2)}$.

That is there are heterogeneities across firms in the level of uncertainty, growth rate and intercept; however, plants within the same firm are all identical except the idiosyncratic demand/productivity shocks $e_{i,j,t}$. Appendix 2.3 explains how we simulate the data under this aggregation assumption.

Lemma 9 *The effect of any arbitrary choice of m on the level of expected capital stock can be accounted for by adjusting μ_{ζ_0}.*

Proof: See Appendix 1.7.

5 A Structural Estimation

Since the effects of uncertainty on capital accumulation are working through different channels simultaneously, it is difficult to estimate these channels separately and reliably using standard regression techniques for reduced-form investment models. Instead, our strategy is fully parametric, i.e. to recover the structural parameters in the model explicitly by simulated method of moments and to apply counterfactual simulations to gauge the qualitative and quantitative importance of these effects through each channel.

5.1 Simulated Method of Moments

The simulated method of moments (SMM hereafter) aims at estimating a vector of unknown parameters by solving a minimum quadratic distance problem. Formally, following Gouriéroux and Monfort (1996), the **SMM estimator** Θ^* solves

$$\widehat{\Theta} = \arg\min_{\theta} \left(\widehat{\Phi}^D - \frac{1}{H} \sum_{h=1}^{H} \widehat{\Phi}_h^S (\Theta) \right)' \Omega \left(\widehat{\Phi}^D - \frac{1}{H} \sum_{h=1}^{H} \widehat{\Phi}_h^S (\Theta) \right) \tag{27}$$

where Θ is the vector of parameters of our interest; $\widehat{\Phi}^D$is a set of empirical moments estimated from an empirical dataset; $\widehat{\Phi}^S(\Theta)$ is the same set of simulated moments estimated from a simulated dataset of the structural model; H is the number of simulation path; Ω is a positive definite weighting matrix.

Suppose the empirical dataset is a panel with N firms and T years. Given we model unobserved heterogeneities across firms, the asymptotics is for fixed T and $N \to \infty$. At the efficient choice for the weighting matrix Ω^*, the SMM procedure provides a global specification test of the overidentifying restrictions of the model, i.e. if the model is well-specified, the test statistics OI follows a chi-square distribution with degree of freedom equal to the difference between the number of moments and the number of parameters:

$$\begin{aligned} OI &= \frac{NH}{1+H} \left(\widehat{\Phi}^D - \frac{1}{H} \sum_{h=1}^{H} \widehat{\Phi}_h^S (\Theta) \right)' \Omega^* \left(\widehat{\Phi}^D - \frac{1}{H} \sum_{h=1}^{H} \widehat{\Phi}_h^S (\Theta) \right) \\ &\sim \chi^2 \left[\dim \left(\widehat{\Phi} \right) - \dim (\Theta) \right]. \end{aligned} \tag{28}$$

At the efficient choice for the weighting matrix Ω^*, the SMM estimator is asymptotically normal for fixed H and T, and $N \to \infty$, i.e.

$$\sqrt{N} \left(\widehat{\Theta} - \Theta^* \right) \xrightarrow{D} N \left(0, W \left(H, \Omega^* \right) \right) \tag{29}$$

where

$$W (H, \Omega^*) = \left(1 + \frac{1}{H} \right) \left(E \left[\partial \widehat{\Phi}^{S\prime} \left(\widehat{\Theta} \right) / \partial \Theta \right] \Omega^* E \left[\partial \widehat{\Phi}^S \left(\widehat{\Theta} \right) / \partial \Theta' \right] \right)^{-1}$$

Define binding function as $\widehat{\Phi}^S = \widehat{\Phi}^S (\Theta)$, that is how the simulated moments change with the structural parameters. Define the Jacobian matrix for the binding functions as $J = \left[\partial \widehat{\Phi}^{S\prime} \left(\widehat{\Theta} \right) / \partial \Theta \right]$. The crucial point of SMM is that the simulated moments $\widehat{\Phi}^{S\prime} \left(\widehat{\Theta} \right)$ depend on the structural parameters $\widehat{\Theta}$ used in that particular round of simulation. Therefore identification requires the variation in the simulated moments being informative about the changes in the underlying structural parameters. The sufficient condition for local identification is that the Jacobian matrix J has full row rank.

Appendix 2.4 reports how we estimate the efficient weighting matrix, solve the minimum quadratic distance problem, calculate the numerical derivatives and check the local identification.

5.2 Structural Parameters

Table 1 lists the set of parameters Θ that we aim to estimate, which can be divided into 6 categories. [1] parameter determining the importance of the discount rate effect θ. [2] parameter measuring the importance of the HAC effect $1-\tau$, given the demand elasticity $\varepsilon \neq 2$. [3] parameters measuring the magnitude of capital adjustment costs, i.e. quadratic adjustment costs b_q, partial irreversibility b_i and fixed adjustment costs b_f. Denote $b = (b_q, b_i, b_f)$. [4] parameters characterising technology and demand, i.e. the capital share in production function β and the demand elasticity with respect to price ε. [5] parameters characterising the stochastic process, i.e. the serial correlation of shocks ρ; the mean and standard deviation of the growth rate μ_μ and σ_μ; the mean and standard deviation of the log of level of uncertainty $\mu_{l\sigma}$ and $\sigma_{l\sigma}$; and the mean and standard deviation for the intercept μ_{z_0} and σ_{z_0}. [6] parameters measuring the magnitude of measurement errors in the data, i.e. the standard deviation of transitory and permanent measurement errors in investment rate σ_{IT} and σ_{IP}, and in sales σ_{YT} and σ_{YP}.

Besides these 18 structural parameters, there are another 2 parameters in our investment model that are exogenous but would also affect the investment policy and capital accumulation. First, the depreciation rate δ. We impose $\delta = 0.08$, the number used in constructing the capital stock series with perpetual inventory method in the empirical sample. Second, the risk-free interest rate $\overline{r}$. We impose $\overline{r} = 0.065$, which is in line with the value used in the literature such as Bloom (2007).

5.3 Moments and Identification

Table 2 lists the set of moments $\widehat{\Phi}^D$ that we aim to match. The selection of moments follows the "informativeness" principle and is guided by the properties of the model we discussed in previous sections. In column 1, left panel, Table 3, we report the value of these moments estimated from the empirical sample.

Denote $EK_j = mean_t\left(\widehat{K}_{j,t}\right)$ and $EKY_j = mean_t\left(\widehat{K}_{j,t}/Y_{j,t}\right)$, i.e. the within-group mean of the capital stock and capital-to-sales ratio. Recall $SDdy_i$ is the within-group standard deviation of sales growth rate and the proxies for the level of uncertainty. Hence the first two key moments $corr\left(EK_j, SDdy_j\right)$ and $corr\left(EKY_j, SDdy_j\right)$ calculate the between-group correlation coefficients for capital stock and uncertainty, and for capital intensity and uncertainty. According to our investment model, all else being equal, $corr\left(EK_j, SDdy_j\right)$ is informative about the discount rate effect θ, the HAC effect τ and capital adjustment costs b; $corr\left(EKY_j, SDdy_j\right)$ is informative about the discount rate effect θ, and capital adjustment costs b. Conditional on b being identified by other moments, $corr\left(EKY_j, SDdy_j\right)$ identifies θ. Conditional on b and θ being identified by other moments, $corr\left(EK_j, SDdy_j\right)$ identifies τ. In our empirical sample, both these two correlation coefficients are negative and small. This implies either all these three effects are weak, or these effects are strong individually but basically balance each other, leaving an overall small, negative effect across firms.

The third moment $prop\left(i_{j,t} < -0.01\right)$, fourth moment $prop\left(|i_{j,t}| < 0.01\right)$ and fifth moments $prop\left(i_{j,t} > 0.20\right)$ report the proportion of disinvestment, investment inaction and investment spikes. Recall our investment model predicts that both partial irreversibility b_i and fixed adjustment costs b_f would generate zero investment, however, partial irreversibility would lead to less disinvestment, while fixed adjustment costs more investment spikes. Furthermore, quadratic adjustment costs b_q dampen investment hence predicts less investment spikes. Very few disinvestment and zero investment are recorded in our empirical sample. This either implies the insignificance of the non-convex adjustment costs or reflects the importance of aggregation.

The correlation coefficient $corr\left(i_{j,t}, yk_{j,t}\right)$ reflects the responsiveness of investment rate to the log sales-to-capital ratio. Recall $yk_{j,t}$ is the proxy for the scaled demand/productivity. The investment policies illustrated in Section 2 imply that high values of b_q and b_i decrease this coefficient; while any measurement errors in investment rate $(\sigma_{IT}, \sigma_{IP})$ or sales $(\sigma_{YT}, \sigma_{YP})$ can cause attenuation in this coefficient. The serial correlation of investment rate $corr\left(i_{j,t}, i_{j,t-1}\right)$ reflects both the importance of b_q and the persistence of the stochastic process ρ. Transitory measurement errors (σ_{IT}) attenuate this coefficient while permanent measurement errors (σ_{IP}) blow it up. The serial correlation of the log sales-to-capital ratio $corr\left(yk_{j,t}, yk_{j,t-1}\right)$ is also informative about ρ; meanwhile, transitory (σ_{YT}) and permanent (σ_{YP}) measurement errors affect this coefficient in the opposite direction. Empirically, we observe a low correlation between investment rate and the log sales-to-capital ratio, high serial correlation in investment rate and very high serial correlation in log sales-to-capital ratio, which may reflect the importance of adjustment costs, the persistence of the stochastic process, or the existence of measurement errors.

The next two moments are derived more directly through the first-order conditions of the model. The mean of profit-to-sales ratio $mean\left(\pi_{j,t}/Y_{j,t}\right)$ and capital-to-sales ratio $mean\left(EKY_j\right)$ are informative for the capital share in production function β and the demand elasticity with respect to price ε. First, Lemma 1 claims, no matter whether there is adjustment cost or not, the first-order condition of the short-run profit maximization problem indicates that sales is always proportional to the operating profit. i.e. $\pi_{j,t}/Y_{j,t} = 1/\gamma\varepsilon$, where $\gamma = 1/\left(1+\beta\left(\varepsilon-1\right)\right)$. Furthermore, Lemma 4 claims, in the absence of adjustment costs, the first-order condition of the dynamic programming indicates that capital-to-sales ratio is always a constant, i.e. $K_{j,t}/Y_{j,t} = const2$, where $const2 = \left[\beta\left(\varepsilon-1\right)\left(1+r\right)\right]/\left[\varepsilon\left(r+\delta\right)\right]$. Therefore, for given θ, b, and δ, identifying β and ε is equivalent to solving two equations (6) and (24) for two unknowns simultaneously. Table 3 reports a sample average of 26.6% profit-to-sales ratio and 49.7% capital-to-sales ratio.

For given adjustment costs parameters b and depreciation rate δ, the first moment of investment rate $mean\left(i_{j,t}\right)$ is informative about the mean of the growth rate μ_μ; while the second moment $sd\left(i_{j,t}\right)$ is informative about the heterogeneities in the stochastic process σ_μ and $\sigma_{l\sigma}$, and measurement errors in investment rate $\left(\sigma_{IT}, \sigma_{IP}\right)$. The mean and standard deviation of the investment rate for these U.K. manufacturing firms are both about 12%.

Denote Edy_i and $SDdy_i$ as the within-group mean and standard deviation of the sales growth rate for plant i, hence are the proxies for the growth rate μ_i and level of uncertainty σ_i for plant i. Our aggregation assumption implies plants in the same firm have the same level of growth rate and uncertainty, hence subject to the smoothness during aggregation, Edy_j and $SDdy_j$ are also informative about μ_j and σ_j. Therefore $mean\left(Edy_j\right)$ and $sd\left(Edy_j\right)$ are informative about μ_μ and σ_μ; $mean\left(SDdy_j\right)$ and $sd\left(SDdy_j\right)$ are informative about $\mu_{l\sigma}$ and $\sigma_{l\sigma}$. Figure 4a and 4b plot the empirical distribution of Edy_j and $SDdy_j$, respectively. There is a clear pattern of "normal" for Edy_j and "log-normal" for $SDdy_j$, which is consistent with our assumption about the distribution of μ_j and σ_j in Section 4.

The last three moments $mean\,(EK_j)$, $sd\,(EK_j)$, and $sd\,(EKY_j)$ capture the empirical distribution of capital stock and capital intensity. Given the critical role of these two variables in identifying the effects of uncertainty, it is important that we could match their empirical distribution. In particular, conditional on θ and τ being identified by the first two moments, adjustment costs b being identified by investment dynamics, as Lemma 8 claims, μ_{ζ_0} accounts for any effect of arbitrary choice of w, t, unit of measurement and number of plants m on the level of capital stock $mean\,(EK_j)$; σ_{ζ_0} accounts for any effect of imposing common value for $(\beta, \varepsilon, \delta, w, t)$ on the dispersion of capital stock $sd\,(EK_j)$. Measurement errors in sales $(\sigma_{YT}, \sigma_{YP})$ is a crude way to account for the effect of any unobserved heterogeneity in $(\beta, \varepsilon, \delta)$ on the dispersion of capital-to-sales ratio $sd\,(EKY_j)$. The moments we report in Table 3 is measured in the unit of £$100,000$ for capital stock. There is large dispersion in the empirical distribution for both capital stock and capital intensity.

5.4 Estimates

Table 3 presents our estimation results for the full model, imposing the number of plants within each firm to be 10. The first column in the right panel reports the estimates of the structural parameters and the second column lists the numerical standard errors of these estimates.

The estimate for θ is 0.675, positive and significantly different from zero, which is a strong indication for the empirical relevance of the risk-adjusted discount rate effect. The estimated τ is about 0.5 and significantly below 1. Together with the estimated demand elasticity ε being significantly greater than 2, this implies strong empirical relevance for a positive HAC effect.

The estimates for all three forms of capital adjustment costs are found to be significantly different from zero. In particular, $\widehat{b_q} = 0.319$ implies a quadratic adjustment cost, which is about 0.12% of the total sales, evaluated with an investment rate and capital-to-sales ratio at the sample average. $\widehat{b_i} = 0.284$ implies that resale of a capital goods would incur a sell-loss, which is about 28% of its original purchase price. $\widehat{b_f} = 0.070$ implies any investment or disinvestment would result in a

7% loss of operating profit, which is about 1.86% of the total sales, evaluated with a profit-to-sales ratio at the sample average.

The estimated $\widehat{\beta} = 0.127$, a capital share in the production function that is in line with most empirical research for estimating production function for manufacturing firms. The estimate for the demand elasticity with respect to price is $\widehat{\varepsilon} = 6.387$, which implies a mark-up coefficient $\frac{\widehat{\varepsilon}}{\widehat{\varepsilon}-1} = 1.186$. These two estimates together determine the estimate for the capital coefficient in the operating profit $1 - \widehat{\gamma} = 0.407$, an indication of strong concavity.

