

# Ad hoc Constructions for EKGDDs and EK Packings

## 1. BASE-SETS FOR $(1^{24}t, 2)$ EKGDD

Let  $X = (\mathbb{Z}_3 \times [8]) \cup \{\infty_j : j \in [t]\}$ . Given a base-set  $\mathcal{D} \subseteq \mathcal{E}(X, 2)$ , we obtain a  $(1^{24}t, 2)$  EKGDD with groups  $\{\{i : i \in \mathbb{Z}_3 \times [8]\} \cup \{\{\infty_j : j \in [t]\}\}$  by considering the set-pairs:

$$\{D + i : i \in \mathbb{Z}_3, D \in \mathcal{D}\}.$$

A base-set for  $(1^{24}3, 2)$  EKGDD is given by:

$$\begin{aligned} &\{\{0_3, 0_4\}, \{0_7, 1_5\}\} && \{\{0_6, 2_1\}, \{1_5, 2_3\}\} \\ &\{\{0_7, 1_4\}, \{1_8, 2_8\}\} && \{\{0_2, 2_6\}, \{1_4, 2_8\}\} \\ &\{\{0_6, 2_8\}, \{1_8, \infty_2\}\} && \{\{0_2, 0_3\}, \{2_4, \infty_1\}\} \\ &\{\{1_2, 1_8\}, \{2_3, \infty_3\}\} && \{\{0_1, 0_3\}, \{0_4, 1_4\}\} \\ &\{\{0_2, 0_5\}, \{2_7, \infty_2\}\} && \{\{0_4, 0_6\}, \{2_6, \infty_1\}\} \\ &\{\{0_2, 1_6\}, \{0_5, 1_2\}\} && \{\{0_2, \infty_3\}, \{0_7, 1_1\}\} \\ &\{\{0_4, 1_3\}, \{0_6, \infty_3\}\} && \{\{0_1, 0_2\}, \{1_5, 1_6\}\} \\ &\{\{0_6, 2_5\}, \{2_7, \infty_3\}\} && \{\{1_1, 2_4\}, \{1_5, \infty_2\}\} \\ &\{\{0_5, 1_8\}, \{1_5, \infty_1\}\} && \{\{0_3, 0_5\}, \{0_6, 1_7\}\} \\ &\{\{0_7, 0_8\}, \{1_4, 2_1\}\} && \{\{0_3, 2_2\}, \{0_8, 1_1\}\} \\ &\{\{0_1, 1_1\}, \{0_6, 0_8\}\} && \{\{0_6, 1_1\}, \{1_2, 1_7\}\} \\ &\{\{0_2, 2_3\}, \{0_3, 0_8\}\} && \{\{0_4, 1_2\}, \{0_5, 1_4\}\} \\ &\{\{0_8, 2_3\}, \{1_5, 2_5\}\} && \{\{0_3, 2_7\}, \{1_2, \infty_2\}\} \\ &\{\{1_7, 2_6\}, \{1_8, 2_7\}\} && \{\{0_1, 1_7\}, \{2_1, \infty_1\}\} \\ &\{\{0_4, 2_3\}, \{1_1, 1_7\}\} && \end{aligned}$$

A base-set for  $(1^{24}5, 2)$  EKGDD is given by:

