Optimal Binary Switch Codes with Small Query Size

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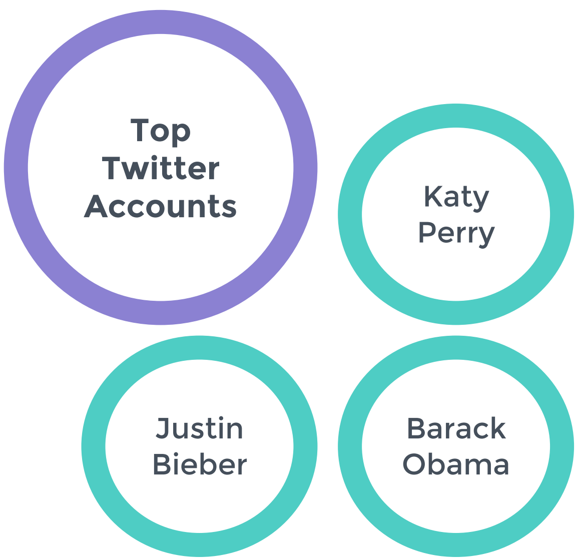
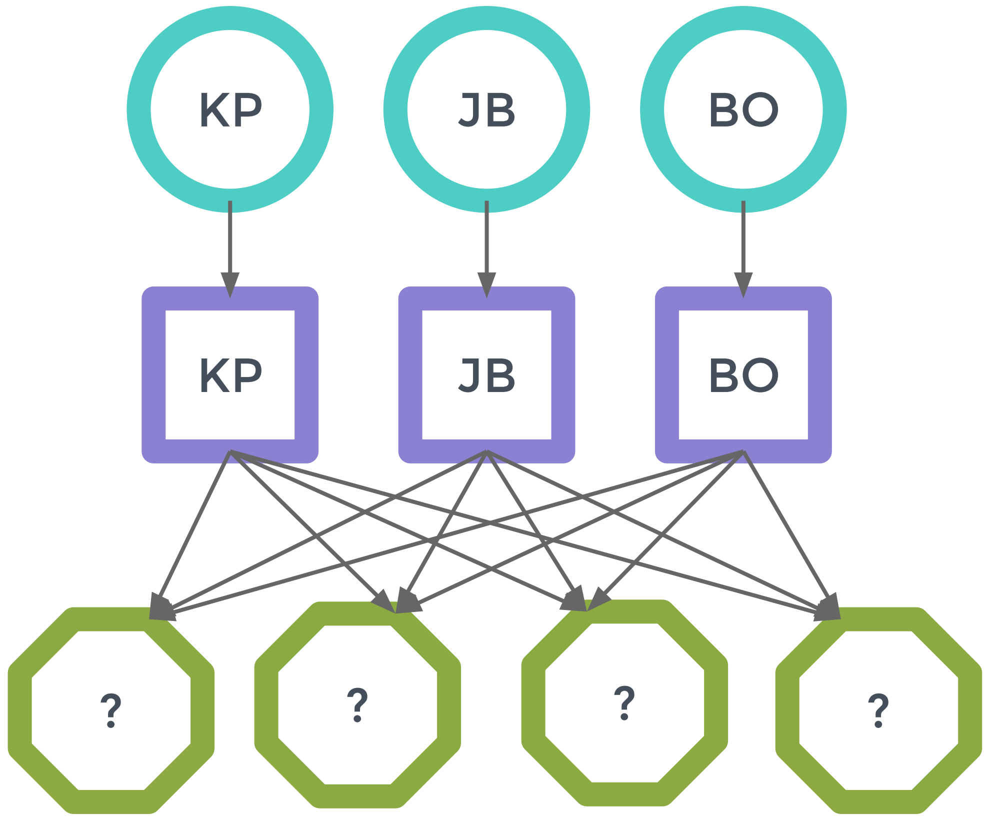
Joint work with Zhiying Wang and Yuval Cassuto

