Cooling Codes

Thermal-Management Coding for High-Performance Interconnects

Han Mao Kiah, Nanyang Technological University, Singapore

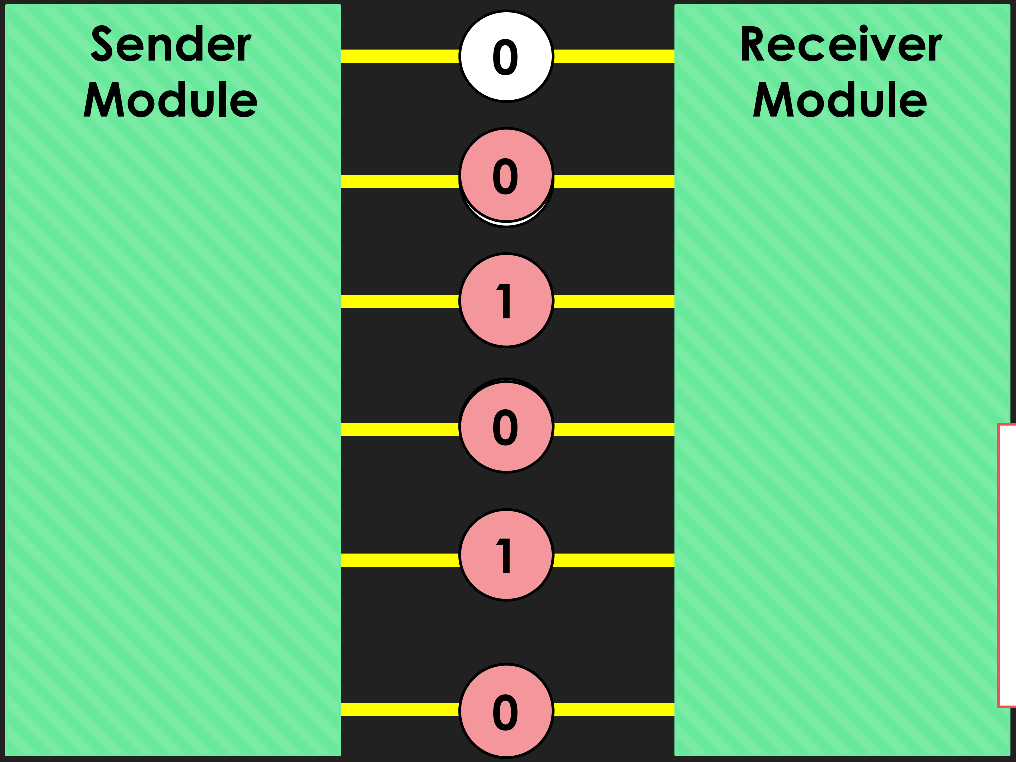
Joint work with:

Yeow Meng Chee, Nanyang Technological University

Tuvi Etzion, Technion, Israel Institute of Technology

Alexander Vardy, University of California, San Diego

DSM Bus Communication



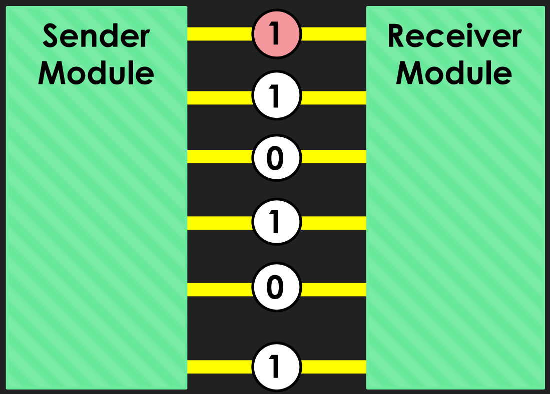
- Modules are connected via wires.
- Each wire has two states to represent a bit of information.

Problem

When a wire switches state, or when there is a state transition, the wire heats up.

Minimizing Switching Activity

- Previous work focus on reducing the number of state transitions.
- Encoding techniques:
  - Bus-Invert (Stan and Burleson 1995)
  - Thermal Spreading (Wang et al. 2007)
- Information theoretic analysis (Sotiriadis et al. 2003, Koch et al. 2009)



Bus-Invert

- Introduces one redundant wire.
- Chooses to send x or its complement $\bar{x}$ so that the number of state transitions is at most $n/2$ for n wires.