$$\begin{aligned} &\{\{0_1, 2_6\}, \{2_4, 2_7\}\} && \{\{0_2, 0_4\}, \{2_1, 2_8\}\} \\ &\{\{0_4, 1_1\}, \{1_4, \infty_2\}\} && \{\{0_2, 1_7\}, \{0_8, \infty_2\}\} \\ &\{\{0_3, 2_8\}, \{1_1, \infty_5\}\} && \{\{0_4, 1_7\}, \{1_3, 2_3\}\} \\ &\{\{0_3, 1_5\}, \{2_8, \infty_1\}\} && \{\{0_7, 1_5\}, \{1_8, \infty_4\}\} \\ &\{\{0_2, 0_5\}, \{1_3, \infty_3\}\} && \{\{0_3, 1_2\}, \{1_5, 1_7\}\} \\ &\{\{0_6, 2_2\}, \{0_8, 2_6\}\} && \{\{0_7, 2_1\}, \{1_2, \infty_5\}\} \\ &\{\{0_1, 1_8\}, \{2_1, \infty_3\}\} && \{\{0_2, 1_5\}, \{1_7, 2_6\}\} \\ &\{\{1_2, 2_1\}, \{2_2, \infty_4\}\} && \{\{0_2, 2_6\}, \{0_3, \infty_1\}\} \\ &\{\{1_5, 2_4\}, \{2_5, \infty_5\}\} && \{\{1_6, 1_7\}, \{2_7, \infty_3\}\} \\ &\{\{0_4, 0_5\}, \{1_2, 2_2\}\} && \{\{0_8, 2_5\}, \{1_6, \infty_2\}\} \\ &\{\{1_1, 1_4\}, \{1_3, 2_7\}\} && \{\{0_6, 2_8\}, \{1_4, 1_8\}\} \\ &\{\{0_1, 1_4\}, \{0_7, \infty_1\}\} && \{\{0_1, 0_7\}, \{0_8, 1_5\}\} \\ &\{\{1_3, 1_4\}, \{1_8, \infty_3\}\} && \{\{1_3, \infty_4\}, \{2_3, 2_8\}\} \\ &\{\{0_1, 1_1\}, \{0_5, 2_6\}\} && \{\{0_1, \infty_2\}, \{0_6, 1_3\}\} \\ &\{\{0_7, 1_8\}, \{2_5, \infty_1\}\} && \{\{0_4, 2_6\}, \{2_5, \infty_4\}\} \\ &\{\{0_2, 0_7\}, \{0_4, 1_6\}\} && \{\{0_3, 2_4\}, \{0_5, 0_6\}\} \\ &\{\{0_3, \infty_5\}, \{1_2, 1_6\}\} && \end{aligned}$$

A base-set for  $(1^{24}7, 2)$  EKGDD is given by:

$$\begin{aligned} &\{\{0_5, 2_3\}, \{1_1, \infty_5\}\} && \{\{0_8, \infty_6\}, \{1_4, 2_7\}\} \\ &\{\{0_6, 2_7\}, \{1_7, \infty_1\}\} && \{\{1_5, \infty_4\}, \{2_4, 2_6\}\} \\ &\{\{0_1, 1_2\}, \{1_4, \infty_4\}\} && \{\{0_6, \infty_4\}, \{0_7, 1_5\}\} \\ &\{\{0_6, 2_4\}, \{0_8, 1_2\}\} && \{\{0_2, 2_7\}, \{2_3, \infty_7\}\} \\ &\{\{0_1, 2_2\}, \{0_8, 2_3\}\} && \{\{1_1, 1_8\}, \{1_3, \infty_1\}\} \\ &\{\{0_1, 0_3\}, \{1_7, 2_4\}\} && \{\{0_4, \infty_3\}, \{1_4, 2_3\}\} \\ &\{\{0_2, 0_3\}, \{2_5, \infty_6\}\} && \{\{1_1, 2_8\}, \{2_5, \infty_5\}\} \\ &\{\{0_1, \infty_6\}, \{0_6, 2_1\}\} && \{\{0_2, 1_3\}, \{1_5, \infty_1\}\} \\ &\{\{0_1, 1_1\}, \{2_2, 2_8\}\} && \{\{1_7, \infty_7\}, \{1_8, 2_4\}\} \\ &\{\{0_2, \infty_5\}, \{1_2, 2_4\}\} && \{\{1_3, 2_7\}, \{1_4, \infty_2\}\} \\ &\{\{1_4, 2_5\}, \{2_6, \infty_1\}\} && \{\{1_1, 1_6\}, \{1_2, 1_4\}\} \\ &\{\{0_5, \infty_3\}, \{0_7, 2_5\}\} && \{\{1_4, \infty_6\}, \{1_8, 2_5\}\} \\ &\{\{0_2, 2_5\}, \{1_3, 1_7\}\} && \{\{0_1, 0_2\}, \{0_7, \infty_3\}\} \\ &\{\{1_3, 2_1\}, \{1_6, \infty_7\}\} && \{\{0_2, 0_7\}, \{1_6, 2_8\}\} \\ &\{\{0_1, 2_4\}, \{2_7, \infty_2\}\} && \{\{1_2, 2_8\}, \{1_5, \infty_2\}\} \\ &\{\{0_6, 2_5\}, \{2_1, \infty_2\}\} && \{\{1_6, \infty_3\}, \{2_6, 2_8\}\} \\ &\{\{1_6, 2_5\}, \{2_4, \infty_7\}\} && \{\{0_5, 2_2\}, \{1_7, 2_8\}\} \\ &\{\{1_3, 1_8\}, \{2_3, 2_6\}\} && \{\{0_3, 2_8\}, \{1_8, \infty_4\}\} \\ &\{\{0_6, 0_7\}, \{1_3, \infty_5\}\} && \end{aligned}$$