The estimated serial correlation parameter for the stochastic process is 0.931 but significantly different from 1, which implies the effect of the shocks are very persistent but not permanent. The estimates for the mean and standard deviation of the growth rate are 0.017 and 0.044, respectively, and are both significantly different from zero. This implies on average the demand/productivity grows at 1.7% per year, meanwhile there is large heterogeneity in the growth rate across firms in this sample. The estimated mean and standard deviation for the log of σ can be transformed into mean and standard deviation for σ itself, the measure of overall uncertainty in our model, which are 0.219 and 0.233, respectively. Since a value of σ of 0.2 has been considered as "typical" for simulation purposes in many theoretical research, for example, Pindyck (1988), our estimates confirm this typical choice but also highlight the existence of large heterogeneity in the level of uncertainty across firms.

Evaluation for the estimates of μ_{ζ_0} and σ_{ζ_0} depends on how well the first two moments of capital stock are matched. In particular, $\widehat{\sigma}_{\zeta_0}$ reflects heterogeneity across firms in their average capital stock during our sample period, and accounts for some of the persistence in firm-size differences. Three out of four types of measurement errors under our consideration are significant. In particular, measurement errors in both investment rate and sales have a large permanent component. Besides the pure recording errors in the firm-level data, this also implies the existence of other "unobserved heterogeneity" across the firms that we have not modelled "structurally".

The left panel of Table 3 lists both empirical moments and the simulated moments generated from the investment model when the above structural estimates are utilized.

The first two simulated moments imply both the expected capital stock and expected capital intensity are negatively correlated with the measure of uncertainty in our simulated data, as they are in the empirical data. Hence our investment model generates the right prediction about the overall sign of the effects of uncertainty on capital accumulation. Compared with the value of empirical moments, the predicted magnitude is slightly low for the capital stock and relatively too high for the capital intensity. This could be the result that we have oversimplified the technology on the side of variable input, or we have not fully accounted for those heterogeneities that may affect the capital-to-sales ratio.

Among the rest set of the moments, the simulated mean of sales growth rate is relatively low compared with the empirical mean, while all the other moments are very well matched. Overall, the overidentifying restriction test statistics is 165 with one degree of freedom under this specification.

5.5 Comparison with the Literature

Since this is the first paper that offers structural estimates of the discount rate effect, the HAC effect and heterogeneities in the stochastic process, there is no existing literature that we can use to compare our findings. However, the pioneering work of Cooper and Haltiwanger (2006) and Bloom (2007) provide the possibility to compare our estimates on capital adjustment costs.

Table 4 lists the estimates on capital adjustment costs from each paper, together with assumed number of plants aggregated within each firm, and common moments reported in our paper and in either of their papers. Across these research, Cooper and Haltiwanger (2006) estimates a large fixed adjustment costs while Bloom (2007) gets a large value for the partial irreversibility parameter. In contrast, our estimates for these two non-convex adjustment costs are somewhere in between. For quadratic adjustment costs, the estimates in Bloom (2007) depend on whether labour adjustment costs have been included, whether aggregation has been taken over time and the number of plants for cross-sectional aggregation. Compared with empirical research inferring quadratic adjustment costs from the "Q-model", for example, Hayashi (1982), the structural estimates from these three papers are indeed very close to each other and significantly lower than those traditional findings.

5.6 Specification Tests

Table 5 reports specification tests for several restricted models, where our preferred full model is listed in the first column as benchmark.

Column (2) lists the result by imposing $\theta = 0$, i.e. imposing no discount rate effect. Compared with our preferred full model, the overall fit deteriorates a lot, mainly due to the restricted model cannot fit the negative correlation between capital stock, capital intensity and measure of uncertainty. This restriction also leads to a higher estimate for the HAC effect ($1 - \widehat{\tau}$ decreases) and a higher estimate for the quadratic and fixed adjustment costs.

Column (3) lists the result by imposing $\tau = 1$, i.e. imposing no HAC effect. Compared with the benchmark model, this restricted model generates a lower estimate for the non-convex adjustment costs and makes the simulated correlation between capital intensity and measure of uncertainty further away from its empirical value.

Column (4) lists the result by imposing $b = 0$, i.e. imposing no capital adjustment costs. Using the first column results as benchmark, this restricted model estimates a higher discount rate effect and a lower HAC effect. Furthermore, given the investment dynamics would be very volatile in the absence of adjustment costs, the estimated stochastic process has to be much more stable to match the dampened investment behaviour in the data.

Column (5) lists the result by imposing $\sigma_\mu = 0$, i.e. imposing no heterogeneity in the growth rate. As we may expect, under this restriction, the model first, cannot fit the large dispersion of the growth rate in the empirical data and second, estimates a higher quadratic and fixed adjustment costs. This highlights the importance of allowing for heterogeneity in growth rate in getting consistent estimates for adjustment costs.

Column (6) lists the result by imposing $\sigma_{IT} = \sigma_{IP} = \sigma_{YT} = \sigma_{YP} = 0$, i.e. imposing no measurement errors in investment rate and sales. Given the full model has estimated a large permanent component and a small transitory component of measurement errors in both variables, this restricted model mainly tests the effect of not allowing for permanent measurement errors. Not surprising, this restricted specification is massively rejected, mainly because the simulated correlation between both capital stock and capital intensity and measure of uncertainty are too large compared with their empirical counterparts, besides the model cannot fit the large dispersion in the empirical capital-to-sales ratio. Since allowing for permanent measurement errors is one way to generate persistent differences in the data, these results highlight the importance of taking into account "unobserved heterogeneity" in firm-level data: even modelling it in a crude way is much better than ignoring it at all.

Table 6 lists the results for estimating the same full model but imposing the number of plants within each firm to be $1, 5, 10$ and 15, respectively. As we may expect, since aggregation is one of the important sources of smoothing, assuming fewer number of plants for aggregation results in a higher estimate for b_q, lower estimates for b_i and b_f, and lower estimates for all parameters measuring dispersion σ_μ, $\sigma_{l\sigma}$ and σ_{ζ_0}. The estimate for the mean of the intercept μ_{ζ_0} increases, since with fewer number of plants for aggregation, each plant has to be "larger" to match the empirical mean of the capital stock. According to the overidentifying restriction test statistics, a model imposing $m = 5$ or $m = 10$ would be preferred to those imposing $m = 1$ or $m = 15$. The fact that we allow for $m = 10$ plants to take into account the effect of aggregation while Bloom (2007) allows for $m = 250$, is partly due to larger firm size in the sample used by Bloom (2007), but mainly because the shocks in our model are idiosyncratic, while the shocks in Bloom(2007) have both idiosyncratic and aggregate components. Finally, comparing the two columns for $m = 5$ and $m = 10$, we find the estimates for θ and τ are very stable, which implies the robustness of the discount rate effect and the HAC effect within our preferred specifications that take into account the effect of aggregation.

5.7 Robustness Tests

Table 7 presents results for two robustness tests.

Column (1) is our preferred full model, imposing risk-free interest rate $\overline{r} = 0.065$ and using a set of 19 moments discussed in Section 5.3. Column (2) lists the results for the same full model, using the same set of 19 moments, but imposing $\overline{r} = 0.04$. As we see, the estimated $\widehat{\theta}$ increases. This implies given the linear specification for the discount rate scheme, imposing a lower level of the intercept has to be compensated by a higher level of the slope, so as to match the simulated moments with the empirical moments. In spite of the differences in the estimates by imposing different values for $\overline{r}$, the results in Column (2) also indicate the empirical importance of all three effects: the discount rate effect, the HAC effect and the adjustment costs effect. Hence using different values for the risk-free interest rate may lead to quantitative differences in the results, but not qualitative.

Column (3) lists the results for the same full model, imposing $\overline{r} = 0.065$, but using a larger set of 36 moments. These 36 moments are chosen from a statistic point of view. We have 2 basic variables for which we simulate firm-year values (K and Y), from which we can also derive 2 growth rates (i and dy) and 1 ratio (K/Y). This gives us 5 variables for which we can compute means, standard deviations, serial correlation coefficients, and cross-correlation coefficients, giving 25 moments. We also compute the within-group standard deviation of sales growth, which is time-invariant but varies across firms. We can use the between-group mean and standard deviation, and the correlation coefficients with the within-group average levels of the 5 variables above, giving a further 7 moments. We also use the mean of profit-to-sales ratio. Finally we use the three proportions of disinvestment, investment inaction and investment spikes. Together this suggests a set of 25+7+1+3=36 moments. Comparing Column (3) with Column (1), the estimates for some of the structural parameters do vary, as a result to match the additional 17 moments. However, the estimates for θ, τ, and b still highlight the empirical importance for each of the channels: the discount rate effect, the HAC effect and the adjustment costs effect.

6 Counterfactual Simulations

Definition 4 ***Aggregate Capital Intensity*** $sum[\widehat{K}_t(\sigma)]/sum[Y_t(\sigma)]$ *is the ratio of the aggregated optimal productive capital stock to the aggregated sales for a given sample in period t.*

Although the expected capital intensity is a key variable for identification in firm-level data, it is the aggregate capital intensity that is widely calculated and reported in the studies of economic growth and economic development.

Figures 5a and Figure 5b illustrate how the level of uncertainty affects expected capital stock and aggregate capital intensity in our simulated data, using the estimated parameter values reported in Table 3 and the same random numbers employed during estimation. We take the estimated mean of σ as our reference level of uncertainty, i.e. $\sigma = 0.22$, keeping $\sigma_{l\sigma}$ constant and decreasing $\mu_{l\sigma}$ gradually so that the average level of uncertainty decreases from 0.22 to 0.11. The average capital stock and aggregate capital intensity levels are both scaled by the levels in the simulation using the reference level of uncertainty, so that the values on the vertical axis can be read as percentage changes in the average capital stock and aggregate capital intensity as we reduce the level of uncertainty below this reference value. As we read from these two figures, all else being equal, a permanent reduction in the average level of uncertainty by 50% is estimated to increase average capital stock levels by about 25% and to increase aggregate capital intensity by about 13%. This implies the overall effects of uncertainty on capital stock and capital intensity are both negative, and the magnitude tends to be substantial.

The second step is to estimate the relative importance of the discount rate effect, the HAC effect, and the capital adjustment costs effect in generating the overall effects of uncertainty. This is done by two nested control experiments.

In Figure 6a and 6b, we impose the discount rate at the estimated sample average, i.e. $r = \overline{r} + \widehat{\theta} \cdot mean\left(\widehat{\sigma}\right) = 0.065 + 0.675 \times 0.219 = 0.213$ when we vary the level of uncertainty, so that the effects illustrated here are only due to the HAC effect and capital adjustment costs effect. According to these two figures, if the discount rate did not change with the level of uncertainty, and if the level of uncertainty was halved permanently, the average capital stock would decrease about 5% and the aggregate capital intensity would decrease about 3%. Together with the estimates in Figure 5a and 5b, this implies the elasticity of capital stock and capital intensity with respect to the level of uncertainty are -0.6 and -0.32, respectively, through the channel of discount rate effect.

In Figure 7a and 7b, besides imposing the discount rate at $r = 0.213$, we also impose $\tau = 1$, when we vary the level of uncertainty, so that the effects illustrated here are only due to capital adjustment costs. According to these two figures, if the discount rate and the HAC effect did not change with the level of uncertainty, and if the level of uncertainty was halved permanently, the average capital stock would increase about 5%. Together with the estimates in Figure 6a, this implies the elasticity of capital stock with respect to the level of uncertainty are +0.2, through the channel of HAC effect. In contrast, there is very little change in the aggregate capital intensity, comparing Figure 7b with Figure 6b. This is because the HAC effect only affects capital stock but not capital intensity, as we proved in Proposition 2.

Since Figure 7a and 7b illustrate the net effect of uncertainty through the channel of capital adjustment costs, this implies through this channel, the elasticity of capital stock and capital intensity with respect to the level of uncertainty are -0.1 and +0.06, respectively.

Therefore comparison between these nested experiments highlights the importance of the discount rate effect in determining the effects of uncertainty on capital accumulation.

Finally, to check whether the findings in Figure 5a and 5b are robust to the choice of the number of plants, the risk-free interest rate, and the set of moments to match, in Figure 8a and 8b, 9a and 9b, and 10a and 10b, we implement the counterfactual simulations based on the estimates in Column (2) of Table 6, Column (2) and (3) of Table 7, respectively. As we see, all these figures also present a negative effect of uncertainty on both expected capital stock and aggregate capital intensity. Therefore, in spite of some quantitative differences, the effects of uncertainty on capital accumulation we find in Figure 5a and Figure 5b are qualitatively robust.

7 Conclusions

This paper provides a structural framework to estimate the effects of uncertainty on capital accumulation at the firm level. Our investment model allows for uncertainty to affect capital accumulation through three possible channels that have been highlighted by the uncertainty-investment literature. The sign and magnitude of each of these mechanisms are illustrated by counterfactual simulations, based on a set of optimal estimates for the structural parameters of the model, using the simulated method of moments.

The findings of this paper include that, first, there is significant empirical evidence of both a risk-adjusted discount rate effect and a HAC effect from our structural estimation. Second, both convex and non-convex adjustment costs are necessary in modelling firm-level investment. Third, the estimated model suggests that a moderate, positive HAC effect dominates a small, negative adjustment costs effect, leaving a big, negative effect of uncertainty on both average capital stock levels and aggregate capital intensity through a risk-adjusted discount rate effect.

The robustness of the last finding is subject to the following considerations.

First of all, among all the 19 moments, the match for the correlation between capital intensity and our measure of uncertainty is relatively poor. This might be due to the assumption of costless adjustment of variable input, or perhaps more importantly, because our investment model has not fully accounted for other unobserved heterogeneities that affect capital intensity. The robustness of our finding about the effects of uncertainty on capital intensity requires the assumption that these unobserved heterogeneities are uncorrelated with the level of uncertainty.

Second, like most of the uncertainty-investment literature, the investment model we set up in this paper is explicitly partial equilibrium in nature. This allows us to model a very rich structure in the stochastic process for the purpose of identifying the HAC effect and capital adjustment costs, while the cost is that we have simplified the structure of the risk-adjusted discount rate effect. Our empirical finding implies that this effect could be substantial. Given that firms in Datastream are large publicly-traded firms in the UK stock market, this raises the interesting question of why the risk-adjusted discount rate effect that we have modelled appears to be so strong, and whether this finding is robust if the effects of uncertainty on risk premium are modelled and estimated in a consumption-CAPM framework.

Finally, in a general equilibrium framework, the analysis of uncertainty on capital accumulation should focus not only on the effects of stochastic process, but also on technology and relative factor prices. This is particularly important, since in the presence of capital adjustment costs, the optimal response of the firms is to substitute away from capital towards variable input, which implies technology adoption and relative prices are both potentially endogenous.

All these analyses are beyond the scope of this paper, while we hope this paper is the first step towards interesting and more challenging future research.