ISIT 2015

Network Switches - Toy Example

- k input ports
- n memory banks
- R output ports





k=3, R=4, n=3

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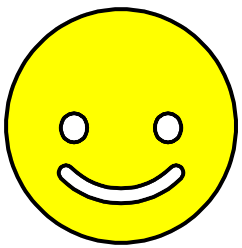
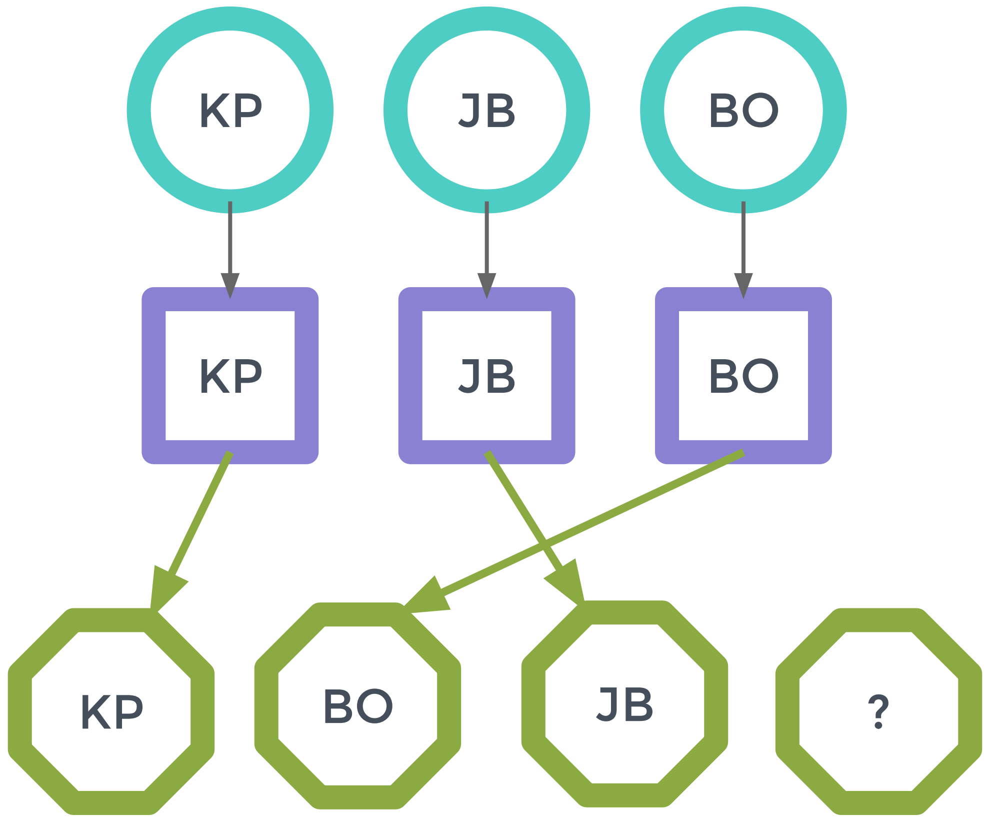