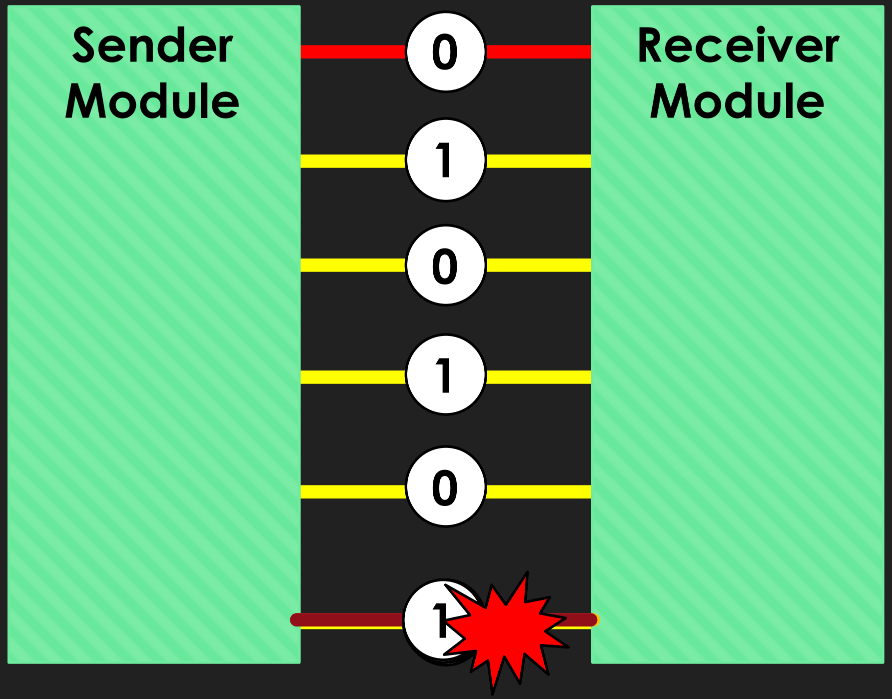
Controlling Peak Temperature

How?

- Avoiding state transitions on the hottest wires.

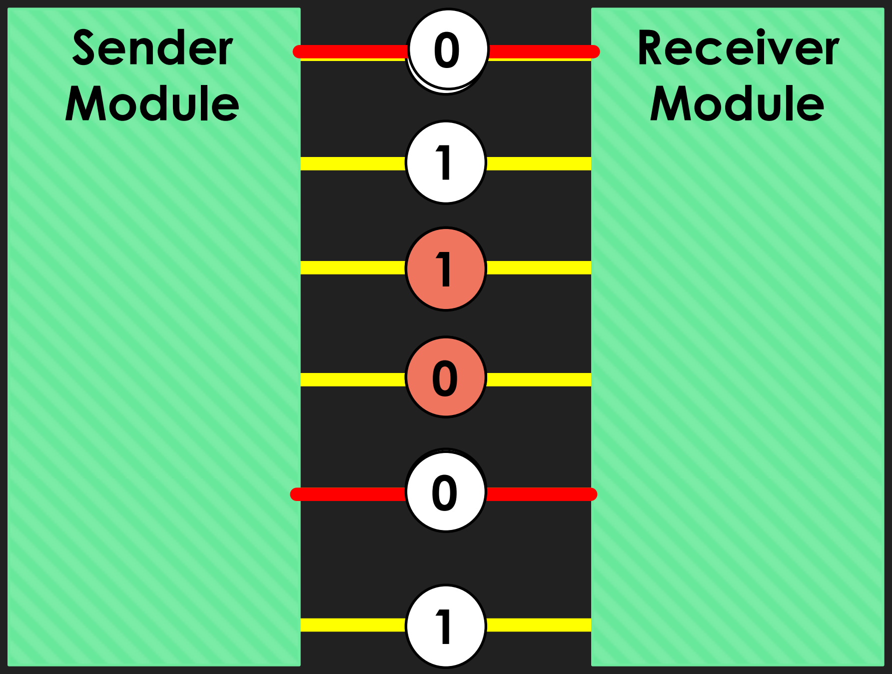
Why?

- To handle anomalous events.
- Source signals are not usually uniformly distributed.



Key Features of Coding Scheme

- Consider a bus comprising n wires.
- **Property A(t)**: Every transmission does not cause state transitions on the t hottest wires.
- **Property B(w)**: Every transmission causes state transitions on at most w wires.
- **Property C(e)**: Correct up to at most e transmission errors.



$n = 6$ $t = 2$ $w = 2$

This talk

Property A(t): Every transmission does not cause state transitions on the t hottest wires.

Property B(w): Every transmission causes state transitions on at most w wires.