Appendices

1 Proof for Lemmas and Propositions

1.1 Proof for Lemma 1

The short-run profit maximization problem is

$$\begin{aligned}
\pi(X_t, A_t, \widehat{K}_t) &= \max_{L_t} P_t Q_t - wL_t \\
&= \max_{L_t} X_t^{\frac{1}{\varepsilon}} Q_t^{\frac{\varepsilon-1}{\varepsilon}} - wL_t \\
&= \max_{L_t} X_t^{\frac{1}{\varepsilon}} A_t^{\frac{\varepsilon-1}{\varepsilon}} L_t^{\frac{(\varepsilon-1)(1-\beta)}{\varepsilon}} \widehat{K}_t^{\frac{(\varepsilon-1)\beta}{\varepsilon}} - wL_t
\end{aligned}$$

First-order condition leads to

$$\begin{aligned}
L_t &= \left(\frac{\gamma\varepsilon - 1}{w\gamma\varepsilon}\right)^{\gamma\varepsilon} X_t^{\gamma} \left(A_t^{\gamma}\right)^{\varepsilon-1} \widehat{K}_t^{1-\gamma} \\
\pi_t &= \frac{wL_t}{\gamma\varepsilon - 1} = const0 \cdot X_t^{\gamma} \left(A_t^{\gamma}\right)^{\varepsilon-1} \widehat{K}_t^{1-\gamma} \\
Y_t &= P_t Q_t = \frac{\gamma\varepsilon w L_t}{\gamma\varepsilon - 1} = \gamma\varepsilon\pi_t
\end{aligned}$$

where γ and $const0$ are given by equation (7) and (8).

1.2 Proof for Lemma 2

Taking the log on both sides of equation (4) leads to

$$\begin{aligned}
\log \pi_t &= \log const0 + \gamma \log X_t + \gamma\left(\varepsilon - 1\right) \log A_t + \left(1-\gamma\right) \log \widehat{K}_t \quad \text{(A1)} \\
&= \log const0 + \gamma\left[\log X_t + \left(\varepsilon - 1\right) \log A_t\right] + \left(1-\gamma\right) \log \widehat{K}_t
\end{aligned}$$

Substituting (9) and (10) into (A1), its stochastic part can be written as

$$\begin{aligned}
&\gamma\left\{\left[c_x + \mu_x t + \zeta_t^x\right] + \left(\varepsilon - 1\right)\left[c_a + \mu_a t + \zeta_t^a\right]\right\} \\
= \ &\gamma \left\{ \begin{array}{c} \left[\zeta_0^x + \left(\varepsilon - 1\right)\zeta_0^a\right] + \left[c_x + \left(\varepsilon - 1\right) c_a\right] + \left[\mu_x + \left(\varepsilon - 1\right)\mu_a\right] t \\ + \left[\sum_{s=0}^{t-1} \rho_x^i e_{t-s}^x + \left(\varepsilon - 1\right) \sum_{s=0}^{t-1} \rho_a^i e_{t-s}^a\right] \end{array} \right\}
\end{aligned}$$

By imposing $\rho_x = \rho_a = \rho$, since e_t^x and e_t^a are independently normally distributed, a linear combination of these two random variables is still normally distributed with proper mean and variance, i.e.

$$\sum_{s=0}^{t-1}\rho_x^i e_{t-s}^x + (\varepsilon-1)\sum_{s=0}^{t-1}\rho_a^i e_{t-s}^a = \sum_{s=0}^{t-1}\rho^i\left[e_{t-s}^x + (\varepsilon-1)\,e_{t-s}^a\right] = \sum_{s=0}^{t-1}\rho^i e_{t-s}$$

where $e_t \overset{i.i.d}{\sim} N\left(0,\, \sigma_x^2 + (\varepsilon-1)^2\,\sigma_a^2\right)$.

This implies that the log of operating profit (A1) can be rewritten as

$$\log \pi_t = \log const0 + \gamma \log Z_t + (1-\gamma)\log \widehat{K}_t$$

or equivalently the operating profit as

$$\pi(Z_t, \widehat{K}_t) = const0 \cdot Z_t^{\gamma}\widehat{K}_t^{1-\gamma}$$

The law of motion for the combined random variable Z_t is given by (12), with parameters in this stochastic process defined in (13).

1.3 Proof for Lemma 3

In the frictionless case $G(Z_t, K_t; I_t) = 0$, so that the Bellman equation is

$$V(Z_t, K_t) = \max_{I_t}\{const0 \cdot Z_t^{\gamma}\,(K_t + I_t)^{1-\gamma} - I_t + \frac{1}{1+r}E_t\left[V(Z_{t+1}, K_{t+1})\right]\}$$

with the law of motion for capital stock

$$K_{t+1} = (1-\delta)\,(K_t + I_t)$$

Taking derivative with respect to K_t on both sides of the Bellman equation, we get the quantity known as marginal q

$$\frac{\partial V_t}{\partial K_t} = 1 \tag{A2}$$

which is a constant due to our timing assumption.

The first order condition with respect to K_{t+1} is

$$\frac{\partial V_t}{\partial K_{t+1}} = const0 \cdot \frac{1-\gamma}{1-\delta}\left[\frac{Z_t}{K_t + I_t}\right]^{\gamma} - \frac{1}{1-\delta} + \frac{1}{1+r}E_t\left[\frac{\partial V_{t+1}}{\partial K_{t+1}}\right] = 0 \tag{A3}$$

Replacing $\frac{\partial V_{t+1}}{\partial K_{t+1}}$ in (A3) with (A2) leads to the Euler equation

$$\left[\frac{Z_t/K_t}{1 + I_t/K_t}\right]^{\gamma} = \frac{1 - \frac{1-\delta}{1+r}}{const0 \cdot (1-\gamma)}$$

Rearranging this equation gives the optimal investment rate in equation (21) and optimal capital stock in equation (22).

1.4 Proof for Lemma 4

For capital stock, taking expectation on both sides of equation (22) directly gives

$$E\left[\widehat{K}_t^*(\sigma)\right] = const1 \cdot E[Z_t]$$

For capital intensity,

$$\begin{aligned}
\frac{\widehat{K}_t^*}{Y_t} &= \frac{\widehat{K}_t^*}{\gamma\varepsilon \cdot const0 \cdot Z_t^{\gamma}\widehat{K}_t^{*1-\gamma}} \\
&= \frac{1}{\gamma\varepsilon \cdot const0}\left(\frac{\widehat{K}_t^*}{Z_t}\right)^{\gamma} \\
&= \frac{1}{\gamma\varepsilon \cdot const0}(const1)^{\gamma} \\
&= \frac{(1-\gamma)(1+r)}{\gamma\varepsilon(r+\delta)} \\
&= \frac{\beta(\varepsilon-1)(1+r)}{\varepsilon(r+\delta)}
\end{aligned}$$

which is $const2$ given in equation (24).

1.5 Proof for Lemma 6

According to equation (9)

$$\begin{aligned}
E[x_t] &= E[c_x + \mu_x t + \zeta_t^x] \\
&= E[c_x + \mu_x t + \zeta_0^x + \sum_{s=0}^{t-1}\rho_x^i e_{t-s}^x] \\
&= \zeta_0^x + c_x + \mu_x t \\
V(x_t) &= V[c_x + \mu_x t + \zeta_t^x] \\
&= V[\sum_{s=0}^{t-1}\rho_x^i e_{t-s}^x] \\
&= \sigma_x^2/\left(1-\rho^2\right) \text{ when } 0<\rho<1 \text{ and } t\to\infty
\end{aligned}$$

Then

$$\begin{aligned}
E[X_t] &= \exp\left[E(x_t) + 0.5V(x_t)\right] \\
&= \exp\left[\zeta_0^x + c_x + \mu_x t + 0.5\sigma_x^2/\left(1-\rho^2\right)\right] \\
&= \exp(\zeta_0^x + \mu_x t) \text{ if } c_x = -0.5\sigma_x^2/\left(1-\rho^2\right)
\end{aligned}$$

Similarly, we have $E[A_t] = \exp(\zeta_0^a + \mu_a t)$, if $c_a = -0.5\sigma_a^2/(1-\rho^2)$.

1.6 Proof for Lemma 7

Given $c = c_x + (\varepsilon - 1)\, c_a$, $c_x = -0.5\sigma_x^2/\left(1-\rho^2\right)$, $c_a = -0.5\sigma_a^2/\left(1-\rho^2\right)$ and $\sigma^2 = \sigma_x^2 + (\varepsilon - 1)^2 \sigma_a^2$, we can derive that

$$\begin{aligned} E\left[Z_t\right] &= \exp\left[E\left(z_t\right) + 0.5V\left(z_t\right)\right] \\ &= \exp\left[\zeta_0 + c + \mu t + 0.5\sigma^2/\left(1-\rho^2\right)\right] \\ &= \exp\left[\zeta_0 + \mu t + \left(c_x + \frac{\sigma_x^2}{2\left(1-\rho^2\right)}\right) + (\varepsilon-1)\left(c_a + \frac{(\varepsilon-1)\,\sigma_a^2}{2\left(1-\rho^2\right)}\right)\right] \\ &= \exp\left[\zeta_0 + \mu t + \frac{(\varepsilon-1)(\varepsilon-2)}{2\left(1-\rho^2\right)}\sigma_a^2\right] \\ &= \exp\left[\zeta_0 + \mu t + \frac{(\varepsilon-2)(1-\tau)}{2(\varepsilon-1)\left(1-\rho^2\right)}\sigma^2\right] \end{aligned}$$

1.7 Proof for Lemma 8 and 9

It is well-known that for any random variable $x_i \overset{i.i.d}{\sim} N\left(\mu, \sigma^2\right)$, if $X_i = \exp\left(x_i\right)$, the first moment for X_i is

$$\begin{aligned} E\left[X_i\right] &= \exp\left(E\left[x_i\right] + 0.5V\left[x_i\right]\right) \\ &= \exp\left(\mu + 0.5\sigma^2\right) \end{aligned} \tag{A4}$$

And the second moment is

$$\begin{aligned} V\left[X_i\right] &= E\left[X_i^2\right] - \left(E\left[X_i\right]\right)^2 \\ &= E\left[\exp\left(2x_i\right)\right] - \left[\exp\left(\mu + 0.5\sigma^2\right)\right]^2 \\ &= \exp\left[2\mu + 0.5\cdot(2\sigma)^2\right] - \exp\left(2\mu + \sigma^2\right) \\ &= \exp\left(2\mu + 2\sigma^2\right) - \exp\left(2\mu + \sigma^2\right) \\ &= \left[\exp\left(2\mu + \sigma^2\right)\right]\left[\exp\left(\sigma^2\right) - 1\right] \end{aligned}$$

or equivalently, the standard deviation is given be

$$sd\left[X_i\right] = \exp\left(\mu + 0.5\sigma^2\right)\sqrt{\left[\exp\left(\sigma^2\right) - 1\right]} \tag{A5}$$

Assume $\theta = 0$ (no discount rate effect) and $\tau = 1$ (no HAC effect) for now. Denote operation taking over firm and time with subscript j and t, respectively. Suppose the m plants within the same firms have similar size. Then the within-group capital stock for firm j is

$$E\widehat{K}_j = E_t\left[\widehat{K}_{j,t}\right] = E_t\left[\sum\nolimits_{i=1}^{m} \widehat{K}_{i,j,t}\right] \simeq E_t\left[m\cdot\widehat{K}_{i,j,t}\right] = mE_t\left[\widehat{K}_{i,j,t}\right]$$

In the absence of adjustment costs,

$$
\begin{aligned}
E_j\left[E\widehat{K}_j\right] &= E_j\left[mE_t\left[\widehat{K}_{i,j,t}\right]\right] = m \cdot const1 \cdot E_j\left[E_t\left[Z_{i,j,t}\right]\right] \\
sd_j\left[E\widehat{K}_j\right] &= sd_j\left[mE_t\left[\widehat{K}_{i,j,t}\right]\right] = m \cdot const1 \cdot sd_j\left[E_t\left[Z_{i,j,t}\right]\right]
\end{aligned}
$$

The law of motion for $Z_{i,j,t}$ is given by equation (26), by which we get

$$
\begin{aligned}
E_t[z_{i,j,t}] &= c_j + \zeta_{0j} + \mu t \\
Var_t[z_{i,j,t}] &= \frac{\sigma_j^2}{1-\rho^2}
\end{aligned}
$$

Using the result in (A4)

$$
\begin{aligned}
E_t\left[Z_{i,j,t}\right] &= \exp\left(E_t[z_{i,j,t}] + 0.5Var_t[z_{i,j,t}]\right) \\
&= \exp\left(c_j + \zeta_{0j} + \mu t + 0.5\sigma_j^2/\left(1-\rho^2\right)\right) \\
&= \exp\left(\zeta_{0j} + \mu t\right) \\
&= \exp\left(\mu t\right)\exp\left(\zeta_{0j}\right)
\end{aligned}
$$

Calculating the between-group mean and standard deviation leads to

$$
\begin{aligned}
E_j\left[E_t\left[Z_{i,j,t}\right]\right] &= \exp\left(\mu t\right)E_j\left[\exp\left(\zeta_{0j}\right)\right] \\
sd_j\left[E_t\left[Z_{i,j,t}\right]\right] &= \exp\left(\mu t\right)sd_j\left[\exp\left(\zeta_{0j}\right)\right]
\end{aligned}
$$

Since $\zeta_{0j} \overset{i.i.d}{\sim} N\left(\mu_{\zeta_0}, \sigma_{\zeta_0}\right)$, applying the the result in (A4) and (A5), we have

$$
\begin{aligned}
E_j\left[\exp\left(\zeta_{0j}\right)\right] &= \exp\left(\mu_{\zeta_0} + 0.5\sigma_{\zeta_0}^2\right) \\
sd_j\left[\exp\left(\zeta_{0j}\right)\right] &= \exp\left(\mu_{\zeta_0} + 0.5\sigma_{\zeta_0}^2\right)\sqrt{\left[\exp\left(\sigma_{\zeta_0}^2\right) - 1\right]}
\end{aligned}
$$

Suppose the capital stock is recorded in £ and we normalise it with £M. Empirically, we calculate the first two moments of $E\widehat{K}_j$ being 0.067 and 0.290. This implies conditional on θ, τ, and b being identified by other moments, the identification for μ_{ζ_0} and $\sigma_{\zeta_0}^2$ can be achieved by solving the simultaneous equations

$$
\Gamma \cdot \exp\left(\mu_{\zeta_0} + 0.5\sigma_{\zeta_0}^2\right) = 0.067 \tag{A6}
$$

$$
\Gamma \cdot \exp\left(\mu_{\zeta_0} + 0.5\sigma_{\zeta_0}^2\right)\sqrt{\left[\exp\left(\sigma_{\zeta_0}^2\right) - 1\right]} = 0.290 \tag{A7}
$$

where $\Gamma = m \cdot const1 \cdot \exp\left(\mu t\right)/M$, and $const1$ is a function of $(\beta, \varepsilon, \delta, w, r)$.

Divide equation (A7) by (A6), we get the solution to σ_{ζ_0}, which is independent of Γ and only depends on the coefficient of variation of the capital stock. Substitute this solution into (A6), μ_{ζ_0} can be solved out for any Γ. Hence given $(\mu, \beta, \varepsilon, \delta, r)$ are pinned down by other moments, the effect of any arbitrary choice of w, t, the unit of measurement M, and the number of plants m, on the level of expected capital stock can be accounted for by adjusting μ_{ζ_0}.