## 2. BASE-SETS FOR $(8^{ut}, 2)$ EKGDD

Let  $X = \mathbb{Z}_{8u} \cup \{i_* : i \in \mathbb{Z}_4\}$ . Given a base-set  $\mathcal{D} \subseteq \mathcal{E}(X, 2)$ , we obtain a  $(8^u4, 2)$  EKGDD with groups  $\{\{i, u+i, 2u+i, 3u+i, 4u+i, 5u+i, 6u+i, 7u+i\} : 0 \leq i \leq u-1\} \cup \{\{i_* : i \in \mathbb{Z}_4\}\}$  by considering the set-pairs:

$$\{D + i : i \in \mathbb{Z}_{8u}, D \in \mathcal{D}\},$$

where  $j_* + i := (j + i \bmod 4)_*$ .

A base-set for  $(8^64, 2)$  EKGDD is given by:

$$\begin{aligned} &\{\{0, 1\}, \{2, 4\}\} && \{\{0, 3\}, \{8, 13\}\} \\ &\{\{0, 4\}, \{11, 19\}\} && \{\{0, 7\}, \{21, 32\}\} \\ &\{\{0, 9\}, \{26, 0_*\}\} && \{\{0, 19\}, \{28, 1_*\}\} \end{aligned}$$

A base-set for  $(8^74, 2)$  EKGDD is given by:

$$\begin{aligned} &\{\{0, 3\}, \{8, 55\}\} && \{\{0, 5\}, \{41, 4_*\}\} \\ &\{\{0, 6\}, \{22, 44\}\} && \{\{0, 12\}, \{39, 43\}\} \\ &\{\{0, 13\}, \{2, 19\}\} && \{\{0, 27\}, \{53, 1_*\}\} \\ &\{\{0, 1\}, \{10, 33\}\} && \end{aligned}$$

Let  $X = \mathbb{Z}_{48} \cup \{i_* : i \in \mathbb{Z}_6\}$ . Consider the base-set  $\mathcal{D}$ :

$$\begin{aligned} & \{\{0, 4_*\}, \{44, 45\}\} \quad \{\{1, 46\}, \{8, 41\}\} \\ & \{\{0, 21\}, \{17, 20\}\} \quad \{\{1, 12\}, \{10, 4_*\}\} \\ & \{\{0, 2_*\}, \{11, 37\}\} \quad \{\{0, 15\}, \{25, 34\}\} \\ & \{\{1, 5\}, \{26, 34\}\} \quad \{\{1, 21\}, \{16, 23\}\} \\ & \{\{0, 27\}, \{40, 2_*\}\} \quad \{\{1, 1_*\}, \{17, 40\}\} \\ & \{\{1, 4_*\}, \{2, 18\}\} \quad \{\{0, 10\}, \{13, 32\}\} \\ & \{\{1, 35\}, \{15, 28\}\} \end{aligned}$$

We obtain a  $(8^6 6, 2)$  EKGDD with groups  $\{\{i, 6 + i, 12 + i, 18 + i, 24 + i, 30 + i, 36 + i, 42 + i\} : 0 \leq i \leq 5\} \cup \{i_* : i \in \mathbb{Z}_6\}$  by considering the set-pairs:

$$\{D + 2i : 2i \in 2\mathbb{Z}_{48}, D \in \mathcal{D}\},$$

where  $j_* + 2i := (j + i \bmod 6)_*$ .