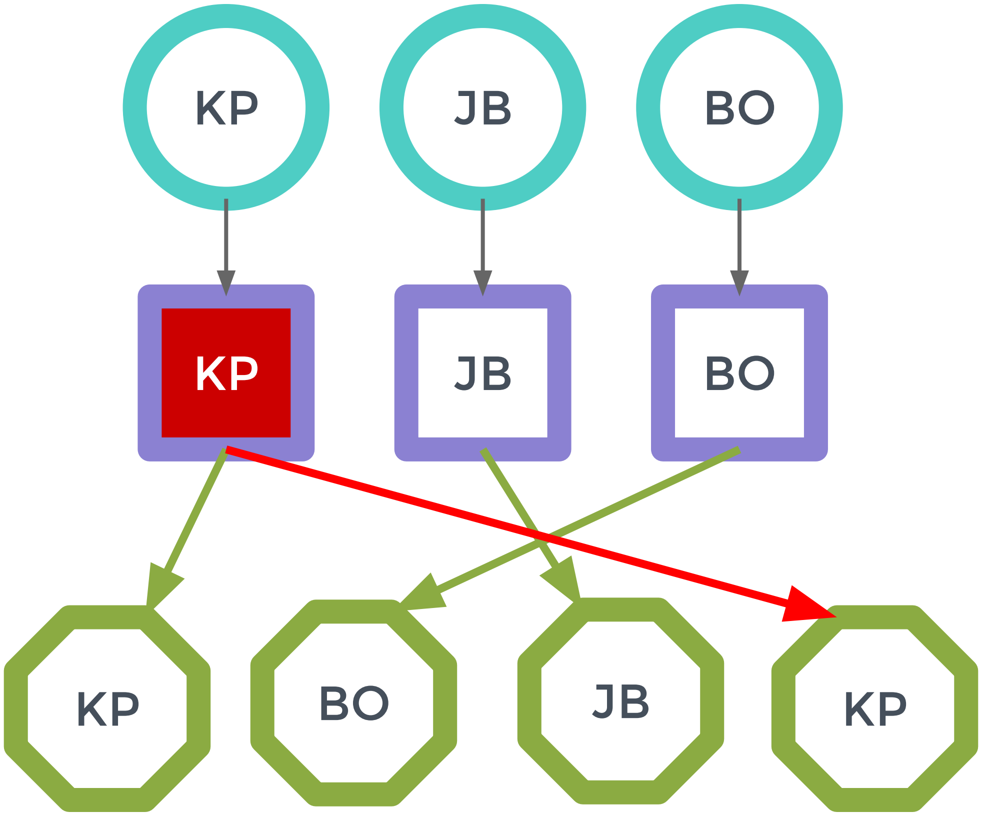


satisfy all requests in parallel

k=3, R=4, n=3

Network Switches - Toy Example

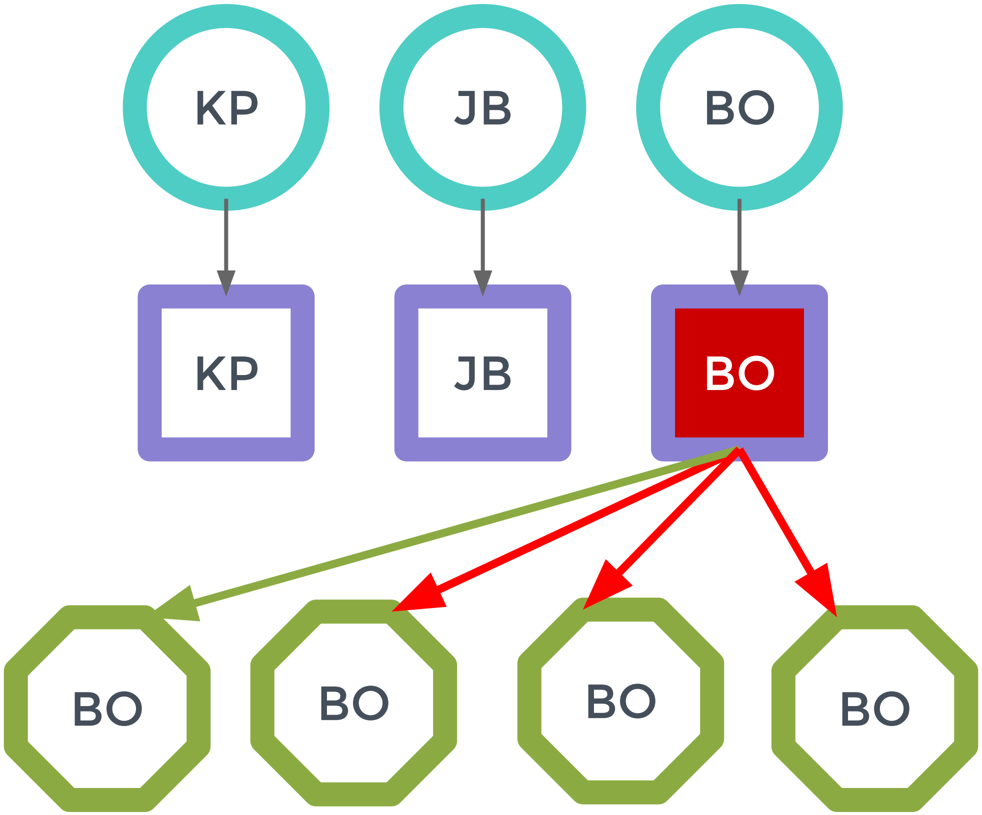
latency!



k=3, R=4, n=3