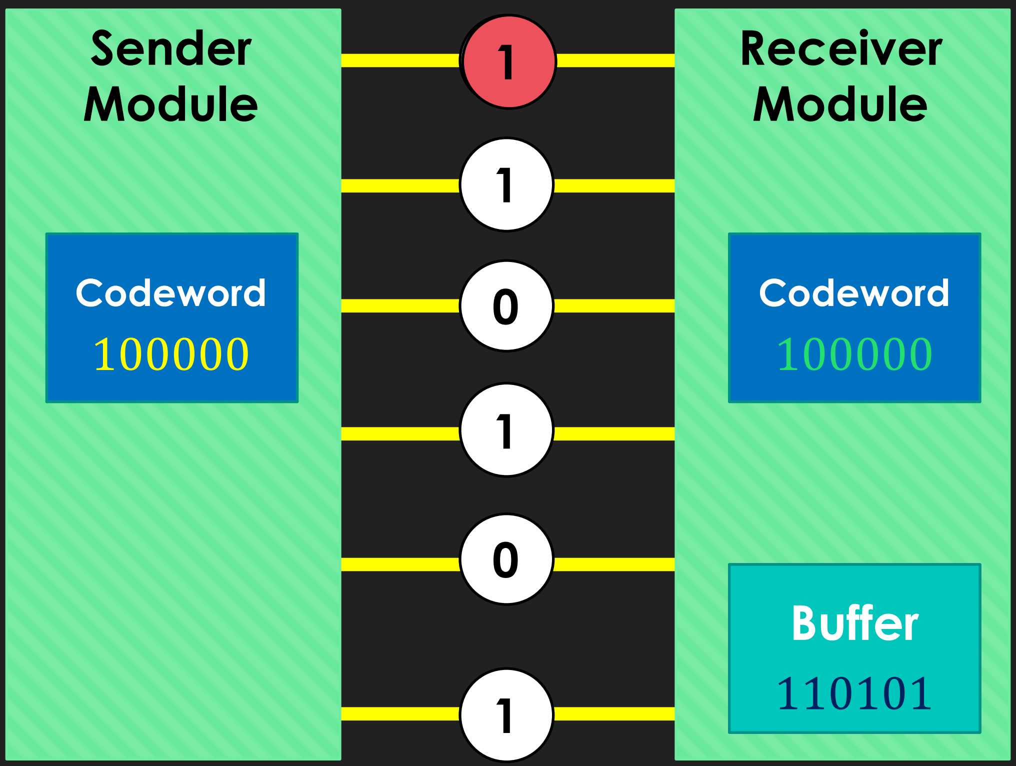
Property C(e): Correct up to at most e transmission errors.

- Code constructions that achieve
  - **Property B(w)** only - Optimal
  - **Property A(t)** only – Optimal redundancy
  - **Property A(t)** + **Property B(w)** only – Asymptotically optimal redundancy
- Adaptive coding schemes.

In the full paper on arxiv,

- More constructions that achieve
  - **Property A(t)** + **Property C(e)** only
  - **Property B(w)** + **Property C(e)** only
  - **Property A(t)** + **Property B(w)** + **Property C(e)**
- Nonadaptive coding schemes