Let  $X = \mathbb{Z}_{56} \cup \{i_{l,*} : i \in \mathbb{Z}_2, l \in [3]\}$ . Consider the base-set  $\mathcal{D}$ :

$$\begin{aligned} & \{\{2, 38\}, \{21, 54\}\} \quad \{\{1, 51\}, \{17, 42\}\} \\ & \{\{0, 32\}, \{5, 17\}\} \quad \{\{1, 54\}, \{55, 0_{2,*}\}\} \\ & \{\{2, 1_{3,*}\}, \{4, 47\}\} \quad \{\{0, 50\}, \{20, 37\}\} \\ & \{\{1, 5\}, \{11, 13\}\} \quad \{\{4, 9\}, \{27, 38\}\} \\ & \{\{0, 1_{1,*}\}, \{25, 44\}\} \quad \{\{1, 32\}, \{24, 26\}\} \\ & \{\{1, 18\}, \{31, 1_{3,*}\}\} \quad \{\{1, 42\}, \{37, 53\}\} \\ & \{\{1, 28\}, \{25, 0_{1,*}\}\} \quad \{\{0, 27\}, \{24, 0_{2,*}\}\} \\ & \{\{3, 50\}, \{4, 12\}\} \end{aligned}$$

We obtain a  $(8^7 6, 2)$  EKGDD with groups  $\{\{i, 7 + i, 14 + i, 21 + i, 28 + i, 35 + i, 42 + i, 49 + i\} : 0 \leq i \leq 6\} \cup \{i_{l,*} : i \in \mathbb{Z}_2, l \in [3]\}$  by considering the set-pairs:

$$\{D + 2i : 2i \in 2\mathbb{Z}_{56}, D \in \mathcal{D}\},$$

where  $j_{l,*} + 2i := (j + i \bmod 2)_{l,*}$ .

### 3. BASE-SET FOR $(3^{10}, 3)$ EKGDD

Let  $X = (\mathbb{Z}_9 \times [3]) \cup \{i_* : i \in \mathbb{Z}_3\}$ . Consider the base-set  $\mathcal{D}$ :

$$\begin{aligned} & \{\{2_1, 7_1, 0_*\}, \{5_2, 6_2, 8_3\}\} \quad \{\{0_1, 0_*, 4_2\}, \{7_1, 8_1, 1_2\}\} \\ & \{\{1_2, 4_2, 6_2\}, \{2_2, 7_3, 8_3\}\} \quad \{\{1_*, 3_3, 7_3\}, \{4_1, 1_3, 8_3\}\} \\ & \{\{0_1, 2_1, 8_2\}, \{6_1, 4_3, 7_3\}\} \end{aligned}$$

We obtain a  $(3^{10}, 3)$  EKGDD with groups  $\{\{i_1, i_2, i_3\} : i \in \mathbb{Z}_9\} \cup \{i_* : i \in \mathbb{Z}_3\}$  by considering the set-pairs:

$$\{D + i : i \in \mathbb{Z}_9, D \in \mathcal{D}\},$$

where  $j_{l,*} + i := (j + i \bmod 3)_*$ .

### 4. BASE-SETS FOR $(18^u 27, 3)$ EKGDD

Let  $X = \mathbb{Z}_{18u} \cup \{i_{l,*} : i \in \mathbb{Z}_9, l \in [3]\}$ . Given a base-set  $\mathcal{D} \subseteq \mathcal{E}(X, 3)$ , we obtain a  $(18^u 27, 3)$  EKGDD with groups  $\{\{tu + i : 0 \leq t \leq 17 : 0 \leq i \leq u - 1\} \cup \{i_{l,*} : i \in \mathbb{Z}_9, l \in [3]\}$  by considering the set-pairs:

$$\{D + i : i \in \mathbb{Z}_{18u}, D \in \mathcal{D}\},$$

where  $j_{l,*} + i := (j + i \bmod 9)_{l,*}$ .