2 Numerical Methods

2.1 Solving Dynamic Programming (19) Numerically

The problem we need to solve is

$$\begin{aligned}
V(Z_t, K_t) &= \max_{I_t}\{\Pi(Z_t, K_t; I_t) + \frac{1}{1+r} E_t\left[V(Z_{t+1}, K_{t+1})\right]\} \\
s.t.\ K_{t+1} &= (1-\delta)(K_t + I_t) \\
Z_t &= \exp(z_t) \\
z_t &= c + \mu t + \zeta_t \\
\zeta_t &= \rho\zeta_{t-1} + e_t = \zeta_0 + \sum_{s=0}^{t-1} \rho^s e_{t-s} \text{ where } e_t \overset{i.i.d}{\sim} N\left(0,\, \sigma^2\right)
\end{aligned}$$

According to Lemma 3, in the absence of adjustment costs,

$$K_{t+1}^* = (1-\delta) \cdot \widehat{K}_t^* = (1-\delta) \cdot const1 \cdot Z_t$$

Furthermore, in the absence of uncertainty,

$$\begin{aligned}
Z_t &= \exp(c + \zeta_0 + \mu t) \\
K_{t+1}^{**} &= K_t^{**} \exp(\mu)
\end{aligned}$$

Together, in the absence of both adjustment costs and uncertainty, the non-stochastic frictionless capital stock would be

$$K_t^{**} = K_{t+1}^{**} / \exp(\mu) = const3 \cdot Z_t \tag{B1}$$

where

$$const3 = (1-\delta) \cdot const1 / \exp(\mu)$$

This means the exogenous state variable Z_t is a trend stationary process; the state variable K_t endogenously follows Z_t hence is also a trend stationary process. For any $\mu > 0$, (19) a dynamic programming with unbounded return function and there is in general no theory about the existence and uniqueness of the solution.

However, notice due to our choice of the functional forms, the net revenue $\Pi(Z_t, K_t; I_t)$ is linear homogeneous in $(Z_t, K_t; I_t)$. If there was a solution $I_t = h(Z_t, K_t)$, the linear homogeneity of Π implies the linear homogeneity of V in $(Z_t, K_t; I_t)$, while the linear homogeneity of V in $(Z_t, K_t; I_t)$ guarantees the linear homogeneity of I_t in (Z_t, K_t), by the property of constant return to scale problems (Theorem 4.13, Stokey and Lucas, 1989).

Define $\Psi_t \equiv \exp(\zeta_0 + \mu t)$. Denote $\widetilde{Z}_t = \frac{Z_t}{\Psi_t}$, $\widetilde{K}_t = \frac{K_t}{\Psi_t}$, $\widetilde{I}_t = \frac{I_t}{\Psi_t}$, $\widetilde{Z}_{t+1} = \frac{Z_{t+1}}{\Psi_{t+1}}$, and $\widetilde{K}_{t+1} = \frac{K_{t+1}}{\Psi_{t+1}}$.

Then due to the linear homogeneity of V, we have

$$\begin{aligned}
V(\widetilde{Z}_t, \widetilde{K}_t) &= V(\frac{Z_t}{\Psi_t}, \frac{K_t}{\Psi_t}) \\
&= \frac{1}{\Psi_t} V(Z_t, K_t) \\
&= \max_{I_t/\Psi_t} \{ \frac{1}{\Psi_t} \Pi(Z_t, K_t; I_t) + \frac{\exp(\mu)}{1+r} \frac{1}{\Psi_{t+1}} E_t [V(Z_{t+1}, K_{t+1})] \} \\
&= \max_{I_t/\Psi_t} \{ \Pi \left[\frac{Z_t}{\Psi_t}, \frac{K_t}{\Psi_t}; h\left(\frac{Z_t}{\Psi_t}, \frac{K_t}{\Psi_t} \right) \right] + \frac{\exp(\mu)}{1+r} E_t \left[V(\frac{Z_{t+1}}{\Psi_{t+1}}, \frac{K_{t+1}}{\Psi_{t+1}}) \right] \} \\
&= \max_{I_t/\Psi_t} \{ \Pi(\widetilde{Z}_t, \widetilde{K}_t; \widetilde{I}_t) + \frac{\exp(\mu)}{1+r} E_t \left[V(\widetilde{Z}_{t+1}, \widetilde{K}_{t+1}) \right] \}
\end{aligned}$$

where we have used the fact that $\Psi_{t+1}/\Psi_t = \exp(\mu)$. Hence, we have normalised the dynamic programming (19) into the following problem

$$\begin{aligned}
V(\widetilde{Z}_t, \widetilde{K}_t) &= \max_{I_t/\Psi_t} \{ \Pi(\widetilde{Z}_t, \widetilde{K}_t; \widetilde{I}_t) + \frac{\exp(\mu)}{1+r} E_t \left[V(\widetilde{Z}_{t+1}, \widetilde{K}_{t+1}) \right] \} && \text{(B2)} \\
s.t.\ \widetilde{K}_{t+1} &= \exp(-\mu)(1-\delta)\left(\widetilde{I}_t + \widetilde{K}_t \right) && \text{(B3)} \\
\widetilde{Z}_t &= \exp\left(c + \widetilde{\zeta}_t \right) && \text{(B4)} \\
\widetilde{\zeta}_t &= \rho \widetilde{\zeta}_{t-1} + e_t = \sum_{s=0}^{t-1} \rho^s e_{t-s} \text{ where } e_t \overset{i.i.d}{\sim} N\left(0, \sigma^2\right)
\end{aligned}$$

where $\widetilde{Z}_t$ and $\widetilde{K}_t$ are the two state variables, and $\widetilde{I}_t$ is the control variable, which are all stationary. The investment rate $I_t/K_t = \widetilde{I}_t/\widetilde{K}_t$, which is convenient. The effective discount factor is now $\frac{\exp(\mu)}{1+r}$.

Since conditional expectations need to be formed based on $\widetilde{Z}_t$, we use the approximation method in Tauchen (1986) to discretise the continuous $AR(1)$

process for $\widetilde{\zeta}_t$ into a 9-state Markov process for given parameters ρ and σ. Then we get $\widetilde{Z}_t(i)$ by multiplying $\exp(\widetilde{\zeta}_t(i))$ with the constant $\exp(c)$, where $i = 1, 2, \cdots 9$.

Since in the absence of both adjustment costs and uncertainty, the non-stochastic frictionless capital stock is given by (B1), we define the support of $\widetilde{K_t}$ as

$$\exp\left[\log\left(const3 \cdot \widetilde{Z}_t(1)\right) - 0.5,\ \ \log\left(const3 \cdot \widetilde{Z}_t(9)\right) + 0.5\right]$$

We then discretise this state space with 200 grid points, so that the grids for $\widetilde{K_t}$ are $\widetilde{K}_t(j)$, where $j = 1, 2, \cdots 200$.

Now $\Pi(\widetilde{Z}_t, \widetilde{K}_t; \widetilde{I}_t)$ is real valued, continuous, concave and bounded; the set $\Omega \equiv \{(\widetilde{Z}_t(i), \widetilde{K}_t(j))\}_{(i=1,2,\cdots 9; j=1,2,\cdots 200)}$ is compact and convex. As long as $0 < \frac{\exp(\mu)}{1+r} < 1$, by the Contraction Mapping Theorem (Theorem 9.8 in Stokey and Lucas, 1989), we can always get a unique investment policy $\widetilde{I}_t = h(\widetilde{Z}_t, \widetilde{K}_t)$ using value function iteration.

In practice, within each value function iteration, we adopt a policy improvement algorithm (Chapter 20, Ljungqvist and Sargent, 2000). This costs more time for each value function iteration but substantially saves overall numbers of iterations, hence in total it saves about two-third of the time compared with simple value function iteration in solving (B2). Since at the early stage of estimation, condition $0 < \frac{\exp(\mu)}{1+r} < 1$ might be violated in case of a high value of μ, we set the termination condition as either the difference in the value function between two consecutive iterations is smaller than the tolerance $1e-5$ or the number of value function iterations exceeds 100.

After getting the optimal solution $\widetilde{I}_t = h(\widetilde{Z}_t, \widetilde{K}_t)$, we interpolate so that the final state space for $\widetilde{Z}_t$ has 100 grid points, that for $\widetilde{K}_t$ has 2000 grid points, and the final policy space for $\widetilde{I}_t$ has the dimension of (100×2000).

2.2 Finite Mixture of Types

Both the level of uncertainty σ and the growth rate μ would affect the investment policy. Hence the dynamic programming must be solved for each type of σ and μ. In other words, σ and μ are two additional state variables besides $\widetilde{Z}_t$ and $\widetilde{K}_t$. Given $\log \sigma_j \overset{i.i.d}{\sim} N\left(\mu_{l\sigma}, \sigma^2_{l\sigma}\right)$, and $\mu_j \overset{i.i.d}{\sim} N\left(\mu_{\mu}, \sigma^2_{\mu}\right)$, we discretise these two continuous distribution by Tauchen (1989) method. Due to "the curse of dimensionality", we have to be conservative about the number of grids and set them to be 3 for both of these two state variables. We experiment with higher number of grids for these two state variables and find the simulated moments are not very sensitive. The procedure of how to solve the dynamic programming can be describe as:

for $\sigma = \exp(\log\sigma_1, \log\sigma_2, \log\sigma_3)$
    for $\mu = (\mu_1, \mu_2, \mu_3)$
        for value function iteration converges
            policy improvement algorithm
        end
        interpolate for $\widetilde{I}_t = h(\widetilde{Z}_t, \widetilde{K}_t)$
    end
end

In other words, the dynamic programming (B2) is solved for each type of firms from the type set $F = \{(\sigma_u, \mu_v) : u = 1, \cdots, 3; v = 1, \cdots, 3\}$.

2.3 Simulate the Data

Under our assumption, each plant makes its own investment decision while unobserved heterogeneities and measurement errors are at the firm level. The procedure to simulate the data includes four steps.

Step 1: Simulate data for each plant i of firm j in period t. When $t = 1$, we endow all simulated plants of firm j with the initial condition $\widetilde{\zeta}_{i,j,1} = -\sqrt{\sigma_j^2/(1-\rho^2)}$ and the corresponding initial capital stock $\widetilde{K}_{i,j,1} = const3 \cdot \widetilde{Z}_{i,j,1} = const3 \cdot \exp(c + \widetilde{\zeta}_{i,j,1})$. For all subsequent periods, we randomly draw demand/productivity shocks $e_{i,j,t}(\sigma_j)$ for each plant i of firm j in period t. Given the realization of $\widetilde{Z}_{i,j,t}$ and the inherited $\widetilde{K}_{i,j,t}$, we find the optimal investment $\widetilde{I}_{i,j,t}$ using the policy rule derived above. Then $\widetilde{Z}_{i,j,t}$ or equivalently $\widetilde{\zeta}_{i,j,t}$ evolves exogenously according to (B4), $\widetilde{K}_{i,j,t}$ evolves endogenously according to (B3), operating profit $\widetilde{\pi}_{i,j,t}$ is calculated according to equation(14). Finally, the control variable in this period becomes the state variable in next period.

Step 2: Plant-level data are aggregated into firm-level data. For firm j in period t, the normalised investment data is $\widetilde{I}_{j,t} = \sum_{i=1}^{m} \widetilde{I}_{i,j,t}$; capital stock is $\widetilde{K}_{j,t} = \sum_{i=1}^{m} \widetilde{K}_{i,j,t}$; operating profit is $\widetilde{\pi}_{j,t} = \sum_{i=1}^{m} \widetilde{\pi}_{i,j,t}$.

Step 3: Recover the intercept $\exp(\zeta_{0j})$ and time trend $\exp(\mu_j t)$ in the variables of the original model. For firm j in period t, the actual investment is therefore $I_{j,t} = \widetilde{I}_{j,t}\exp(\zeta_{0j} + \mu_j t)$; the actual capital stock is $K_{j,t} = \widetilde{K}_{j,t}\exp(\zeta_{0j} + \mu_j t)$; the actual operating profit is $\pi_{j,t} = \widetilde{\pi}_{j,t}\exp(\zeta_{0j} + \mu_j t)$; the sales is $Y_{j,t} = \gamma\varepsilon \cdot \pi_{j,t}$ according to equation (6). The investment rate is $i_{j,t} = I_{j,t}/K_{j,t}$.

Step 4: Add measurement errors, so that the observed investment rate is $i_{j,t} = i^*_{j,t}\exp(e^I_{j,t})$, and the observed level of sales is $Y_{j,t} = Y^*_{j,t}\exp(e^Y_{j,t})$.

With these data, the simulated moments listed in Table 2 are calculated to match the empirical moments.

2.4 Simulated Method of Moments

The empirical sample we use is an unbalanced panel with $N = 672$ firms and on average $T = 11$ years of observation for each firm. As suggested by Michaelides and Ng (2000), for SMM, simulating H path of $(N \times T)$ is equivalent to simulate $(HN \times T)$. In addition, each firm has m plants, and we allow for 10 years to start from the ergodic distribution and discard them in calculating the moments. Therefore the simulated data panel is of size $[HNm \times (10 + T)]$ where we use $H = 10$ in our application.

We use the optimal weighting matrix given by a bootstrap estimate for the inverse of the variance-covariance matrix of the empirical moments, i.e.

$$\Omega^* = \left[Nvar\left(\widehat{\Phi}^D\right)\right]^{-1}$$

Due to the discretisation of the state spaces and the discontinuities of the investment policy in the presence of non-convex adjustment costs, we adopt a simulated annealing algorithm described in Goffe, Ferrier and Rogers (1994) to avoid local minima in solving the minimisation problem (27).