Adaptive + Differential Coding



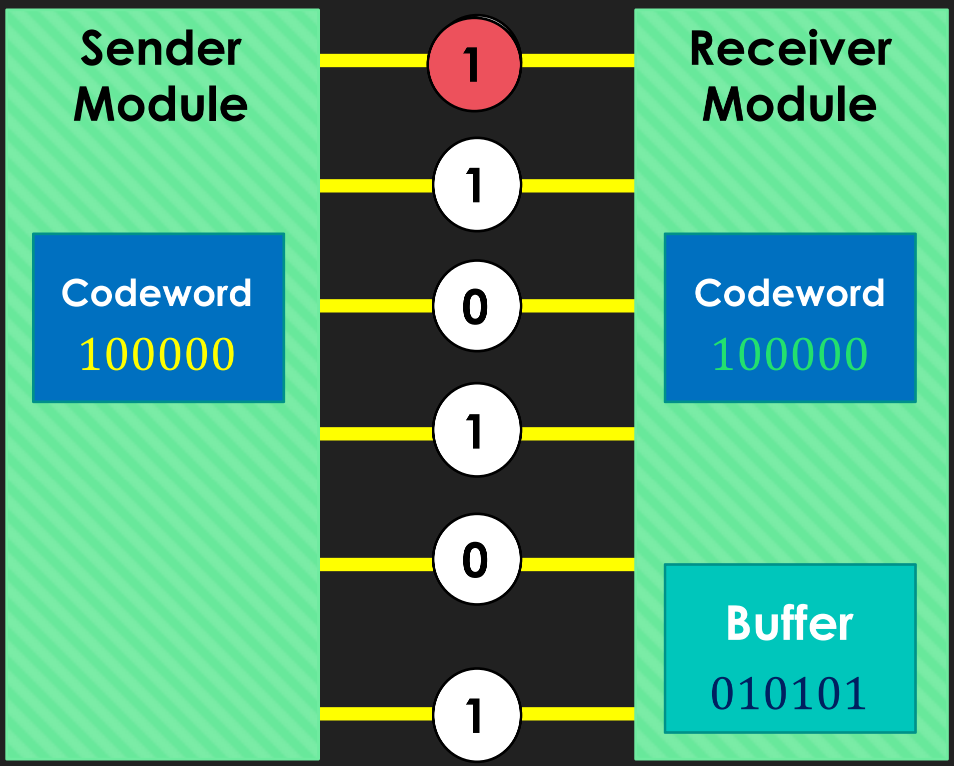
- Suppose Sender wants to communicate

  100000.
- We change the states to

  110101.
- Receiver adds the current state vector to the vector in the buffer to retrieve the message

  100000.
- Receiver updates the buffer.

Differential Coding and Property B



Property B(w): Every transmission causes state transitions on at most w wires.

- A code satisfies **Property B(w)** if and only if

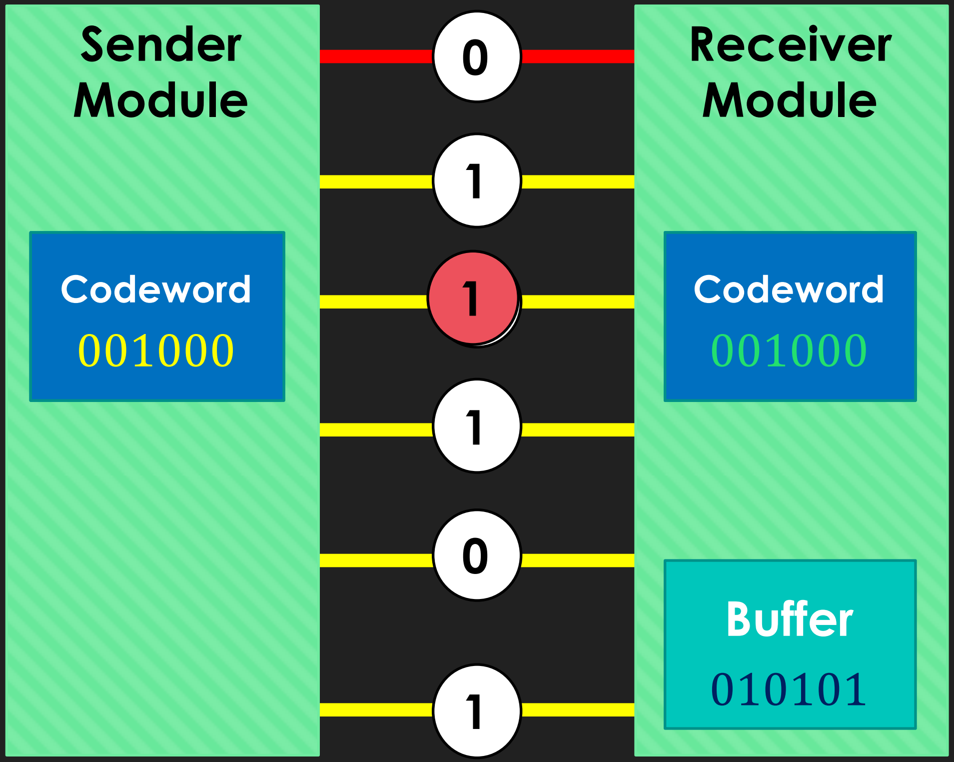
$$\mathrm{wt}(\boldsymbol{x}) \leq w \text{ for codewords } \boldsymbol{x}.$$

- In other words, the optimal code that satisfies **Property B(w)** is

$$J^{+}(n, w) = \{\boldsymbol{x} : \mathrm{wt}(\boldsymbol{x}) \leq w\}.$$

Differential Coding and Property A

Property A(t): Every transmission does not cause state transitions on the t hottest wires.



$t = 1$

A code satisfies **Property A(t)** if and only if

for all codewords $\boldsymbol{x}$,

for all subsets S of size t,

$$\boldsymbol{x}_i = 0 \text{ for all } i \in S.$$

This implies that $\boldsymbol{x} = \boldsymbol{0}$!!

Cooling Codes

Property A(t): Every transmission does not cause state transitions on the t hottest wires.

