

# Small EK designs, EKGDDs and EK packings

## 1. 3-( $n, 2$ ) EK DESIGNS

A 3-(4, 2) EK design:

$$\{\{0, 1\}, \{2, 3\}\} \quad \{\{0, 2\}, \{1, 3\}\} \quad \{\{0, 3\}, \{1, 2\}\}$$

Consider a base-set  $\mathcal{D}$ :

$$\{\{0, 1\}, \{2, 4\}\} \quad \{\{0, 1\}, \{3, 5\}\} \quad \{\{0, 2\}, \{3, 1\}\}$$

A 3-(6, 2) EK design is obtained by considering the set-pairs:

$$\{D + i : i \in \mathbb{Z}_6, D \in \mathcal{D}\}.$$

## 2. EKGDDs

A ( $2^4, 2$ ) EKGDD with groups  $\{\{i, i + 4\} : i \in \mathbb{Z}_4\}$ :

$$\begin{aligned} &\{\{0, 1\}, \{2, 3\}\} \quad \{\{0, 6\}, \{1, 7\}\} \quad \{\{0, 3\}, \{5, 6\}\} \\ &\{\{4, 5\}, \{6, 7\}\} \quad \{\{4, 2\}, \{5, 3\}\} \quad \{\{4, 7\}, \{1, 2\}\} \end{aligned}$$

A ( $1^8 3, 2$ ) EKGDD with groups  $\{\{i\} : i \in \mathbb{Z}_8\} \cup \{\infty_1, \infty_2, \infty_3\}$ :

$$\begin{aligned} &\{\{\infty_1, 0\}, \{1, 2\}\} \quad \{\{\infty_1, 1\}, \{3, 4\}\} \quad \{\{\infty_1, 2\}, \{5, 6\}\} \\ &\{\{\infty_1, 3\}, \{0, 7\}\} \quad \{\{\infty_2, 0\}, \{4, 5\}\} \quad \{\{\infty_2, 1\}, \{2, 6\}\} \\ &\{\{\infty_2, 5\}, \{1, 7\}\} \quad \{\{\infty_2, 6\}, \{0, 3\}\} \quad \{\{\infty_3, 2\}, \{4, 7\}\} \\ &\{\{\infty_3, 3\}, \{2, 5\}\} \quad \{\{\infty_3, 4\}, \{3, 6\}\} \quad \{\{\infty_3, 7\}, \{0, 1\}\} \\ &\{\{4, 6\}, \{5, 7\}\} \end{aligned}$$

A ( $1^{16} 3, 2$ ) EKGDD with groups  $\{\{i\} : i \in \mathbb{Z}_{16}\} \cup \{\infty_1, \infty_2, \infty_3\}$ :