Network Switches - Toy Example

big latency!



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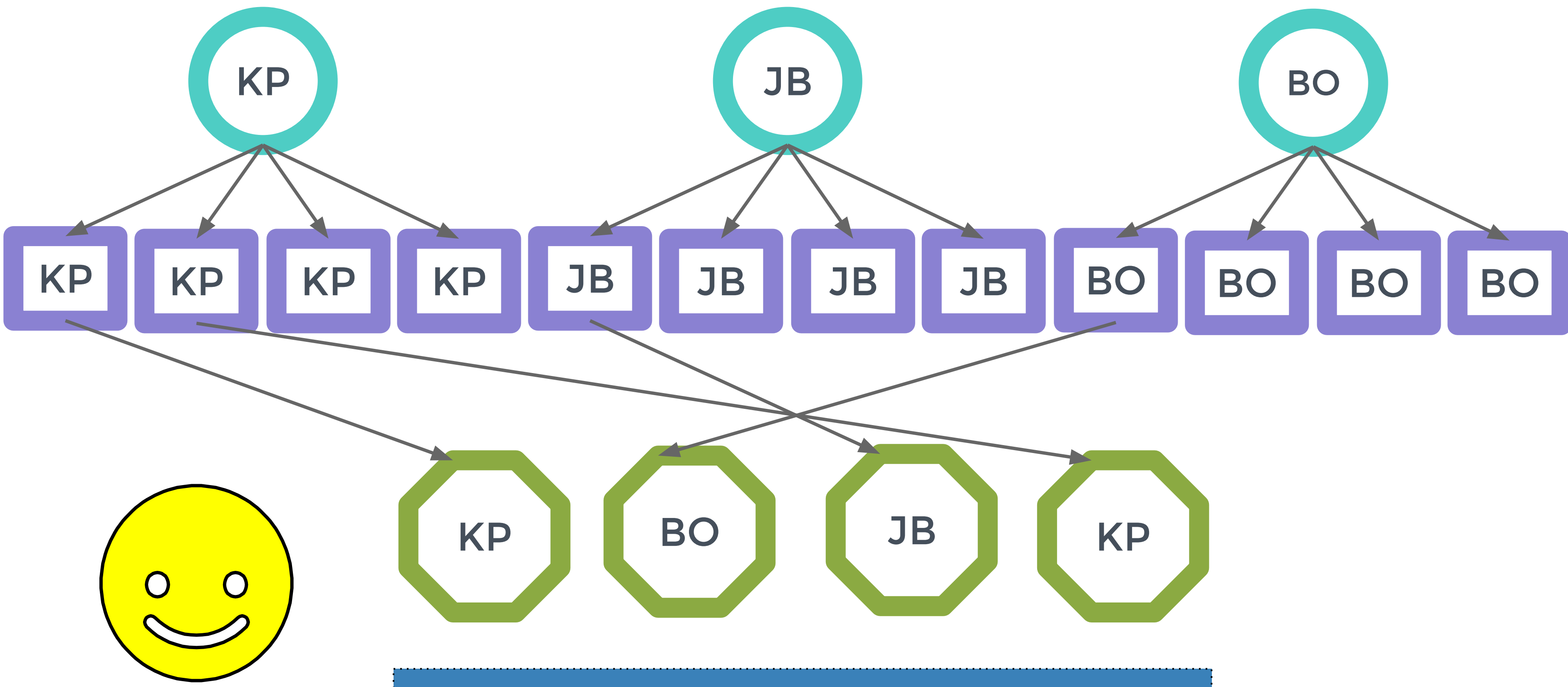
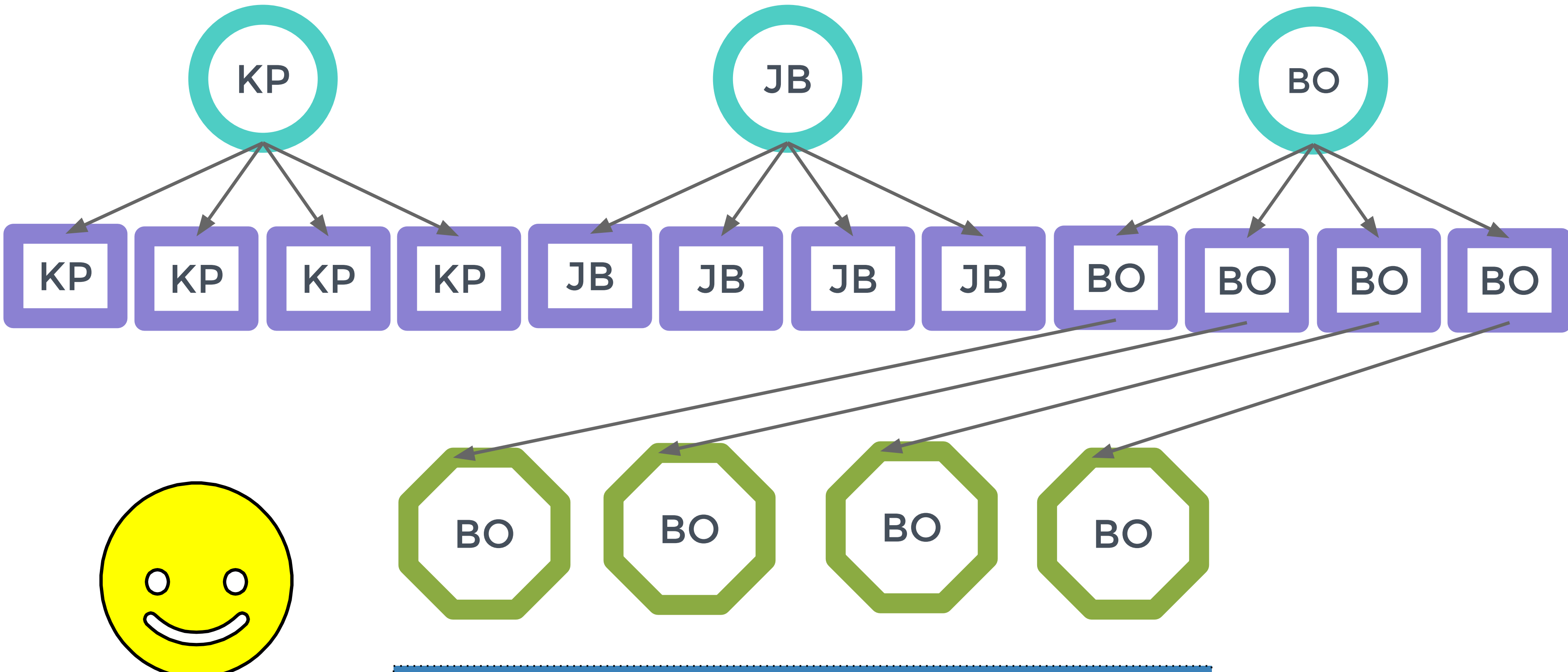