There are 19 moments and 18 structural parameters in our application so that the Jacobian matrix for the binding functions is

$$J = \frac{\partial \widehat{\Phi}^{S\prime}\left(\widehat{\Theta}\right)}{\partial \Theta} = \begin{bmatrix} \frac{\partial \widehat{\Phi}^{S\prime}(\widehat{\Theta})_1}{\partial \Theta_1} & \frac{\partial \widehat{\Phi}^{S\prime}(\widehat{\Theta})_2}{\partial \Theta_1} & \cdots & \frac{\partial \widehat{\Phi}^{S\prime}(\widehat{\Theta})_{19}}{\partial \Theta_1} \\ \frac{\partial \widehat{\Phi}^{S\prime}(\widehat{\Theta})_1}{\partial \Theta_2} & \frac{\partial \widehat{\Phi}^{S\prime}(\widehat{\Theta})_2}{\partial \Theta_2} & \cdots & \frac{\partial \widehat{\Phi}^{S\prime}(\widehat{\Theta})_{19}}{\partial \Theta_2} \\ \vdots & \vdots & \cdots & \vdots \\ \frac{\partial \widehat{\Phi}^{S\prime}(\widehat{\Theta})_1}{\partial \Theta_{18}} & \frac{\partial \widehat{\Phi}^{S\prime}(\widehat{\Theta})_2}{\partial \Theta_{18}} & \cdots & \frac{\partial \widehat{\Phi}^{S\prime}(\widehat{\Theta})_{19}}{\partial \Theta_{18}} \end{bmatrix}$$

When calculating the numerical derivatives, in order to smooth the possible wiggles in the binding function, we use a simple regression techniques as follows. Given parameter set Θ_i $(i = 1, 2, \cdots 18)$, moment set Φ_j $(j = 1, 2, \cdots, 19)$, for each parameter Θ_i, define a range of values $\left[0.8\widehat{\Theta}_i, 1.2\widehat{\Theta}_i\right]$[1] around the optimal estimate $\widehat{\Theta}_i$ and discretise this range into 20 grids $\left[\widehat{\Theta}_i^1, \widehat{\Theta}_i^2, \cdots, \widehat{\Theta}_i^{20}\right]$, meanwhile fix all other parameters at their optimal estimates $\widehat{\Theta}_{-i}$, for each moment Φ_j, we calculate its value at each $\widehat{\Theta}_i^p$ $(p = 1, 2, \cdots 20)$, which will

[1] For the serial correlation parameter, we only consider the range $[0.8\widehat{\rho}, \widehat{\rho}]$, given the estimated $\widehat{\rho}$ is above 0.9 and due to the restriction of $0 < \rho < 1$ in using Tauchen method.

produce 20 values $\widehat{\Phi}_j^p\,(p = 1, 2, \cdots 20)$. Plotting a figure of $\widehat{\Phi}_j^p$ against $\widehat{\Theta}_i^p$ illustrates the shape of the binding function. A flat line implies the moment $\widehat{\Phi}_j$ does not vary with the parameter $\widehat{\Theta}_i$; while a steep line implies $\widehat{\Phi}_j$ is informative about the variation in $\widehat{\Theta}_i$, at least at the local area of our optimal estimates. In most cases, these binding function has a linear shape, hence we run the regression using OLS, e.g.

$$\widehat{\Phi}_j^p = \alpha_{0j,i} + \alpha_{j,i}\widehat{\Theta}_i^p + \varsigma_{j,i}$$

Then the slope coefficient from the regression $\widehat{\alpha}_{j,i}$ fills the element $\frac{\partial \widehat{\Phi}^{S\prime}(\widehat{\Theta})_j}{\partial \Theta_i}$ in the Jacobian matrix J, which turns out to be indeed of full row rank. This implies that first, no column has all zeros—no redundant moment; second, no row has all zeros—all parameters have the possibility to be identified; and third, no rows are linear dependant—none of any two parameters lead to same variation in all moments.

With this Jacobian matrix, the asymptotic variance-covariance matrix of the optimal estimates is calculated according to (29), which produces the standard errors we report in Table 3.

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Table 1 Set of Parameters

Category	Symbol	Definition
Discount Rate Effect	θ	$r = \bar{r} + \theta\sigma$
HAC Effect	τ	$\sigma_x^2 = \tau\sigma^2$
Adjustment Costs	b_q	quadratic adjustment costs
	b_i	partial irreversibility
	b_f	fixed adjustment costs
Technology and Demand	β	capital share in production function
	ε	demand elasticity with respect to price
Stochastic Process	ρ	serial correlation of shocks
	μ_μ	mean of μ, where μ is the growth rate
	σ_μ	standard deviation of μ
	$\mu_{l\sigma}$	mean of log(σ), where σ measures the level of uncertainty
	$\sigma_{l\sigma}$	standard deviation of log(σ)
	μ_{ζ_0}	mean of ζ_0, where ζ_0 is the intercept
	σ_{ζ_0}	standard deviation of ζ_0
Measurement Errors	σ_{IT}	sd of transitory measurement errors in investment rates
	σ_{IP}	sd of permanent measurement errors in investment rates
	σ_{YT}	sd of transitory measurement errors in sales
	σ_{YP}	sd of permanent measurement errors in sales

Table 2 Set of Moments

Symbol	Definition	Informativeness
$corr(EK_j, SDdy_j)$	corr. btw. BG capital stock and measure of uncertainty	τ, θ, b
$corr(EKY_j, SDdy_j)$	corr. btw. BG capital intensity and uncertainty	θ, b
$prop(i_{j,t} < -0.01)$	proportion of negative investment rates	b_i
$prop(\|i_{j,t}\| < 0.01)$	proportion of zero investment rates	b_i, b_f
$prop(i_{j,t} > 0.20)$	proportion of investment spikes	b_f, b_q
$corr(i_{j,t}, yk_{j,t})$	corr. btw. investment rates and log sales-to-capital ratio	$b_q, b_i, \sigma_{IT}, \sigma_{IP}, \sigma_{YT}, \sigma_{YP}$
$corr(i_{j,t}, i_{j,t-1})$	serial correlation of investment rates	$b_q, \rho, \sigma_\mu, \sigma_{IT}, \sigma_{IP}$
$corr(yk_{j,t}, yk_{j,t-1})$	serial correlation of log sales-to-capital ratio	$\rho, \sigma_{YT}, \sigma_{YP}$
$mean(\pi_{j,t}/Y_{j,t})$	mean of profit-to-sales ratio	β, ε
$mean(EKY_j)$	BG mean of WG mean of capital-to-sales ratio	$\beta, \varepsilon, \theta$
$mean(i_{j,t})$	mean of investment rates	b, μ_μ, δ
$sd(i_{j,t})$	standard deviation of investment rates	$b, \sigma_\mu, \mu_{l\sigma}, \sigma_{l\sigma}, \sigma_{IT}, \sigma_{IP}$
$mean(Edy_j)$	BG mean of WG mean of sales growth rates	μ_μ
$sd(Edy_j)$	BG sd of WG mean of sales growth rates	σ_μ, σ_{YT}
$mean(SDdy_j)$	BG mean of WG sd of sales growth rates	$\mu_{l\sigma}, \sigma_{YT}$
$sd(SDdy_j)$	BG sd of WG sd in sales growth rates	$\sigma_{l\sigma}, \sigma_{YT}$
$mean(EK_j)$	BG mean of WG mean of capital stock	$\mu_{\zeta_0}, \tau, \theta$
$sd(EK_j)$	BG sd of WG mean of capital stock	σ_{ζ_0}
$sd(EKY_j)$	BG sd of WG mean of capital-to-sales ratio	$\sigma_{l\sigma}, \sigma_{YT}, \sigma_{YP}$

Note:

BG means "between-group"; WG means "within-group";

corr means "correlation coefficient"; sd means "standard deviation";

$EK_j = mean_t(\widehat{K}_{j,t})$: within-group mean of capital stock for firm *j*;

$EKY_j = mean_t(\widehat{K}_{j,t}/Y_{j,t})$: within-group mean of capital-to-sales ratio for firm *j*;

$Edy_j = mean_t(dy_{j,t})$: within-group mean of sales growth rate for firm *j*;

$SDdy_j = sd_t(dy_{j,t})$: within-group standard deviation of capital stock for firm *j*.

Table 3 Empirical Results

Moments	Empirical	Simulated	Parameters	Estimates	s.e.	Derived Para.	
$corr(EK_j, SDdy_j)$	-0.080	-0.060	θ	0.675	0.021		
$corr(EKY_j, SDdy_j)$	-0.113	-0.278	τ	0.495	0.118	$1-\tau$	0.505
$prop(i_{j,t} < -0.01)$	0.024	0.024	b_q	0.319	0.028		
$prop(\lvert i_{j,t}\rvert < 0.01)$	0.027	0.025	b_i	0.284	0.011		
$prop(i_{j,t} > 0.20)$	0.153	0.145	b_f	0.070	0.003		
$corr(i_{j,t}, yk_{j,t})$	0.139	0.153	β	0.127	0.001		
$corr(i_{j,t}, i_{j,t-1})$	0.392	0.362	ε	6.387	0.042	$1-\gamma$	0.407
$corr(yk_{j,t}, yk_{j,t-1})$	0.968	0.973	ρ	0.931	0.009		
$mean(\pi_{j,t}/Y_{j,t})$	0.266	0.264	μ_μ	0.017	0.000		
$mean(EKY_j)$	0.497	0.513	σ_μ	0.044	0.000		
$mean(i_{j,t})$	0.125	0.111	$\mu_{l\sigma}$	-1.992	0.007	μ_σ	0.219
$sd(i_{j,t})$	0.126	0.093	$\sigma_{l\sigma}$	1.157	0.013	σ_σ	0.233
$mean(Edy_j)$	0.030	0.016	μ_{ζ_0}	-7.654	0.008		
$sd(Edy_j)$	0.062	0.046	σ_{ζ_0}	1.837	0.007		
$mean(SDdy_j)$	0.119	0.116	σ_{IT}	0.009	0.046		
$sd(SDdy_j)$	0.047	0.044	σ_{IP}	0.286	0.002		
$mean(EK_j)$	0.067	0.069	σ_{YT}	0.061	0.000		
$sd(EK_j)$	0.290	0.319	σ_{YP}	0.504	0.002		
$sd(EKY_j)$	0.312	0.318	OI	165			

Table 4 Comparison with the Literature

	This Paper	Cooper and Haltiwanger (2006)	Bloom (2007) (a)	Bloom (2007) (b)	Bloom (2007) (c)	Bloom (2007) (d)
Estimates						
b_q	0.319	0.153	0.000	0.996	0.025	0.616
b_i	0.284	0.019	0.339	0.427	0.453	0.303
b_f	0.070	0.204	0.015	0.011	0.021	0.009
No. of Plants						
	10	**1**	**250**	**250**	**250**	**25**
Data						
$prop\left(i_{j,t} < -0.01\right)$	0.024	0.104	..			
$prop\left(\lvert i_{j,t}\rvert < 0.01\right)$	0.027	0.081	..			
$prop\left(i_{j,t} > 0.20\right)$	0.153	0.186	..			
$corr\left(i_{j,t}, yk_{j,t}\right)$	0.139	0.143	0.260			
$corr\left(i_{j,t}, i_{j,t-1}\right)$	0.392	0.058	0.328			
$sd\left(i_{j,t}\right)$	0.126	0.337	0.139			
$mean\left(SDdy_j\right)$	0.119	..	0.165			

Note:

Bloom(2007) (a): with labour adjustment costs, with time aggregation, No. of plants=250

Bloom(2007) (b): without labour adjustment costs, with time aggregation, No. of plants=250

Bloom(2007) (c): with labour adjustment costs, without time aggregation, No. of plants=250

Bloom(2007) (d): with labour adjustment costs, with time aggregation, No. of plants=25

Table 5 Specification Tests

Column	(1)	(2)	(3)	(4)	(5)	(6)
Restriction	none	$\theta = 0$	$\tau = 1$	$b = 0$	$\sigma_\mu = 0$	*m.e.*= 0
Estimates						
θ	0.675	**0.000**	0.515	1.500	0.552	1.312
τ	0.495	0.801	**1.000**	1.000	0.428	0.156
b_q	0.319	1.766	0.465	**0.000**	0.575	0.244
b_i	0.284	0.193	0.028	**0.000**	0.110	0.276
b_f	0.070	0.144	0.054	**0.000**	0.132	0.036
β	0.127	0.110	0.121	0.113	0.145	0.163
ε	6.387	5.568	6.182	5.850	7.207	7.996
ρ	0.931	0.659	0.750	0.891	0.908	0.746
μ_μ	0.017	0.032	0.013	0.019	0.012	0.019
σ_μ	0.044	0.041	0.046	0.048	**0.000**	0.046
$\mu_{l\sigma}$	-1.992	-1.881	-1.940	-2.500	-1.586	-1.668
$\sigma_{l\sigma}$	1.157	1.202	1.227	0.586	0.943	1.081
μ_{ζ_0}	-7.654	-7.822	-7.570	-7.743	-7.401	-6.964
σ_{ζ_0}	1.837	1.978	1.815	1.965	1.814	1.407
σ_{IT}	0.009	0.002	0.261	0.435	0.003	**0.000**
σ_{IP}	0.286	0.380	0.259	0.162	0.356	**0.000**
σ_{YT}	0.061	0.062	0.061	0.078	0.055	**0.000**
σ_{YP}	0.504	0.505	0.500	0.376	0.553	**0.000**
Moments						
$corr(EK_j, SDdy_j)$	-0.060	-0.039	-0.074	-0.024	-0.070	-0.150
$corr(EKY_j, SDdy_j)$	-0.278	-0.094	-0.309	-0.037	-0.267	-0.822
$prop(i_{j,t} < -0.01)$	0.024	0.003	0.024	0.023	0.022	0.021
$prop(\lvert i_{j,t}\rvert < 0.01)$	0.025	0.019	0.027	0.000	0.020	0.025
$prop(i_{j,t} > 0.20)$	0.145	0.182	0.140	0.161	0.114	0.124
$corr(i_{j,t}, yk_{j,t})$	0.153	0.196	0.124	0.100	0.107	0.252
$corr(i_{j,t}, i_{j,t-1})$	0.362	0.438	0.360	0.337	0.235	0.343
$corr(yk_{j,t}, yk_{j,t-1})$	0.973	0.975	0.974	0.960	0.975	0.961
$mean(\pi_{j,t}/Y_{j,t})$	0.264	0.270	0.263	0.264	0.264	0.268
$mean(EKY_j)$	0.513	0.491	0.497	0.442	0.511	0.462
$mean(i_{j,t})$	0.111	0.129	0.110	0.122	0.105	0.110
$sd(i_{j,t})$	0.093	0.093	0.092	0.096	0.086	0.077
$mean(Edy_j)$	0.016	0.031	0.013	0.019	0.011	0.019
$sd(Edy_j)$	0.046	0.040	0.045	0.045	0.025	0.044
$mean(SDdy_j)$	0.116	0.118	0.117	0.117	0.116	0.070
$sd(SDdy_j)$	0.044	0.045	0.045	0.031	0.046	0.050
$mean(EK_j)$	0.069	0.066	0.064	0.068	0.068	0.091
$sd(EK_j)$	0.319	0.314	0.289	0.338	0.302	0.301
$sd(EKY_j)$	0.318	0.286	0.310	0.191	0.339	0.166
OI	165	356	182	338	284	1398
degree of freedom	1	2	2	4	2	5