$$\begin{aligned} &\{\{1, \infty_2\}, \{9, 13\}\} \quad \{\{3, 6\}, \{5, \infty_2\}\} \quad \{\{4, 15\}, \{6, 12\}\} \\ &\{\{1, 6\}, \{8, 14\}\} \quad \{\{0, 1\}, \{6, 10\}\} \quad \{\{2, 7\}, \{11, 13\}\} \\ &\{\{3, \infty_1\}, \{13, 15\}\} \quad \{\{0, 5\}, \{13, \infty_3\}\} \quad \{\{0, \infty_1\}, \{12, 14\}\} \\ &\{\{8, 9\}, \{11, 15\}\} \quad \{\{2, 8\}, \{4, \infty_1\}\} \quad \{\{4, 7\}, \{10, \infty_2\}\} \\ &\{\{0, 6\}, \{9, \infty_1\}\} \quad \{\{1, \infty_1\}, \{5, 11\}\} \quad \{\{1, 8\}, \{3, \infty_3\}\} \\ &\{\{3, 12\}, \{10, \infty_3\}\} \quad \{\{0, 11\}, \{3, 4\}\} \quad \{\{2, 10\}, \{8, \infty_3\}\} \\ &\{\{2, 9\}, \{3, 10\}\} \quad \{\{0, \infty_2\}, \{1, 11\}\} \quad \{\{0, 12\}, \{7, 8\}\} \\ &\{\{2, \infty_3\}, \{6, 15\}\} \quad \{\{5, 13\}, \{11, 12\}\} \quad \{\{5, 14\}, \{8, 10\}\} \\ &\{\{4, 8\}, \{7, 9\}\} \quad \{\{9, \infty_3\}, \{13, 14\}\} \quad \{\{4, 10\}, \{13, \infty_1\}\} \\ &\{\{0, 7\}, \{5, 15\}\} \quad \{\{6, \infty_3\}, \{7, 11\}\} \quad \{\{2, 12\}, \{9, \infty_2\}\} \\ &\{\{8, 15\}, \{13, \infty_2\}\} \quad \{\{5, 10\}, \{15, \infty_2\}\} \quad \{\{3, 13\}, \{6, 14\}\} \\ &\{\{4, 9\}, \{5, \infty_3\}\} \quad \{\{0, 14\}, \{2, \infty_2\}\} \quad \{\{1, 14\}, \{7, 15\}\} \\ &\{\{1, 3\}, \{4, 12\}\} \quad \{\{3, 9\}, \{7, \infty_1\}\} \quad \{\{2, 14\}, \{5, 12\}\} \\ &\{\{4, 11\}, \{14, 15\}\} \quad \{\{6, 11\}, \{10, 12\}\} \quad \{\{1, 7\}, \{2, \infty_1\}\} \end{aligned}$$

A ( $1^{16} 7, 2$ ) EKGDD with groups  $\{\{i\} : i \in \mathbb{Z}_{16}\} \cup \{\infty_i : i \in [7]\}$ :

$$\begin{aligned} &\{\{0, 6\}, \{3, \infty_6\}\} \quad \{\{2, \infty_5\}, \{5, 6\}\} \quad \{\{2, \infty_2\}, \{8, 12\}\} \\ &\{\{1, 9\}, \{6, \infty_2\}\} \quad \{\{0, 3\}, \{11, \infty_7\}\} \quad \{\{2, 8\}, \{15, \infty_6\}\} \\ &\{\{1, \infty_6\}, \{4, 13\}\} \quad \{\{5, 14\}, \{11, \infty_4\}\} \quad \{\{5, 15\}, \{9, \infty_3\}\} \\ &\{\{8, \infty_3\}, \{9, 12\}\} \quad \{\{0, 10\}, \{15, \infty_2\}\} \quad \{\{0, \infty_5\}, \{13, 14\}\} \\ &\{\{7, \infty_7\}, \{14, 15\}\} \quad \{\{2, 4\}, \{14, \infty_3\}\} \quad \{\{5, 7\}, \{8, \infty_2\}\} \\ &\{\{5, 8\}, \{6, 14\}\} \quad \{\{8, 13\}, \{11, \infty_1\}\} \quad \{\{3, 6\}, \{14, \infty_4\}\} \\ &\{\{2, 12\}, \{4, \infty_4\}\} \quad \{\{10, \infty_2\}, \{11, 13\}\} \quad \{\{1, \infty_4\}, \{8, 11\}\} \\ &\{\{2, 11\}, \{7, \infty_5\}\} \quad \{\{3, 14\}, \{12, \infty_2\}\} \quad \{\{1, 3\}, \{7, \infty_1\}\} \\ &\{\{5, 11\}, \{12, \infty_6\}\} \quad \{\{0, \infty_1\}, \{2, 6\}\} \quad \{\{7, 14\}, \{13, \infty_6\}\} \\ &\{\{1, 8\}, \{3, \infty_7\}\} \quad \{\{3, 7\}, \{4, \infty_3\}\} \quad \{\{0, 8\}, \{4, \infty_5\}\} \\ &\{\{9, 10\}, \{14, \infty_5\}\} \quad \{\{6, \infty_7\}, \{12, 13\}\} \quad \{\{10, 13\}, \{12, \infty_3\}\} \\ &\{\{0, 1\}, \{5, \infty_3\}\} \quad \{\{6, \infty_1\}, \{7, 15\}\} \quad \{\{4, 7\}, \{5, \infty_7\}\} \\ &\{\{7, 9\}, \{10, 12\}\} \quad \{\{11, 14\}, \{15, \infty_3\}\} \quad \{\{2, \infty_6\}, \{3, 9\}\} \\ &\{\{1, 7\}, \{9, \infty_5\}\} \quad \{\{4, 6\}, \{11, \infty_2\}\} \quad \{\{4, 9\}, \{13, \infty_1\}\} \\ &\{\{6, 8\}, \{10, \infty_3\}\} \quad \{\{3, \infty_1\}, \{5, 10\}\} \quad \{\{3, 15\}, \{13, \infty_5\}\} \\ &\{\{3, \infty_4\}, \{9, 15\}\} \quad \{\{0, \infty_6\}, \{1, 12\}\} \quad \{\{0, 11\}, \{9, \infty_1\}\} \\ &\{\{1, \infty_2\}, \{2, 15\}\} \quad \{\{4, 12\}, \{15, \infty_5\}\} \quad \{\{2, 5\}, \{13, \infty_7\}\} \\ &\{\{5, \infty_6\}, \{10, 15\}\} \quad \{\{1, 4\}, \{10, \infty_4\}\} \quad \{\{0, \infty_4\}, \{7, 10\}\} \\ &\{\{1, \infty_1\}, \{12, 14\}\} \quad \{\{2, \infty_7\}, \{10, 11\}\} \quad \{\{4, \infty_7\}, \{6, 9\}\} \\ &\{\{0, 13\}, \{8, \infty_4\}\} \end{aligned}$$