k=3, R=4, n=12

Network Switches - Toy Example



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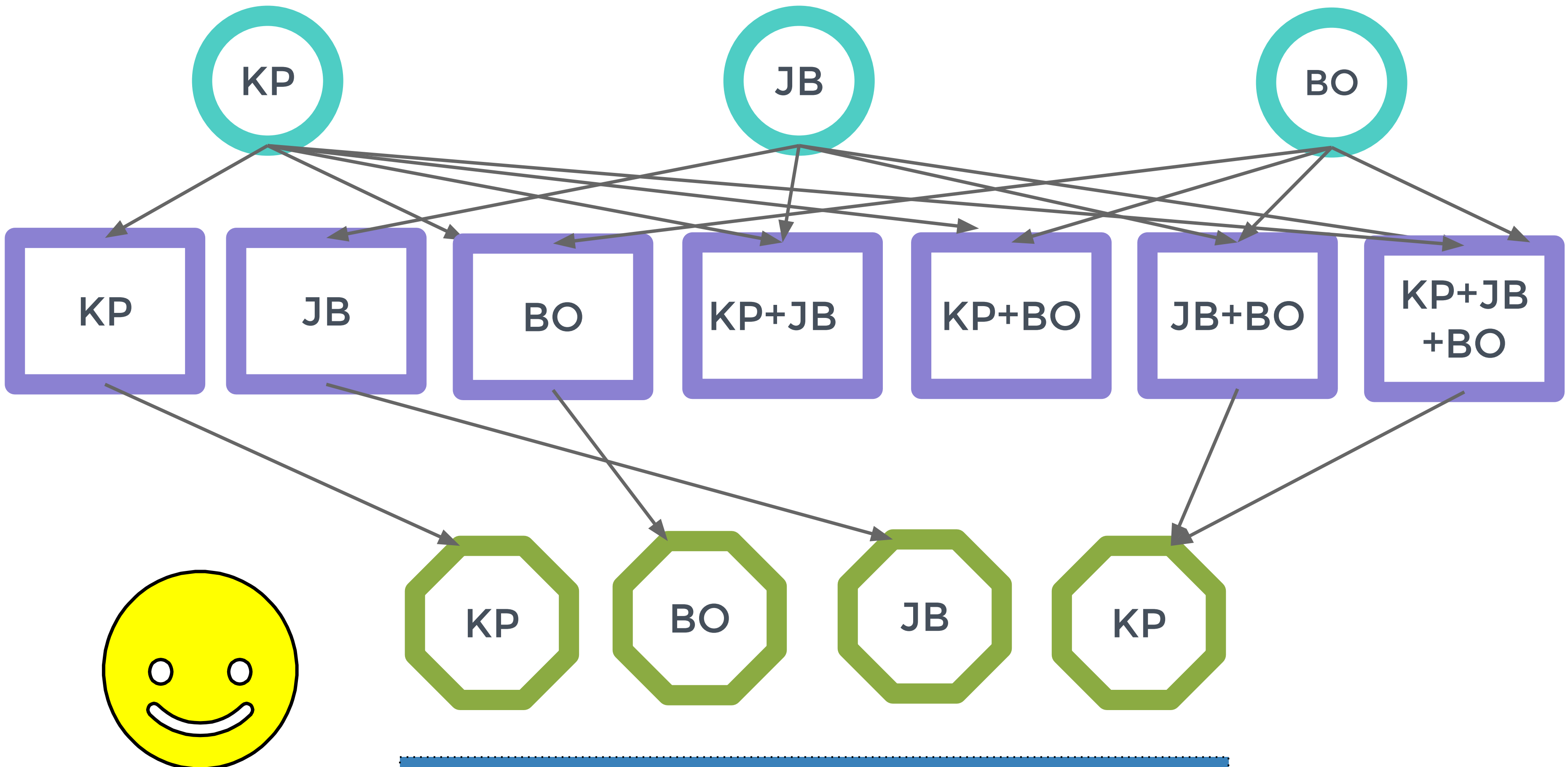
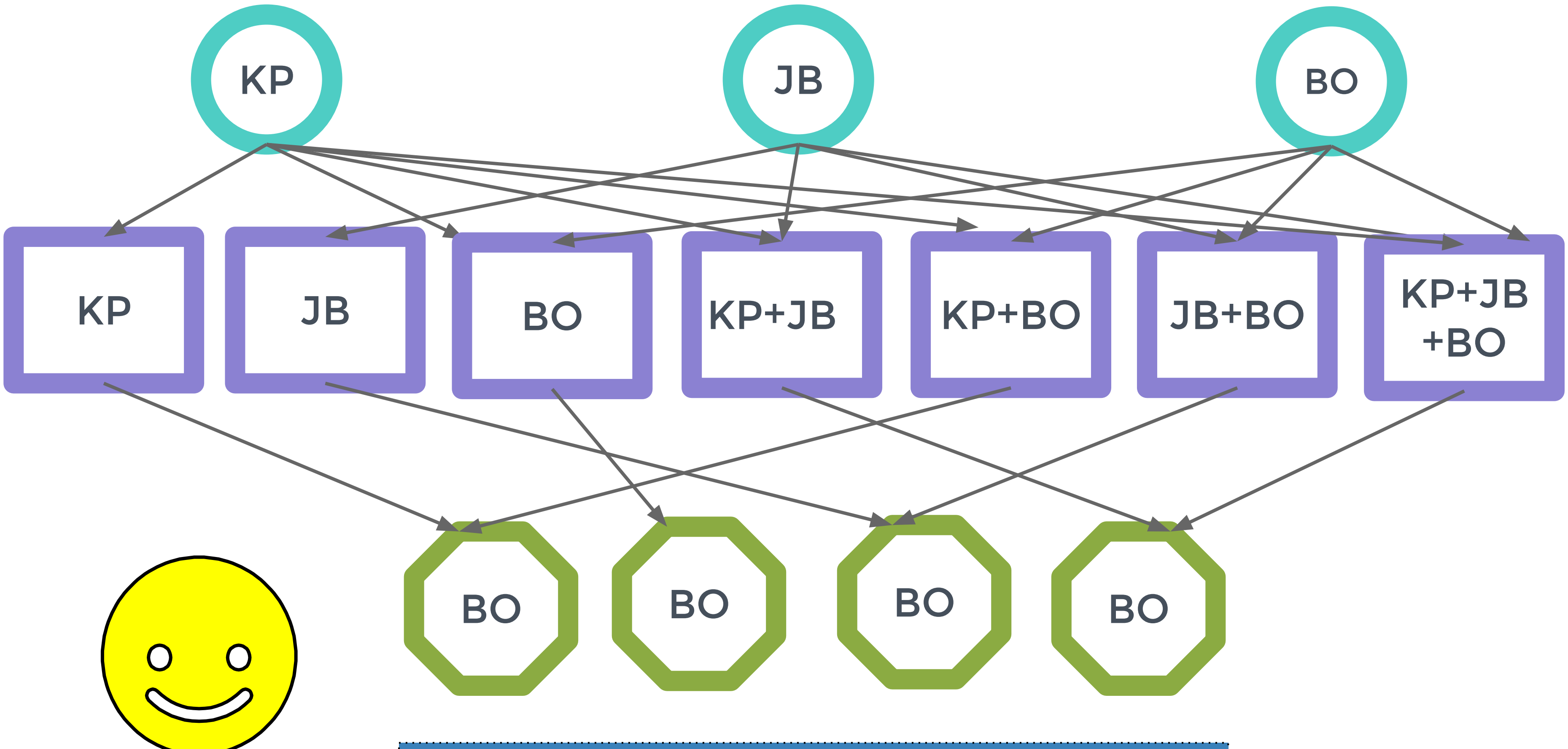