Table 6 Choice for the Number of Plants

Column	(1)	(2)	(3)	(4)
No. of Plants	m=1	m=5	m=10	m=15
Estimates				
θ	0.837	0.503	0.675	0.965
τ	0.503	0.497	0.495	0.488
b_q	1.985	0.563	0.319	0.069
b_i	0.169	0.077	0.284	0.556
b_f	0.003	0.033	0.070	0.051
β	0.126	0.112	0.127	0.128
ε	6.210	5.870	6.387	6.536
ρ	0.565	0.945	0.931	0.895
μ_μ	0.022	0.015	0.017	0.012
σ_μ	0.049	0.040	0.044	0.048
$\mu_{l\sigma}$	-2.132	-2.000	-1.992	-1.890
$\sigma_{l\sigma}$	0.527	0.875	1.157	1.262
μ_{ζ_0}	-5.175	-6.603	-7.654	-8.329
σ_{ζ_0}	1.784	1.664	1.837	2.019
σ_{IT}	0.530	0.302	0.009	0.034
σ_{IP}	0.181	0.152	0.286	0.233
σ_{YT}	0.048	0.060	0.061	0.062
σ_{YP}	0.476	0.511	0.504	0.496
Moments				
$corr(EK_j, SDdy_j)$	-0.048	-0.047	-0.060	-0.056
$corr(EKY_j, SDdy_j)$	-0.193	-0.210	-0.278	-0.289
$prop(i_{j,t} < -0.01)$	0.000	0.025	0.024	0.026
$prop(\lvert i_{j,t}\rvert < 0.01)$	0.021	0.028	0.025	0.029
$prop(i_{j,t} > 0.20)$	0.177	0.145	0.145	0.135
$corr(i_{j,t}, yk_{j,t})$	0.123	0.159	0.153	0.159
$corr(i_{j,t}, i_{j,t-1})$	0.355	0.340	0.362	0.337
$corr(yk_{j,t}, yk_{j,t-1})$	0.972	0.972	0.973	0.975
$mean(\pi_{j,t}/Y_{j,t})$	0.267	0.263	0.264	0.262
$mean(EKY_j)$	0.486	0.497	0.513	0.505
$mean(i_{j,t})$	0.128	0.112	0.111	0.105
$sd(i_{j,t})$	0.105	0.092	0.093	0.091
$mean(Edy_j)$	0.021	0.015	0.016	0.012
$sd(Edy_j)$	0.046	0.044	0.046	0.048
$mean(SDdy_j)$	0.116	0.117	0.116	0.117
$sd(SDdy_j)$	0.044	0.045	0.044	0.046
$mean(EK_j)$	0.074	0.066	0.069	0.061
$sd(EK_j)$	0.356	0.319	0.319	0.293
$sd(EKY_j)$	0.264	0.299	0.318	0.328
OI	286	169	165	197

Table 7 Robustness Tests

Column	(1)	(2)	(3)		
risk-free $\bar{r}$	0.065	0.040	0.065		
No. of Moments	19	19	36		
Estimates					
θ	0.675	0.951	0.373		
τ	0.495	0.042	0.184		
b_q	0.319	0.438	1.070		
b_i	0.284	0.226	0.290		
b_f	0.070	0.053	0.166		
β	0.127	0.131	0.130		
ε	6.387	6.416	6.351		
ρ	0.931	0.839	0.949		
μ_μ	0.017	0.018	0.024		
σ_μ	0.044	0.048	0.020		
$\mu_{l\sigma}$	-1.992	-1.964	-1.583		
$\sigma_{l\sigma}$	1.157	1.252	0.956		
μ_{ζ_0}	-7.654	-7.674	-8.000		
σ_{ζ_0}	1.837	1.754	1.723		
σ_{IT}	0.009	0.271	0.203		
σ_{IP}	0.286	0.262	0.444		
σ_{YT}	0.061	0.061	0.051		
σ_{YP}	0.504	0.506	0.697		
19 Moments				**additional 17 Moments**	
$corr(EK_j, SDdy_j)$	-0.060	-0.082	-0.029	$corr(EY_j, SDdy_j)$	0.005
$corr(EKY_j, SDdy_j)$	-0.278	-0.358	-0.160	$corr(Ei, SDdy_j)$	-0.060
$prop(i_{j,t} < -0.01)$	0.024	0.022	0.017	$corr(Edy_j, SDdy_j)$	-0.021
$prop(\lvert i_{j,t}\rvert < 0.01)$	0.025	0.022	0.010	$mean(Y_{j,t})$	0.100
$prop(i_{j,t} > 0.20)$	0.145	0.157	0.163	$sd(Y_{j,t})$	0.372
$corr(i_{j,t}, yk_{j,t})$	0.153	0.126	0.105	$corr(K_{j,t}, K_{j,t-1})$	0.998
$corr(i_{j,t}, i_{j,t-1})$	0.362	0.388	0.374	$corr(Y_{j,t}, Y_{j,t-1})$	0.991
$corr(yk_{j,t}, yk_{j,t-1})$	0.973	0.980	0.987	$corr(dy_{j,t}, dy_{j,t-1})$	-0.150
$mean(\pi_{j,t}/Y_{j,t})$	0.264	0.266	0.267	$corr(K_{j,t}, Y_{j,t})$	0.828
$mean(EKY_j)$	0.513	0.518	0.506	$corr(K_{j,t}, K_{j,t}/Y_{j,t})$	0.025
$mean(i_{j,t})$	0.111	0.116	0.120	$corr(K_{j,t}, i_{j,t})$	0.009
$sd(i_{j,t})$	0.093	0.098	0.112	$corr(K_{j,t}, dy_{j,t})$	0.005
$mean(Edy_j)$	0.016	0.018	0.021	$corr(Y_{j,t}, K_{j,t}/Y_{j,t})$	-0.110
$sd(Edy_j)$	0.046	0.047	0.060	$corr(Y_{j,t}, i_{j,t})$	0.018
$mean(SDdy_j)$	0.116	0.116	0.120	$corr(Y_{j,t}, dy_{j,t})$	0.021
$sd(SDdy_j)$	0.044	0.045	0.060	$corr(K_{j,t}/Y_{j,t}, dy_{j,t})$	-0.068
$mean(EK_j)$	0.069	0.066	0.033	$corr(i_{j,t}, dy_{j,t})$	0.384
$sd(EK_j)$	0.319	0.302	0.136		
$sd(EKY_j)$	0.318	0.364	0.424		
OI	165	194	1131		