An (6,2)-cooling code

- Instead of vectors, we consider *codesets*

$$C_1 = \{100000, 101011, 111010, 001011, 010001, 110001, 011010\},$$
$$C_2 = \{010000, 100011, 011101, 110011, 111110, 101110, 001101\},$$
$$C_3 = \{001000, 100111, 111000, 101111, 011111, 010111, 110000\},$$
$$C_4 = \{000100, 100101, 011100, 100001, 111001, 111101, 011000\},$$
$$C_5 = \{000010, 100100, 001110, 100110, 101010, 101000, 001100\},$$
$$C_6 = \{000001, 010010, 000111, 010011, 010101, 010100, 000110\},$$
$$C_7 = \{110110, 001001, 110101, 111111, 111100, 001010, 000011\},$$
$$C_8 = \{011011, 110010, 101100, 101001, 011110, 000101, 110111\},$$
$$C_9 = \{111011, 011001, 010110, 100010, 001111, 110100, 101101\}.$$

- An (n,t)-*cooling code* is a collection of M codesets $C_1, C_2, \ldots, C_M$ such that
  - $C_1, C_2, \ldots, C_M$ are disjoint subsets.
  - for all codesets C_i, for all subsets S of size t, there exist a word $\boldsymbol{x} \in C_i$

$$\boldsymbol{x}_j = 0 \text{ for all } j \in S.$$

Cooling Codes

Property A(t): Every transmission does not cause state transitions on the t hottest wires.

An (6,2)-cooling code

- Consider the codeset C_1 and $S = \{1,2\}$.

$$C_1 = \{100000, 101011, 111010, 001011, 010001, 110001, 011010\}.$$

- An (n, t)-*cooling code* is a collection of M codesets $C_1, C_2, \dots, C_M$ such that
  - $C_1, C_2, \dots, C_M$ are disjoint subsets.
  - for all codesets C_i, for all subsets S of size t, there exist a word $\boldsymbol{x} \in C_i$

$$\boldsymbol{x}_j = 0 \text{ for all } j \in S, \text{or}, \boldsymbol{x}\Big|_S = \mathbf{0}\,.$$

Cooling Codes and Related Codes

Definition

An (n, t)-*cooling code* is a collection of M codesets $C_1, C_2, \dots, C_M$ such that

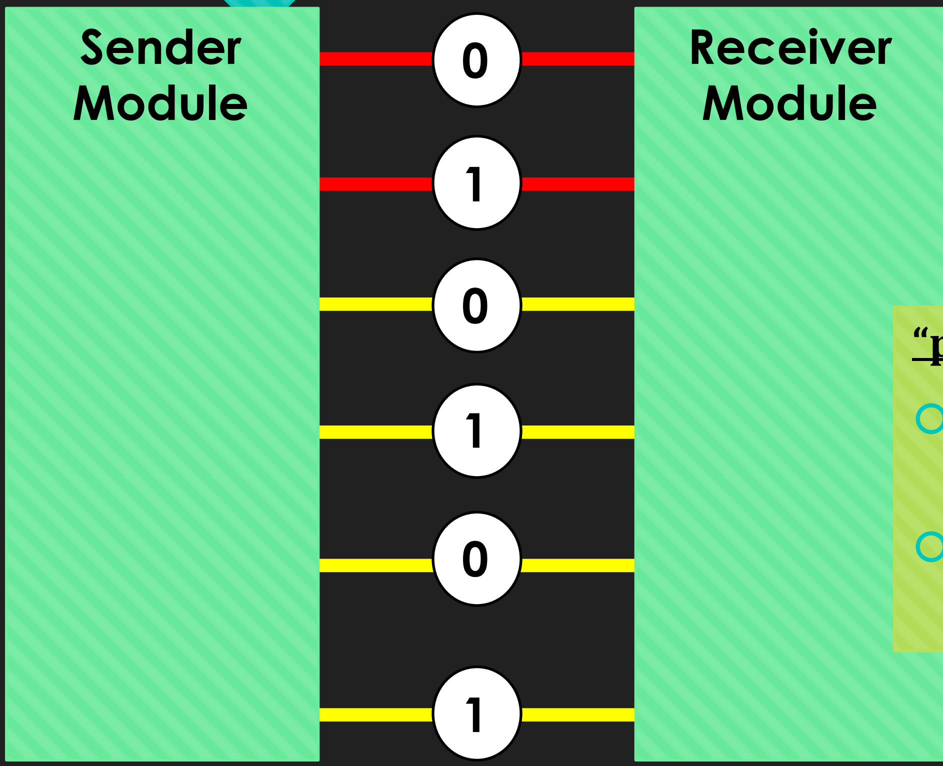
- $C_1, C_2, \dots, C_M$ are disjoint subsets.
- for all codesets C_i, for all subsets S of size t, there exist a word $\boldsymbol{x} \in C_i$