k=4, R=4, n=7

Network Switches - Toy Example



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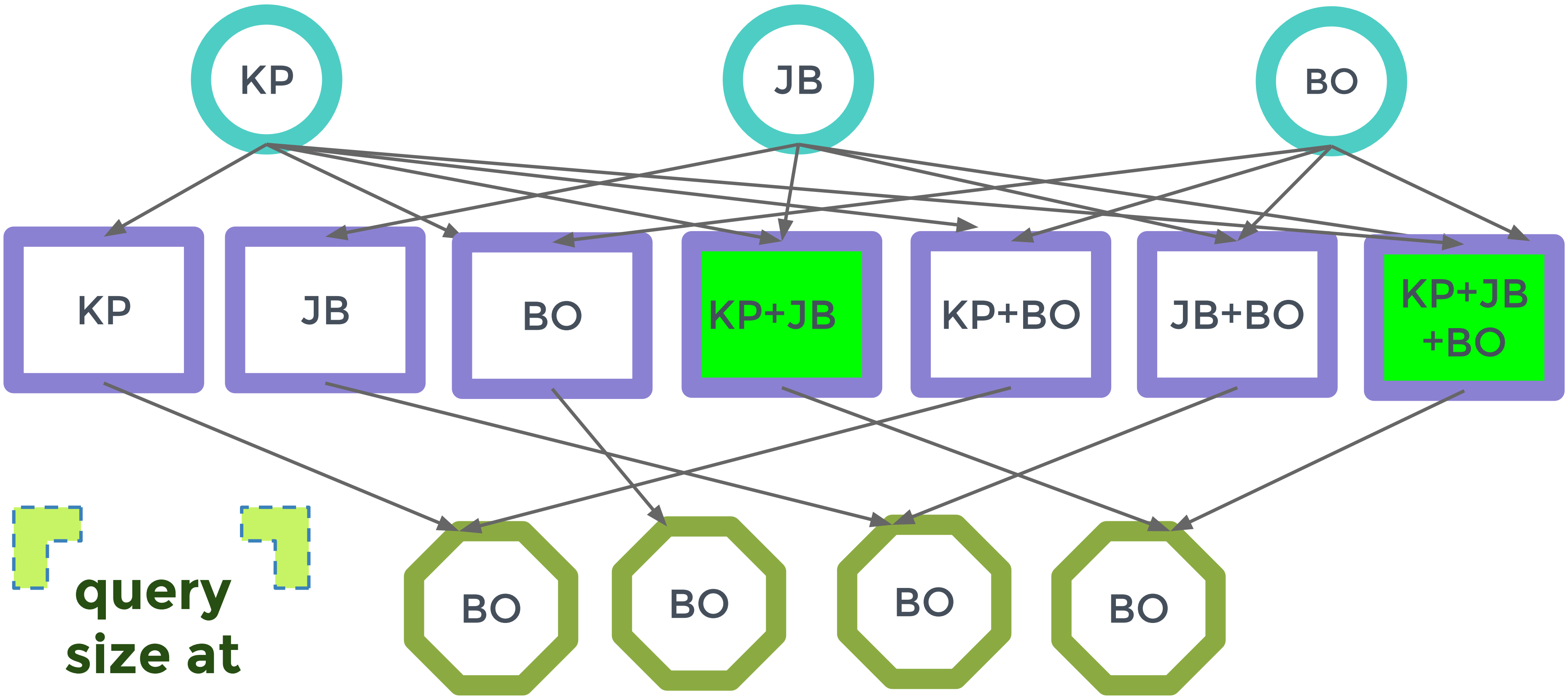
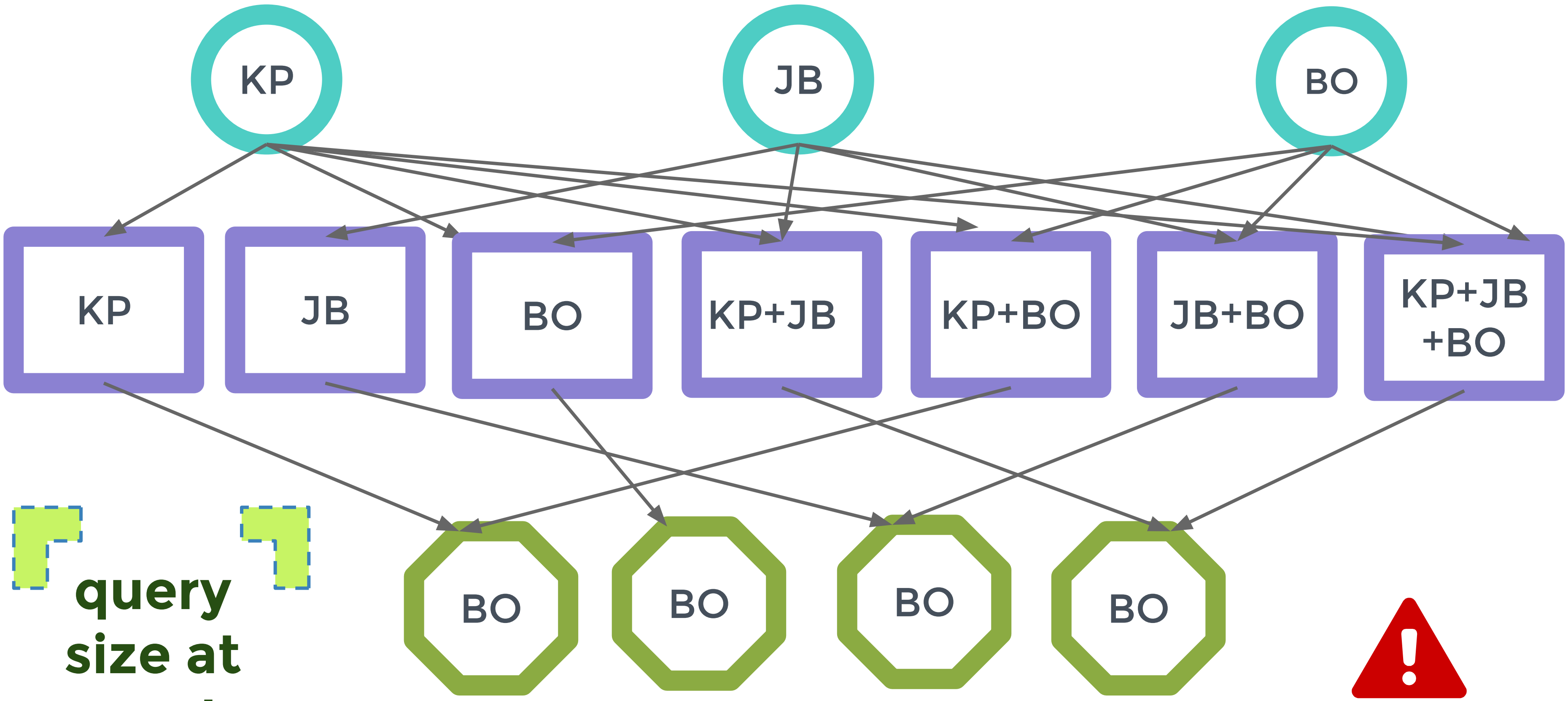