Figure 1: Investment Policy for Quadratic Adjustment Costs Only

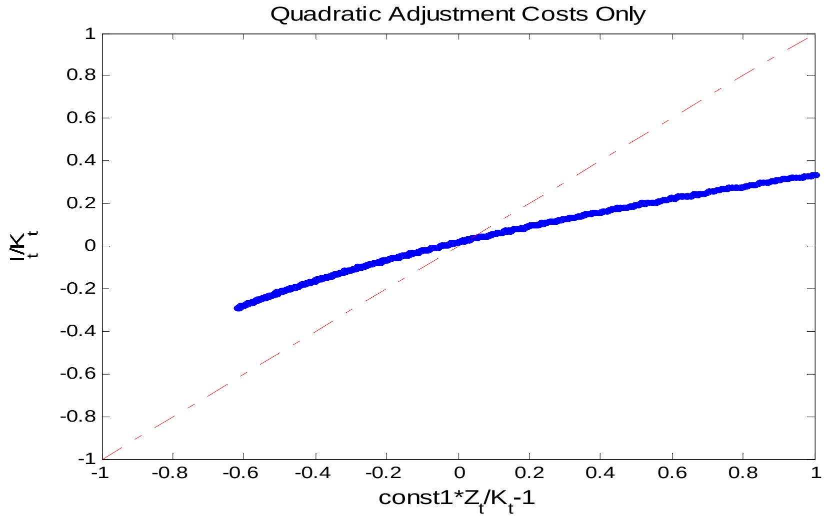

Figure 2: Investment Policy for Partial Irreversibility Only

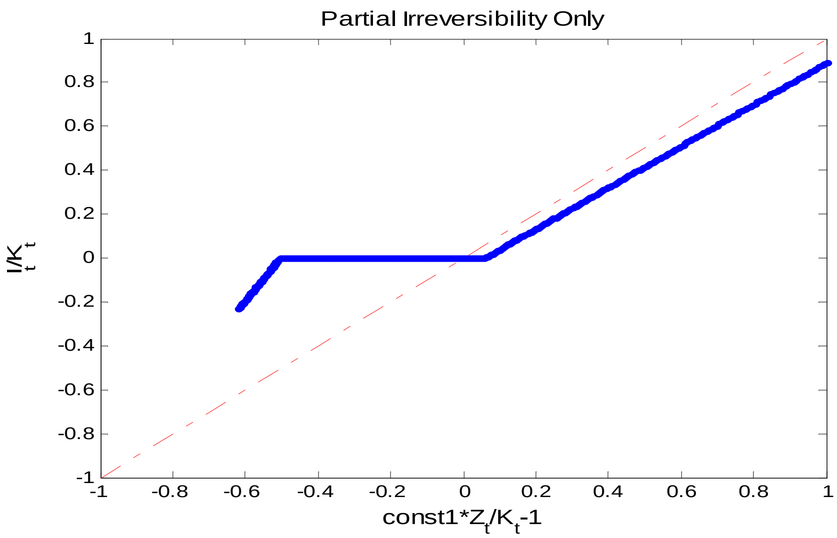

Figure 3: Investment Policy for Fixed Adjustment Costs Only

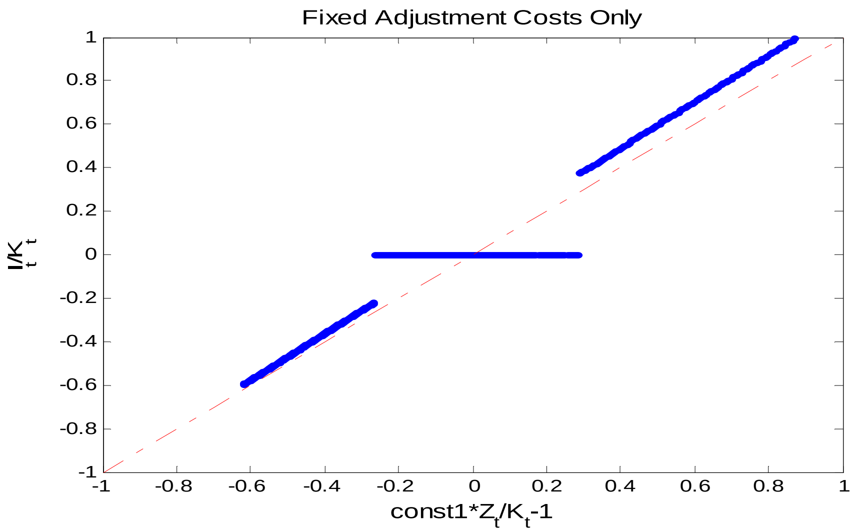

Figure 4a: Empirical Distribution of WG mean of Sales Growth Rates

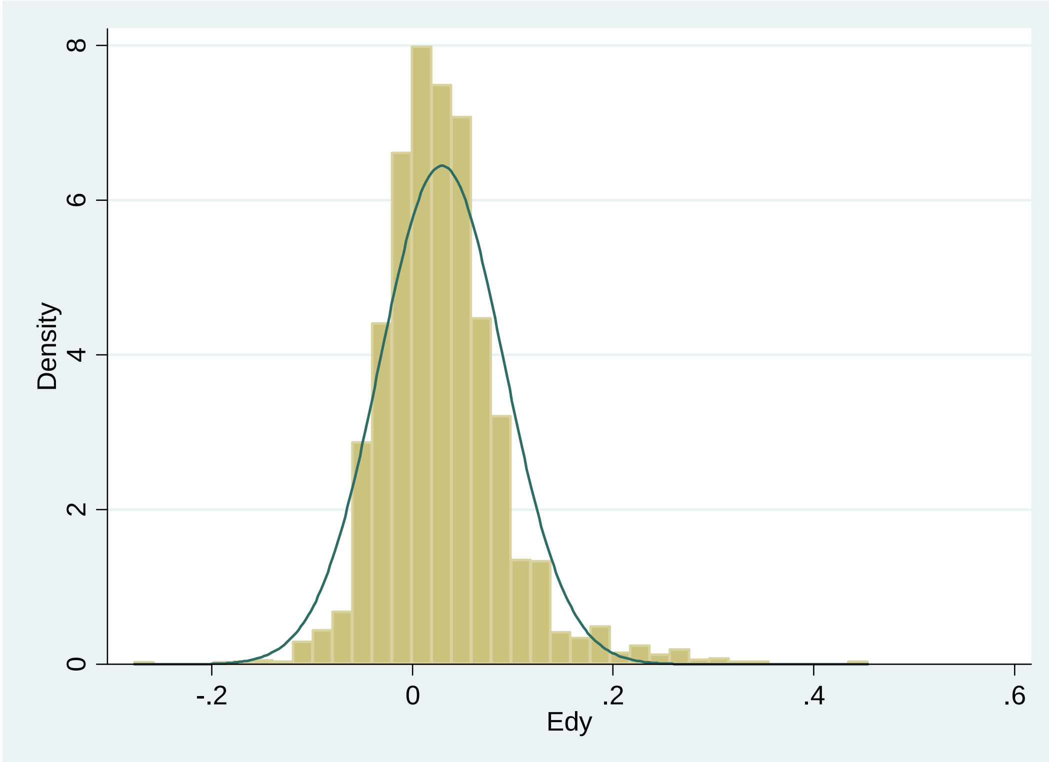

Figure 4b: Empirical Distribution of WG sd of Sales Growth Rates

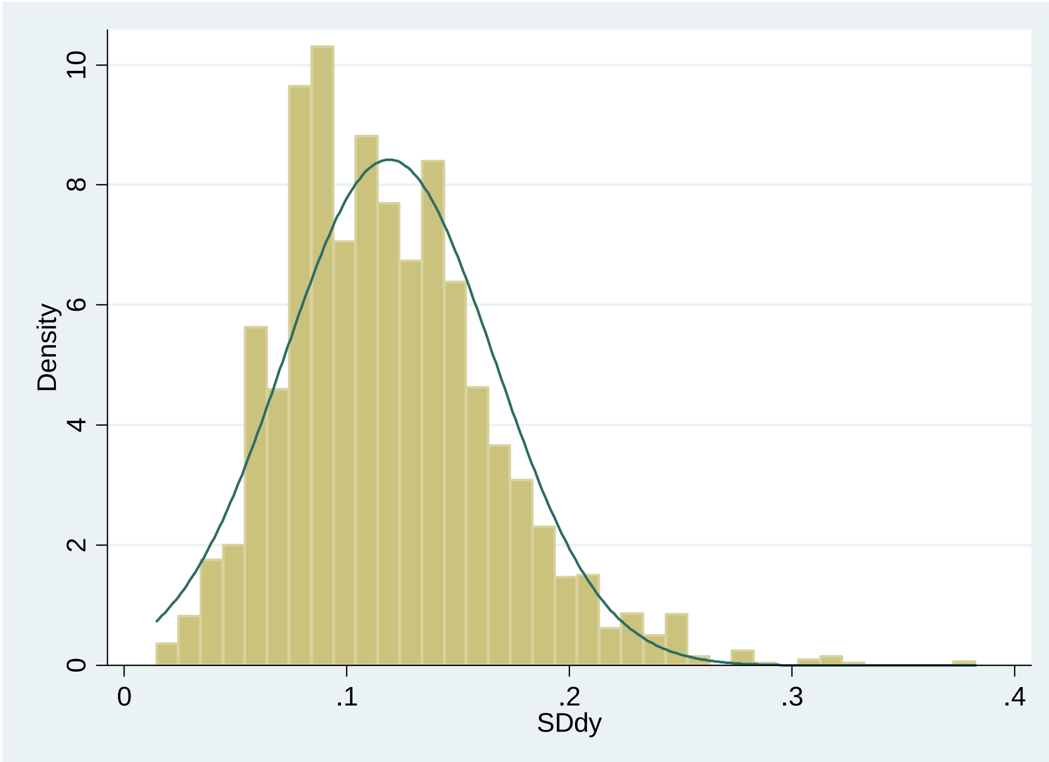

Figure 5a: Overall Effects of Uncertainty on Expected Capital Stock

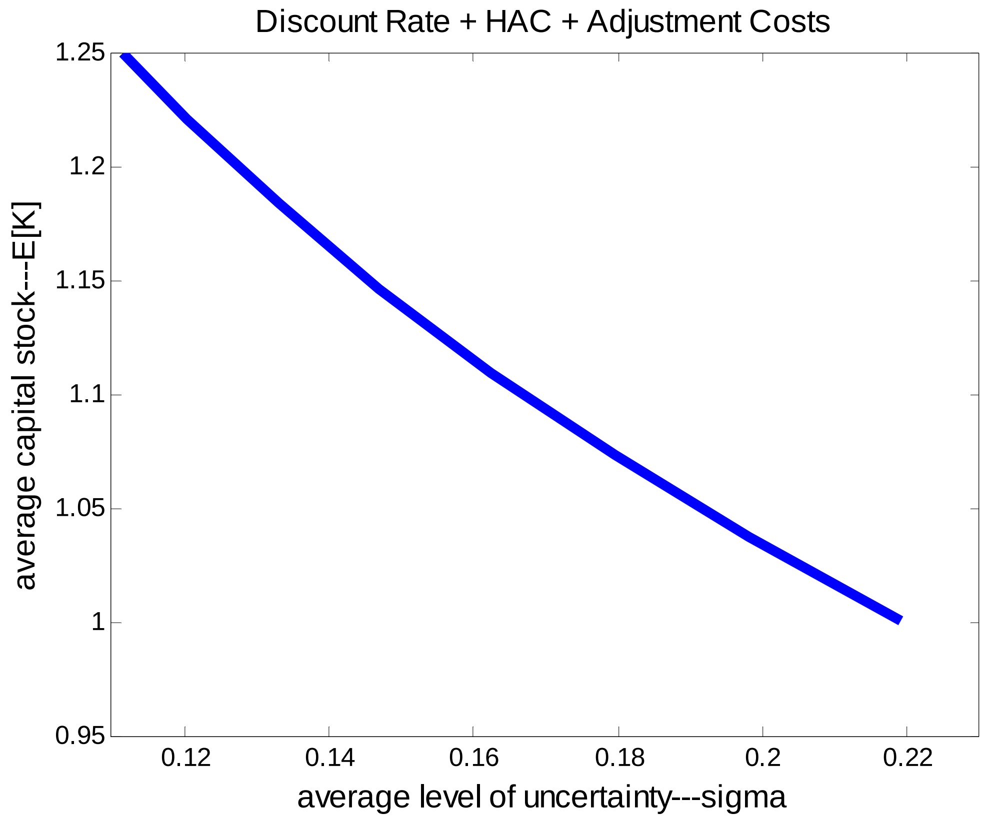

Figure 5b: Overall Effects of Uncertainty on Aggregated Capital Intensity

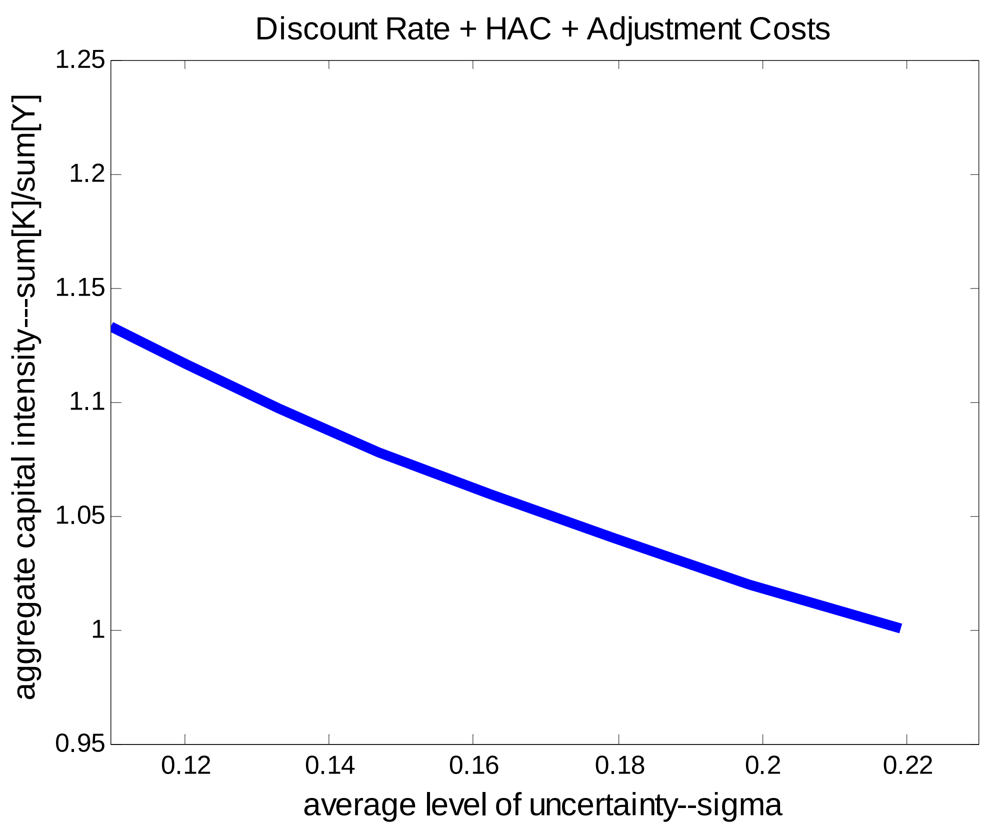

Figure 6a: Effects of HAC and Adjustment Costs on Expected Capital Stock

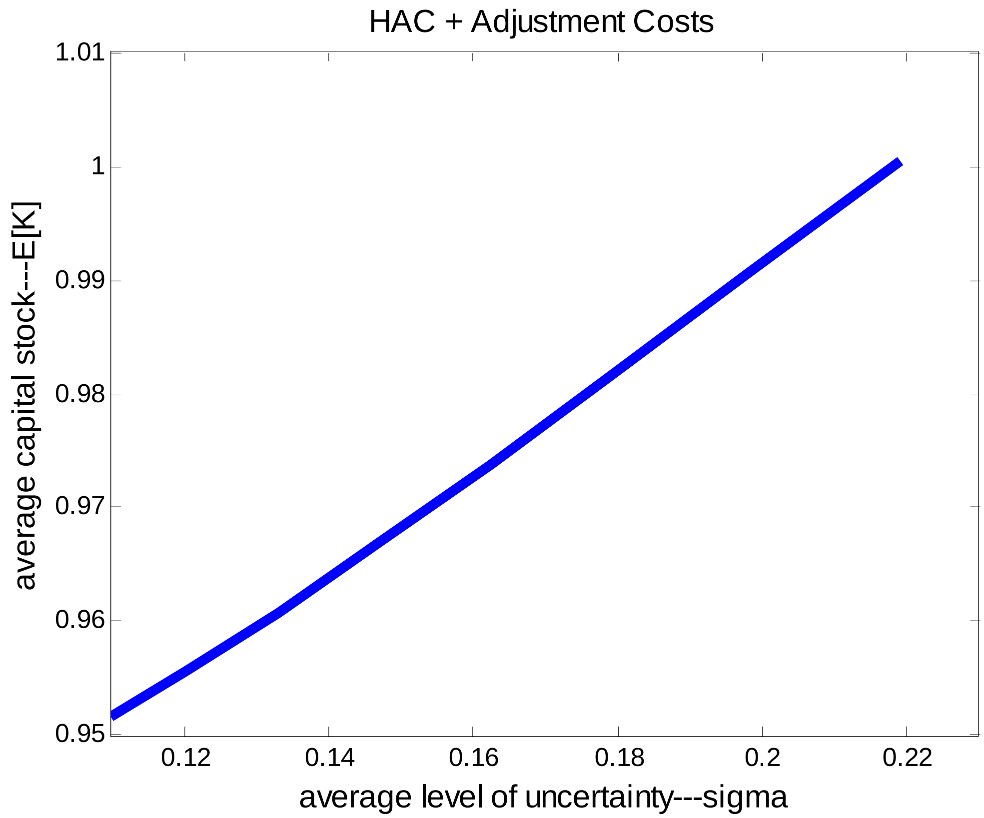