## 3. EK PACKINGS

A 2-(4, 2) EK packing of size one:

$$\{\{0, 1\}, \{2, 3\}\}$$

A 2-(5, 2) EK packing of size one:

$$\{\{0, 1\}, \{2, 3\}\}$$

A 2-(6, 2) EK packing of size two:

$$\{\{0, 1\}, \{2, 3\}\} \quad \{\{0, 2\}, \{4, 5\}\}$$

A 2-(7, 2) EK packing of size four:

$$\begin{aligned} &\{\{0, 1\}, \{2, 3\}\} \quad \{\{0, 2\}, \{4, 5\}\} \quad \{\{0, 4\}, \{1, 6\}\} \\ &\{\{1, 3\}, \{5, 6\}\} \end{aligned}$$

A 2-(13, 2) EK packing of size 18:

$$\begin{aligned} &\{\{0, 1\}, \{2, 3\}\} \quad \{\{0, 2\}, \{4, 5\}\} \quad \{\{0, 3\}, \{6, 7\}\} \\ &\{\{0, 4\}, \{1, 8\}\} \quad \{\{0, 5\}, \{9, 10\}\} \quad \{\{0, 6\}, \{11, 12\}\} \\ &\{\{1, 2\}, \{6, 8\}\} \quad \{\{1, 3\}, \{5, 9\}\} \quad \{\{1, 4\}, \{7, 10\}\} \\ &\{\{2, 4\}, \{3, 9\}\} \quad \{\{2, 5\}, \{7, 11\}\} \quad \{\{2, 9\}, \{10, 12\}\} \\ &\{\{3, 7\}, \{8, 12\}\} \quad \{\{3, 8\}, \{10, 11\}\} \quad \{\{4, 8\}, \{5, 12\}\} \\ &\{\{4, 9\}, \{6, 11\}\} \quad \{\{5, 10\}, \{6, 12\}\} \quad \{\{6, 9\}, \{7, 8\}\} \end{aligned}$$