query size at most two

k=4, R=4, n=7

Network Switches - Toy Example

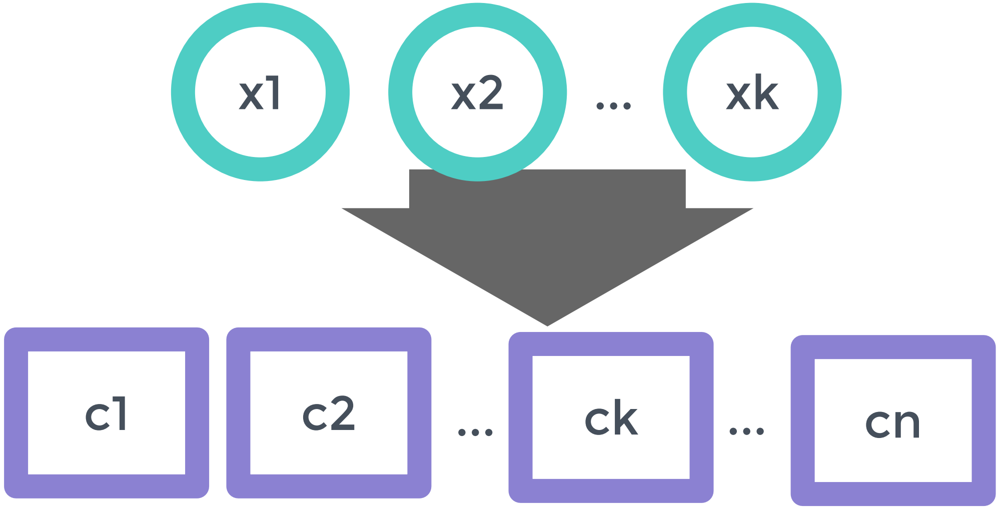


query size at most two

k=4, R=4, n=7

binary alphabet

(n,k,R) Switch Code (Wang et al. ISIT'2013)



Switch code consists of:

- an encoding function $\{0,1\}^k$ to $\{0,1\}^n$,

(n,k,R) Switch Code (Wang et al. ISIT'2013)

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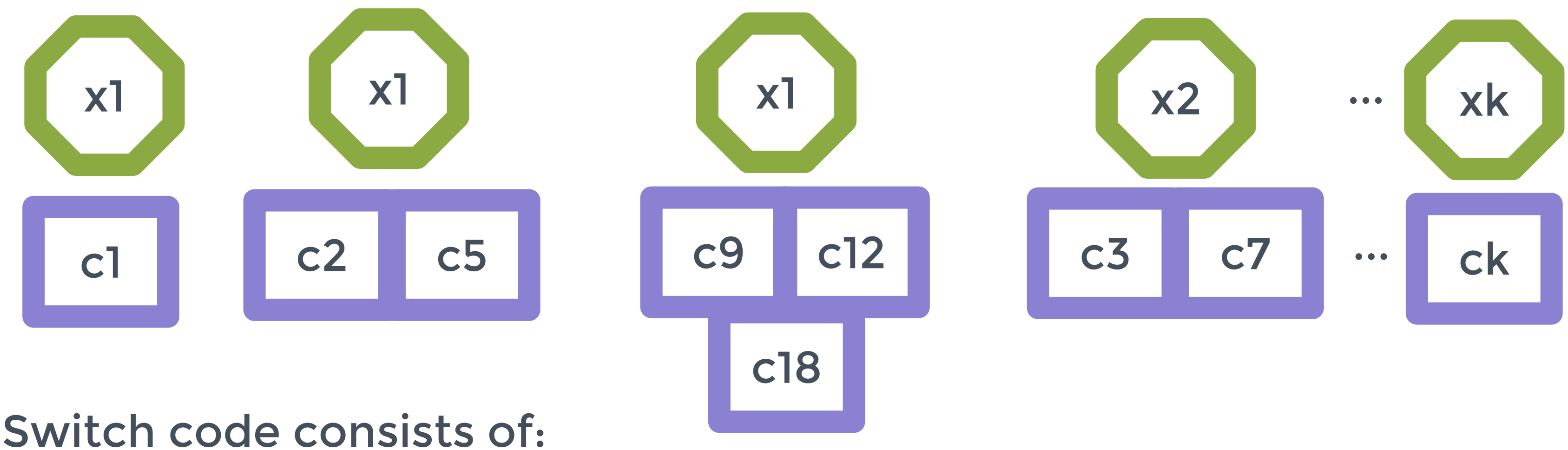
- an encoding function $\{0,1\}^k$ to $\{0,1\}^n$,
- a decoding algorithm such that for any R-multiset of $\{x1, x2, \ldots, xk\}$