  $\boldsymbol{x}_j = 0$ for all $j \in S$, or

  $$\boldsymbol{x}|_S = \mathbf{0}.$$

- Coding for Stuck-At Cells
  - Additional requirement: $\boldsymbol{x}|_S = \boldsymbol{z}$ for all patterns $\boldsymbol{z}$ of length t
  - Cooling codes require significantly lesser redundancy
  - Dumer (1989) constructed a special class of codes for stuck-at-cells
- Write-Once Memories (WOM) codes
  - Different requirement: $\boldsymbol{x}|_S = \mathbf{1}$
  - Different requirement: S need not be a subset of size t
  - Cooling codes can be used to construct WOM codes.
    - "Explicit Constructions of Finite-Length WOM Codes"
    - ISIT, 30 Jun Fri, 12:30pm, Room: K5

Upper Bound on the Code Size



Lemma

A (n, t)-cooling code has at most 2^{n-t} codesets.

“pro

- S
  t

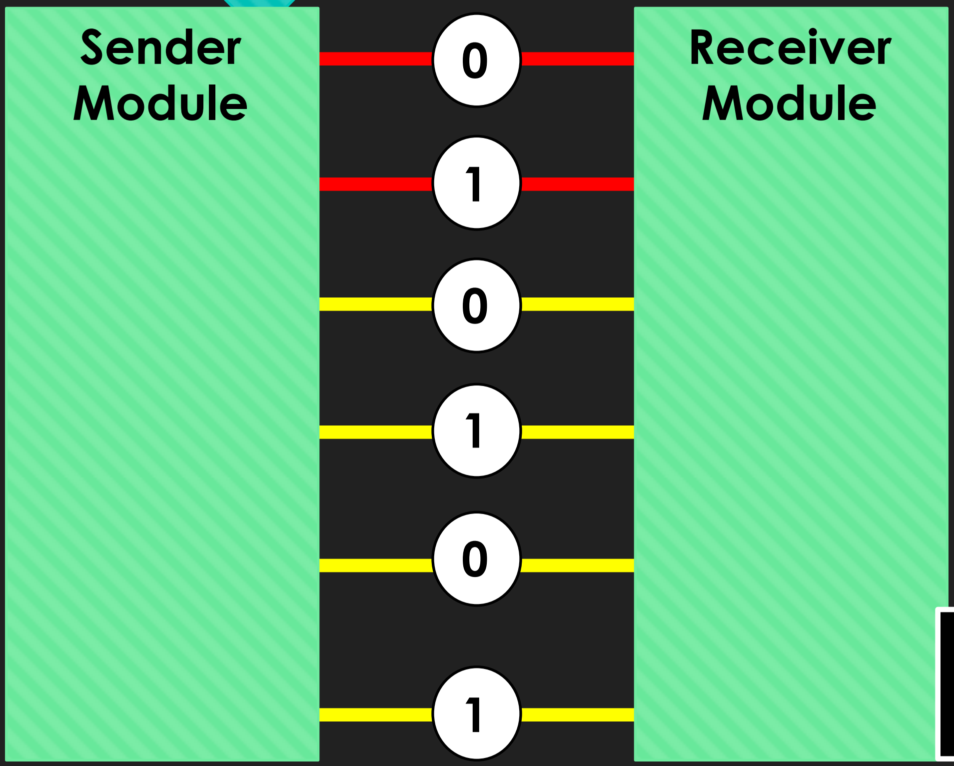
Corollary

If $1 < t < n-1$, an (n, t)-cooling code has at most $2^{n-t} - 1$ codesets.

- Therefore, at most $\mathrm{n} - t$ bits of information can be transmitted.

Property A(t): Every transmission does not cause state transitions on the t hottest wires.

Lower Bound on the Code Size



Lemma

A (n, t)-cooling code has at most 2^{n-t} codesets.

Theorem

If $t + 1 \leq n/2$, there is an (n, t)-cooling code of size at least 2^{n-t-1} codesets.

In other words, construction is optimal in terms of redundancy.

Property A(t): Every transmission does not cause state transitions on the t hottest wires.

Spreads and Partial Spreads

- A partial τ-spread of F_2^n is a collection of τ-dimensional subspaces $V_1, V_2, \ldots, V_M$ of F_2^n such that
  - $V_i \cap V_j = \{\mathbf{0}\}$ for all $i \neq j$.
- If $\bigcup V_i = F_2^n$, then $V_1, V_2, \ldots, V_M$ is called a τ-spread.