A base-set for  $(18^7 27, 2)$  EKGDD is given by:

$$\begin{aligned} & \{\{36, 91, 1_{3,*}\}, \{41, 87, 93\}\} \quad \{\{8, 72, 122\}, \{82, 90, 1_{1,*}\}\} \\ & \{\{16, 64, 108\}, \{42, 61, 1_{3,*}\}\} \quad \{\{60, 77, 100\}, \{89, 115, 4_{1,*}\}\} \\ & \{\{2, 104, 7_{2,*}\}, \{15, 45, 98\}\} \quad \{\{50, 51, 88\}, \{104, 115, 2_{2,*}\}\} \\ & \{\{5, 64, 80\}, \{39, 100, 5_{1,*}\}\} \quad \{\{0, 2, 87\}, \{19, 48, 8_{3,*}\}\} \\ & \{\{5, 20, 30\}, \{29, 123, 5_{2,*}\}\} \end{aligned}$$

A base-set for  $(18^8 27, 2)$  EKGDD is given by:

$$\begin{aligned} & \{\{38, 48, 3_{1,*}\}, \{99, 113, 133\}\} \quad \{\{33, 54, 7_{3,*}\}, \{74, 75, 117\}\} \\ & \{\{34, 107, 120\}, \{126, 132, 4_{2,*}\}\} \quad \{\{1, 24, 52\}, \{138, 141, 0_{3,*}\}\} \\ & \{\{12, 90, 8_{3,*}\}, \{56, 89, 133\}\} \quad \{\{16, 45, 92\}, \{42, 107, 4_{1,*}\}\} \\ & \{\{27, 76, 0_{2,*}\}, \{41, 63, 133\}\} \quad \{\{33, 72, 3_{1,*}\}, \{101, 103, 138\}\} \\ & \{\{0, 5, 94\}, \{127, 139, 3_{2,*}\}\} \quad \{\{2, 101, 137\}, \{4, 11, 30\}\} \end{aligned}$$

## 5. BASE-SETS FOR $(n, 2)$ EK PACKINGS

Let  $X = \mathbb{Z}_5 \times [3]$ . Consider the base-set  $\mathcal{D}$ :

$$\begin{aligned} & \{\{0_1, 0_2\}, \{0_3, 1_1\}\} \quad \{\{0_1, 0_3\}, \{1_2, 1_3\}\} \quad \{\{0_1, 1_2\}, \{2_3, 3_3\}\} \\ & \{\{0_1, 1_3\}, \{3_2, 4_3\}\} \quad \{\{0_1, 3_2\}, \{0_2, 2_2\}\} \end{aligned}$$

We obtain a  $(15, 2)$  EK packing of size 25 by considering the set-pairs:

$$\{D + i : i \in \mathbb{Z}_5, D \in \mathcal{D}\}.$$

Let  $X = \mathbb{Z}_3 \times [7]$ . Consider the base-set  $\mathcal{D}$ :

$$\begin{aligned} & \{\{0_1, 0_2\}, \{0_3, 0_4\}\} \quad \{\{0_1, 0_3\}, \{0_5, 0_6\}\} \quad \{\{0_1, 0_4\}, \{0_7, 1_1\}\} \\ & \{\{0_1, 0_5\}, \{0_2, 1_2\}\} \quad \{\{0_1, 0_6\}, \{1_3, 1_5\}\} \quad \{\{0_1, 0_7\}, \{1_4, 1_6\}\} \\ & \{\{0_1, 1_2\}, \{1_7, 2_2\}\} \quad \{\{0_1, 1_3\}, \{2_3, 2_6\}\} \quad \{\{0_1, 1_4\}, \{2_5, 2_7\}\} \\ & \{\{0_2, 0_3\}, \{1_4, 1_5\}\} \quad \{\{0_2, 0_4\}, \{0_6, 1_3\}\} \quad \{\{0_2, 0_6\}, \{1_6, 2_4\}\} \\ & \{\{0_2, 1_7\}, \{2_3, 2_7\}\} \quad \{\{0_2, 2_5\}, \{1_7, 2_6\}\} \quad \{\{0_3, 1_5\}, \{1_7, 2_5\}\} \\ & \{\{0_3, 2_5\}, \{0_4, 0_7\}\} \quad \{\{0_4, 1_5\}, \{1_4, 2_6\}\} \end{aligned}$$

We obtain a  $(21, 2)$  EK packing of size 51 by considering the set-pairs:

$$\{D + i : i \in \mathbb{Z}_3, D \in \mathcal{D}\}.$$