Figure 6b: Effects of HAC and Adjustment Costs on Aggregated Capital Intensity

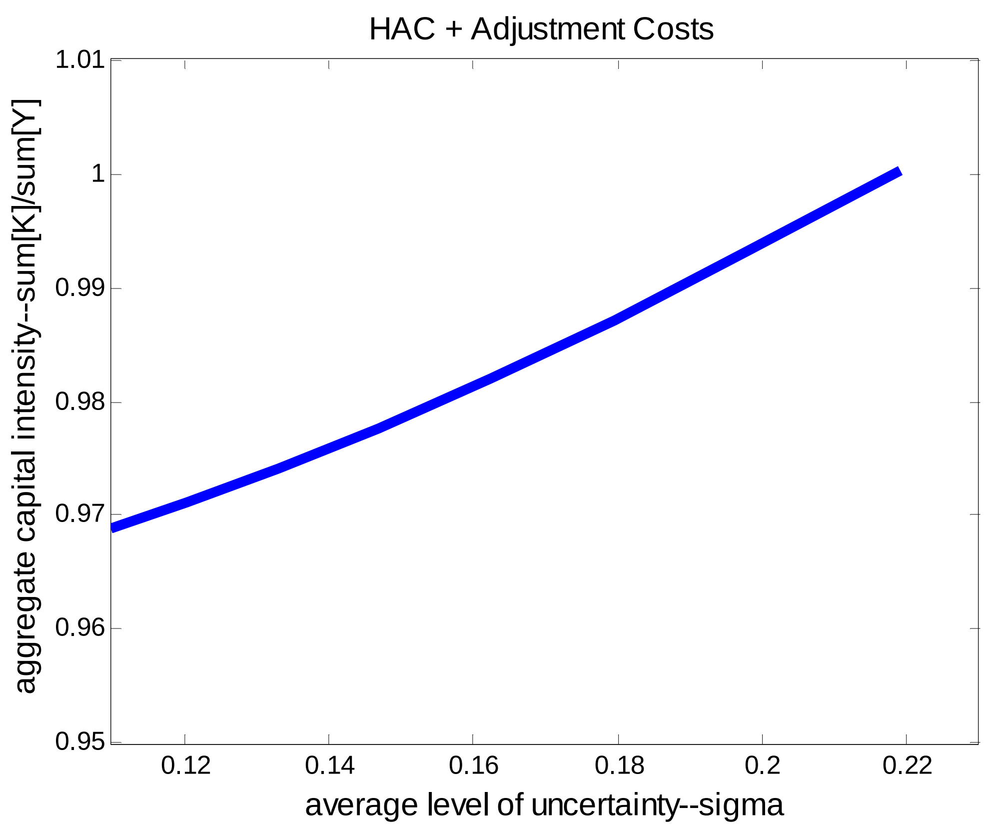

Figure 7a: Effects of Adjustment Costs on Expected Capital Stock

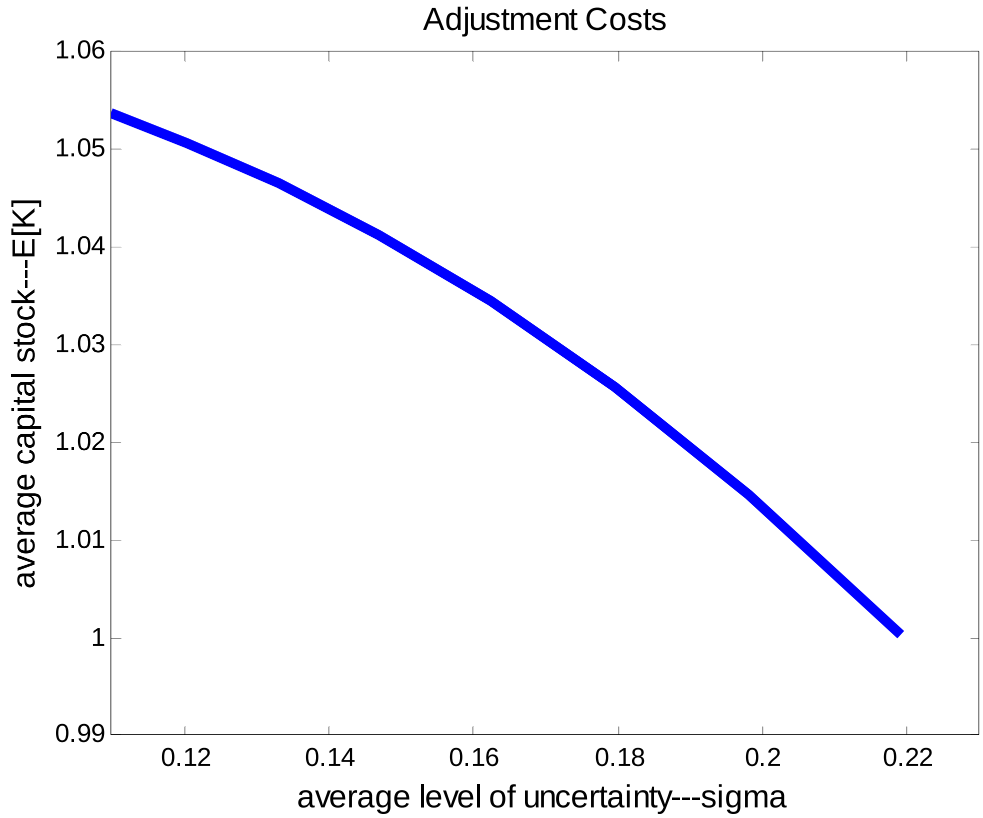

Figure 7b: Effects of Adjustment Costs on Aggregated Capital Intensity

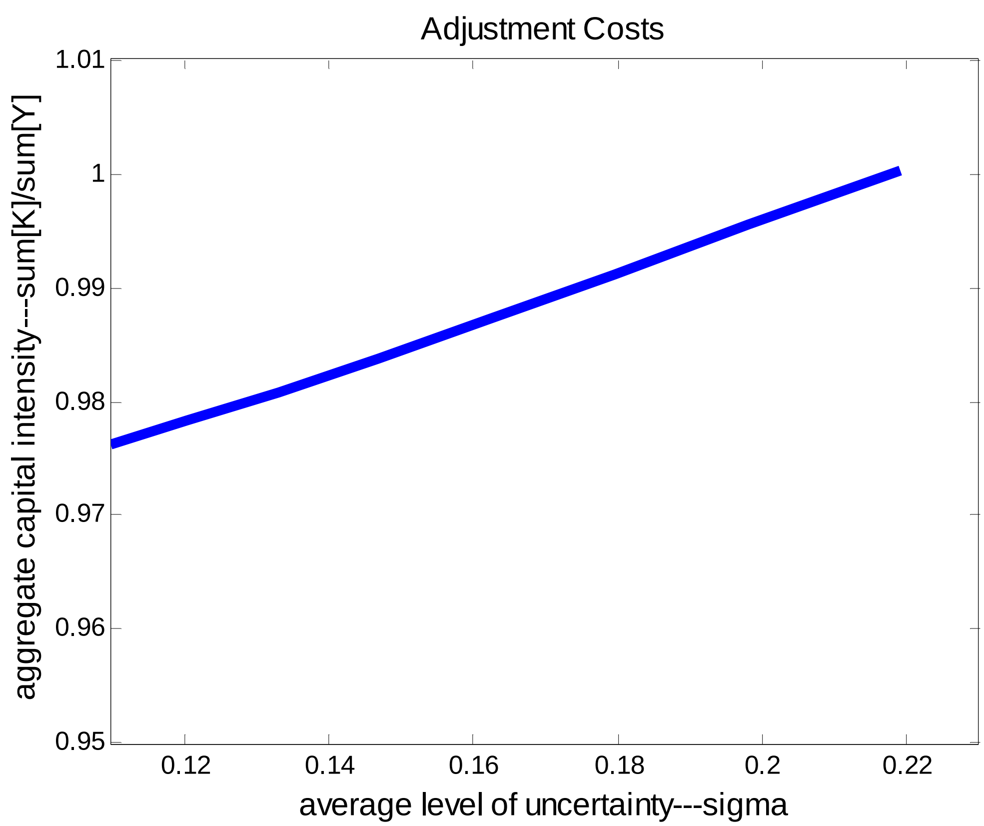

Figure 8a: Assuming m=5 , Column (2) of Table 6

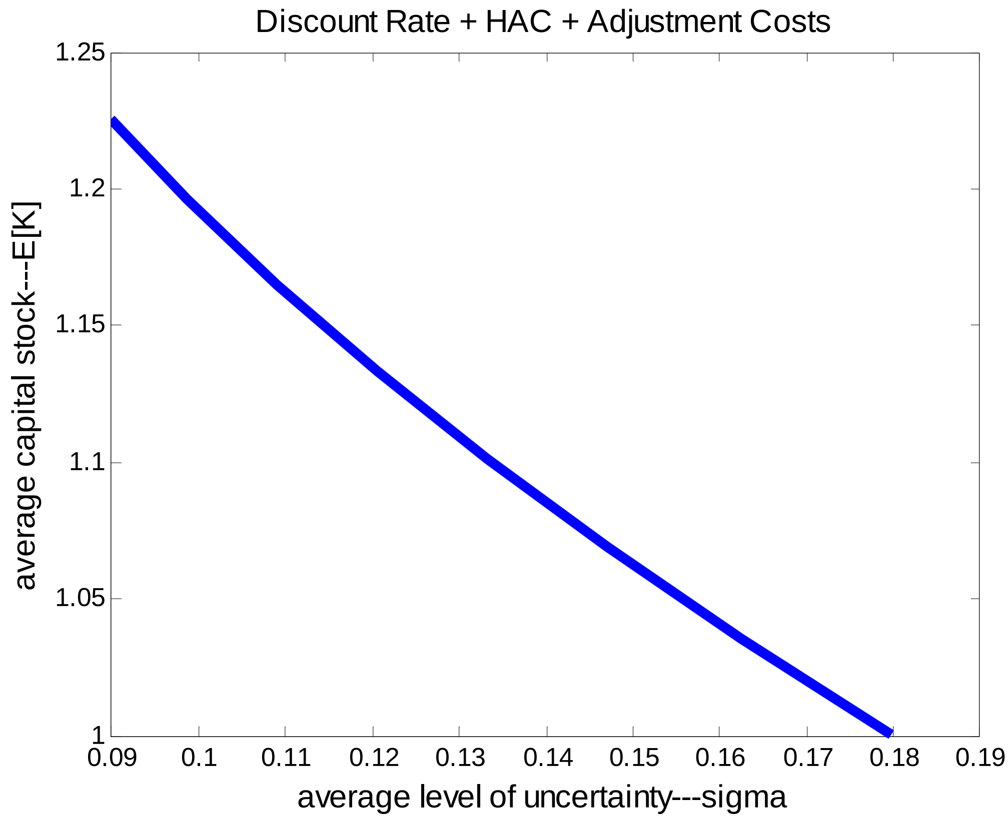

Figure 8b: Assuming m=5 , Column (2) of Table 6

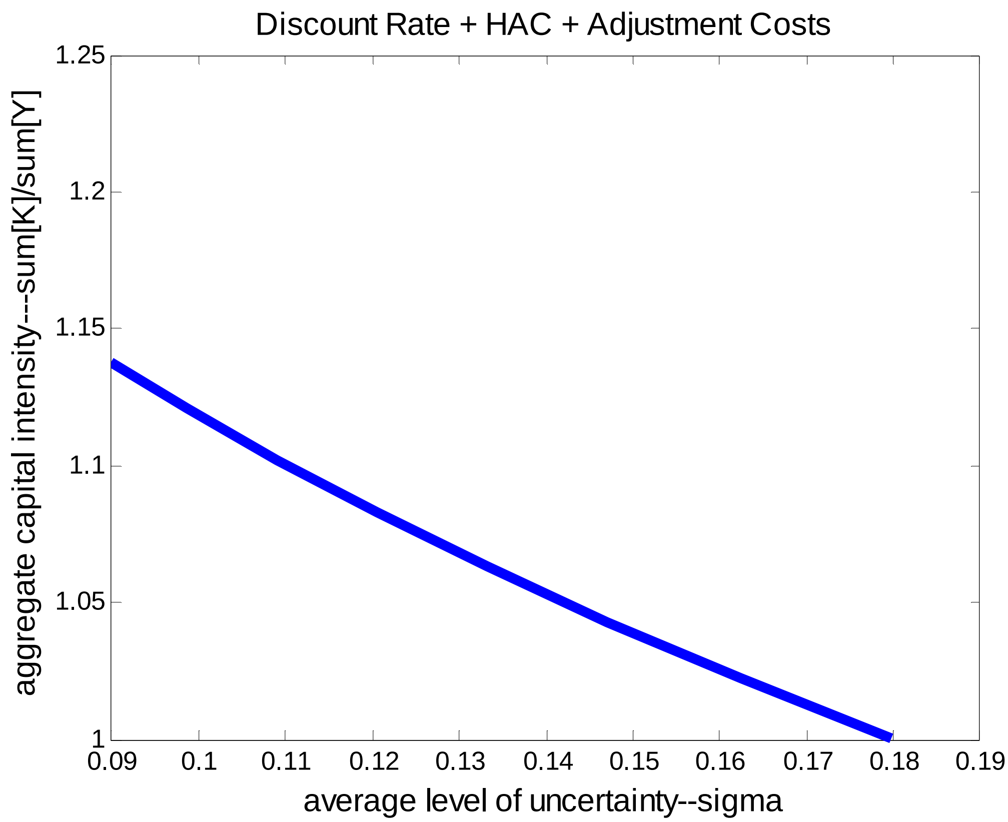

Figure 9a: Imposing $\bar{r}$=0.040 , Column (2) of Table 7

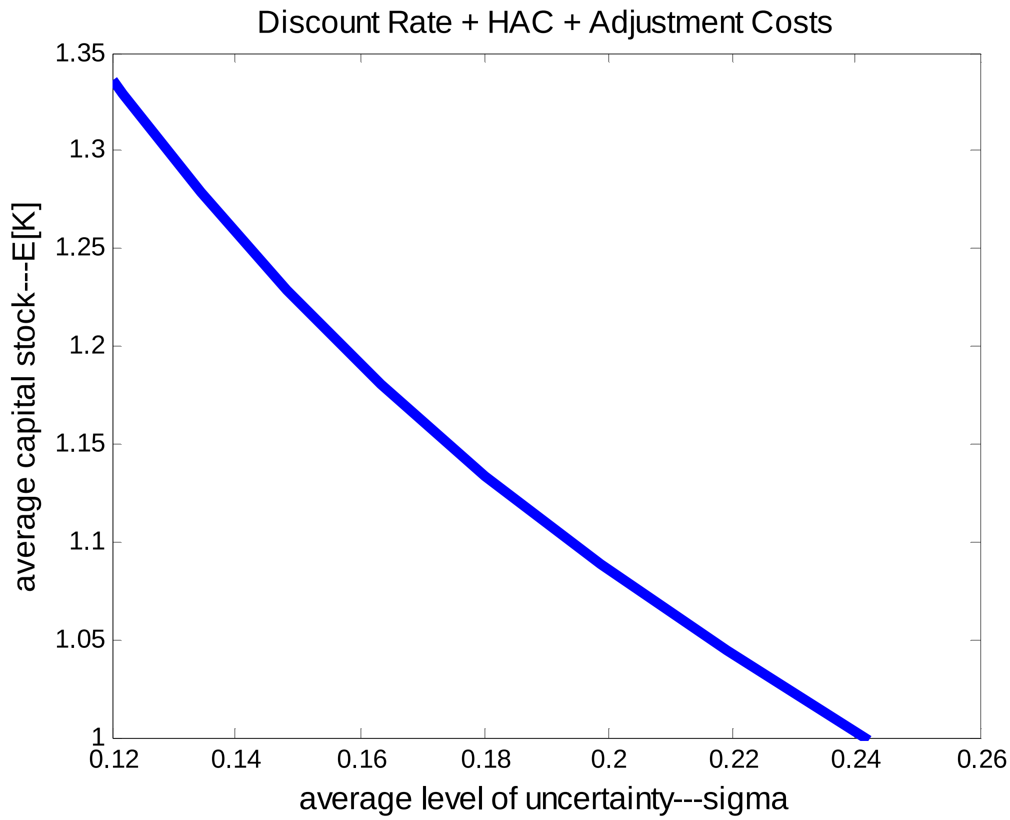

Figure 9b: Imposing $\bar{r}$=0.040 , Column (2) of Table 7

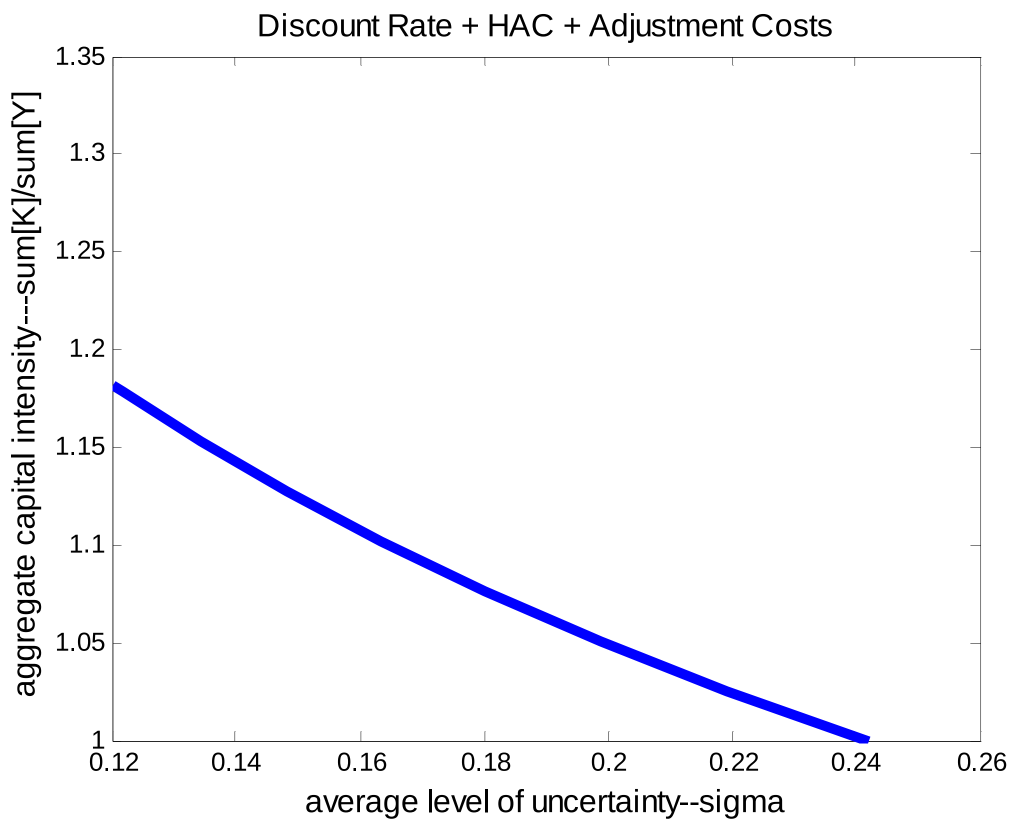

Figure 10a: Matching a Larger Set of Moments , Column (3) of Table 7

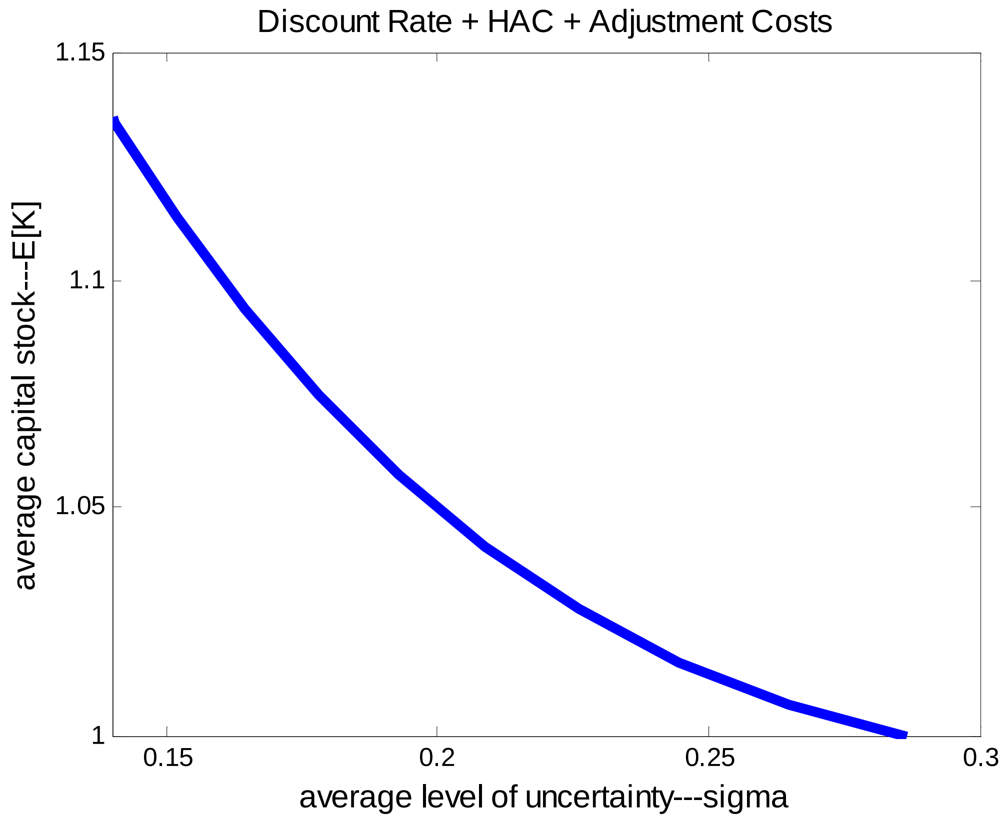

Figure 10b: Matching a Larger Set of Moments , Column (3) of Table 7