A 3-spread of F_2^6

$$
\begin{aligned}
V_1 &= \text{span}\{100000, 101011, 111010\},\\
V_2 &= \text{span}\{010000, 100011, 011101\},\\
V_3 &= \text{span}\{001000, 100111, 111000\},\\
V_4 &= \text{span}\{000100, 100101, 011100\},\\
V_5 &= \text{span}\{000010, 100100, 001110\},\\
V_6 &= \text{span}\{000001, 010010, 000111\},\\
V_7 &= \text{span}\{110110, 001001, 110101\},\\
V_8 &= \text{span}\{011011, 110010, 101100\},\\
V_9 &= \text{span}\{111011, 011001, 010110\}.
\end{aligned}
$$

Spreads yields Cooling Codes

Definition

A partial τ-spread of F_2^n is a collection of τ-dim subspaces $V_1, V_2, \ldots, V_M$ of F_2^n such that $V_i \cap V_j = \{\mathbf{0}\}$ for all $i \neq j$.

Definition

An (n,t)-*cooling code* is a collection of M codesets $C_1, C_2, \ldots, C_M$ such that

- $C_1, C_2, \ldots, C_M$ are disjoint subsets.
- for all codesets C_i, for all subsets S of size t, there exist a word $\boldsymbol{x} \in C_i$
  $\boldsymbol{x}_j = 0$ for all $j \in S$.

Theorem

Let $V_1, V_2, \ldots, V_M$ be a partial $(t+1)$-spread.

Set $V_i^* = V_i \setminus \{\mathbf{0}\}$.

Then $V_1^*, V_2^*, \ldots, V_M^*$ forms an (n,t)-cooling code.

$$
\begin{aligned}
V_1^* &= \mathrm{span}\{100000, 101011, 111010\} \setminus \{\mathbf{0}\},\\
V_2^* &= \mathrm{span}\{010000, 100011, 011101\} \setminus \{\mathbf{0}\},\\
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V_9^* &= \mathrm{span}\{111011, 011001, 010110\} \setminus \{\mathbf{0}\}.
\end{aligned}
$$

A 3-spread of F_2^6 yields a (6,2)-cooling code

‘proof’
Spreads yields Cooling Codes

Theorem

Let $V_1, V_2, \dots, V_M$ be a $(t+1)$-partial spread. Set $V_i^* = V_i \setminus \{\mathbf{0}\}$.

Then $V_1^*, V_2^*, \dots, V_M^*$ forms an (n, t)-cooling code.

Definition

A partial τ-spread of F_2^n is a collection of τ-dim subspaces $V_1, V_2, \dots, V_M$ of F_2^n such that $V_i \cap V_j = \{\mathbf{0}\}$ for all $i \neq j$.

Definition

An (n, t)-*cooling code* is a collection of M codesets $C_1, C_2, \dots, C_M$ such that

- $C_1, C_2, \dots, C_M$ are disjoint subsets.
- for all codesets C_i, for all subsets S of size t, there exist a word $\boldsymbol{x} \in C_i$

$$\boldsymbol{x}_j = 0 \text{ for all } j \in S.$$

$V_i^* \cap V_j^* = \emptyset$ for $i \neq j$ follows from definition.

‘proof’

Spreads yields Cooling Codes

Theorem

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- for all codesets C_i, for all subsets S of size t, there exist a word $\boldsymbol{x} \in C_i$

  $\boldsymbol{x}_j = 0$ for all $j \in S$.

Consider the codeset V_1^* and $S = \{1,2\}$.

$V_1^* = \text{span}\{100000, 101011, 111010\} \setminus \{\mathbf{0}\}$.

Want to find $\boldsymbol{x} \in V_1^*$ such that

$$\boldsymbol{x}_j = 0 \text{ for all } j \in S, \text{or,}$$

$$x_1 = x_2 = 0.$$

‘proof’

Spreads yields Cooling Codes

Consider the codeset V_1^* and $S = \{1,2\}$.

$$V_1^* = \text{span}\{100000, 101011, 111010\} \setminus \{\mathbf{0}\}.$$

Want to find

$$\boldsymbol{x} \in V_1^* \text{ such that } x_1 = x_2 = 0. \qquad (*)$$

- Let K be the collection of vectors that satisfy (*).
- Suppose $\boldsymbol{x}, \boldsymbol{y} \in K$. i.e.
  - $\boldsymbol{x} \in V_1^*$ such that $x_1 = x_2 = 0$.
  - $\boldsymbol{y} \in V_1^*$ such that $y_1 = y_2 = 0$.
- Then $\boldsymbol{x} + \boldsymbol{y} \in K \cup \{\mathbf{0}\}$.
- In other words, $K \cup \{\mathbf{0}\}$ is a vector subspace of V_1.

‘proof’

Spreads yields Cooling Codes

Consider the codeset V_1^* and $S = \{1,2\}$.

$$V_1^* = \text{span}\{100000, 101011, 111010\} \setminus \{\mathbf{0}\}.$$

Want to find

$$\boldsymbol{x} \in V_1^* \text{ such that } x_1 = x_2 = 0. \qquad (*)$$

- Let K be the collection of vectors that satisfy (*).
- In fact, $K \cup \{\mathbf{0}\}$ is the kernel of the map ϕ that projects V_1 onto the coordinates at S.

$$\phi: V_1 \to F_2^{|S|}$$

- Since V_1 has dimension three and its image has dimension two, its kernel $K \cup \{\mathbf{0}\}$ must be nontrivial.
- So, there exists nonzero $\boldsymbol{x}$ that satisfies (*).

Cooling Codes - Property A

Theorem

Let $V_1, V_2, \dots, V_M$ be a $(t+1)$-partial spread.

Set $V_i^* = V_i \setminus \{\mathbf{0}\}$.

Then $V_1^*, V_2^*, \dots, V_M^*$ forms an (n,t)-cooling code.

Theorem (Classic + Etzion and Vardy 2011)

Let $\tau \leq n/2$.

There is a τ-partial spread of size at least $2^{n-\tau}$.

Definition

A partial τ-spread of F_2^n is a collection of τ-dim subspaces $V_1, V_2, \dots, V_M$ of F_2^n such that $V_i \cap V_j = \{\mathbf{0}\}$ for all $i \neq j$.

Definition

An (n,t)-*cooling code* is a collection of M codesets $C_1, C_2, \dots, C_M$ such that

- $C_1, C_2, \dots, C_M$ are disjoint subsets.
- for all codesets C_i, for all subsets S of size t, there exist a word $\boldsymbol{x} \in C_i$

  $\boldsymbol{x}_j = 0$ for all $j \in S$.

Theorem

If $t + 1 \leq n/2$, there is an (n,t)-cooling code of size at least 2^{n-t-1} codesets.

The construction is *optimal in terms of redundancy*.

(Dumer 1989) There exist *efficient encoding* and *decoding* methods for spreads.

Property A and B

Property A(t): Every transmission does not cause state transitions on the t hottest wires.

Property B(w): Every transmission causes state transitions on at most w wires.

- Let $n = 286, t = 30, w = 127$.
- Consider a 40-partial spread of F_2^{286} of size 2^{246}.
- Let $V_1, V_2, \dots, V_{2^{246}}$ be the vector spaces.
- Let $V_i^* = V_i \setminus \{\mathbf{0}\}$ be the codesets.
- Choose a codeset V_i^*.
- For any subset S of size 30, the kernel K of the projection $\phi: V_i \to F_2^{|S|}$ has dimension of at least 10.
- We puncture K at the coordinates in S to obtain a

  subspace K' of dim 10 and length 256.
- By Plotkin bound,

  there exists nonzero $\boldsymbol{x} \in K'$ with $\mathrm{wt}(\boldsymbol{x}) \leq 127$.

Property A and B

Property A(t): Every transmission does not cause state transitions on the t hottest wires.

Property B(w): Every transmission causes state transitions on at most w wires.

- Let $n = 286, t = 30, w = 124$.
- Consider a 40- partial spread of F_2^{286} of size 2^{246}.
- Let $V_1, V_2, \ldots, V_{2^{246}}$ be the vector spaces.
- Let $V_i^* = V_i \setminus \{\mathbf{0}\}$ be the codesets.
- Choose a codeset V_i^*.
- For any subset S of size 30, the kernel K of the projection $\phi: V_i \rightarrow F_2^{|S|}$ has dimension of at least 10.
- We puncture K at the coordinates in S to obtain a

  subspace K' of dim 10 and length 256.
- Checking codetables.de,

  there exists nonzero $\boldsymbol{x} \in K'$ with $\mathrm{wt}(\boldsymbol{x}) \leq 124$.

Property A and B

Theorem

Suppose that $t + r \leq (n + s)/2$. If

i. there is an $[n, s, w + 1]$-linear code, and

ii. an $[n - t, r, w + 1]$-linear code does not exist,

then there is a code that satisfies **Property** $\boldsymbol{A(t)}$ and $\boldsymbol{B(w)}$ of size 2^{n-t-r}.

Property A(t): Every transmission does not cause state transitions on the t hottest wires.

Property B(w): Every transmission causes state transitions on at most w wires.

Cooling Codes:
Thermal-Management Coding for High-Performance Interconnects

Full Paper

- Available at https://arxiv.org/abs/1701.07872

Other Results:

- Construction of (n, t)-cooling code for $t + 1 > n/2$.
- Construction of codes that satisfy **Property** $\boldsymbol{A(t)}$ and $\boldsymbol{B(w)}$ using Baranyai's theorem, concatenation
- Construction of codes that satisfy additional **Property** $\boldsymbol{C(e)}$