(n,k,R) Switch Code (Wang et al. ISIT'2013)



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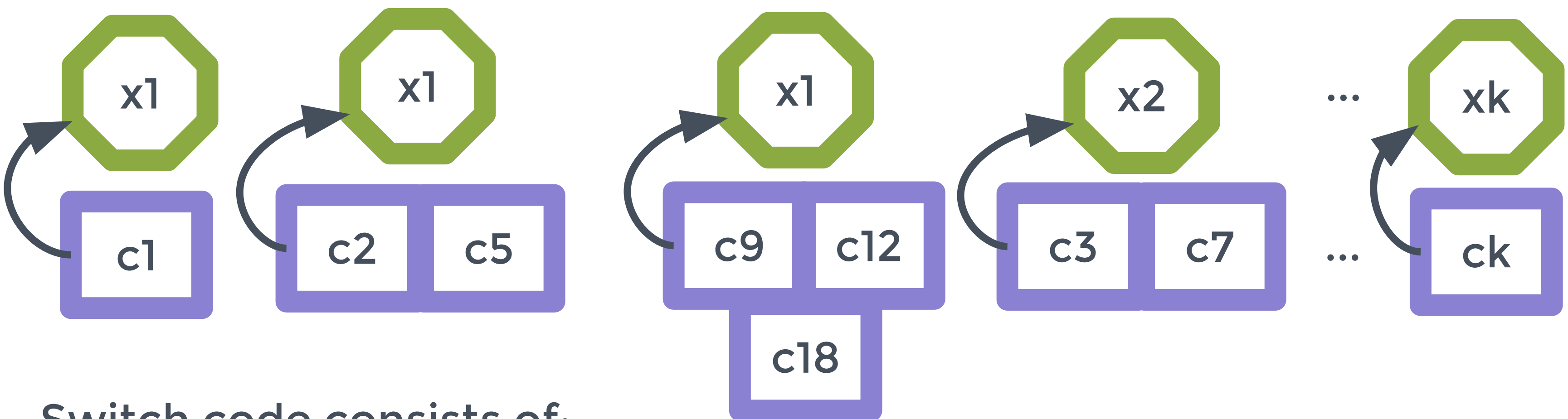

Switch code consists of:

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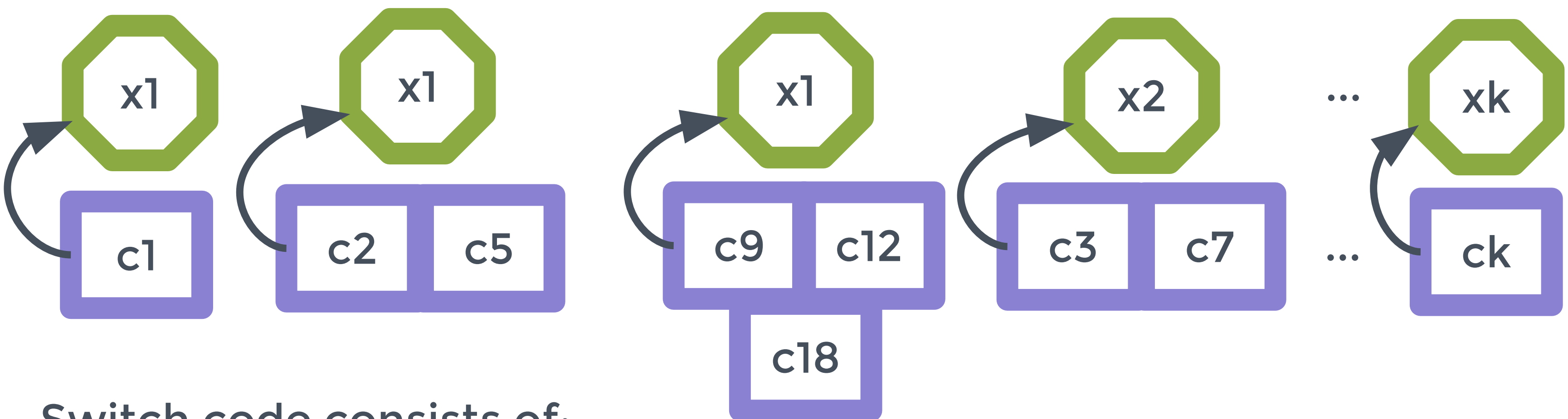

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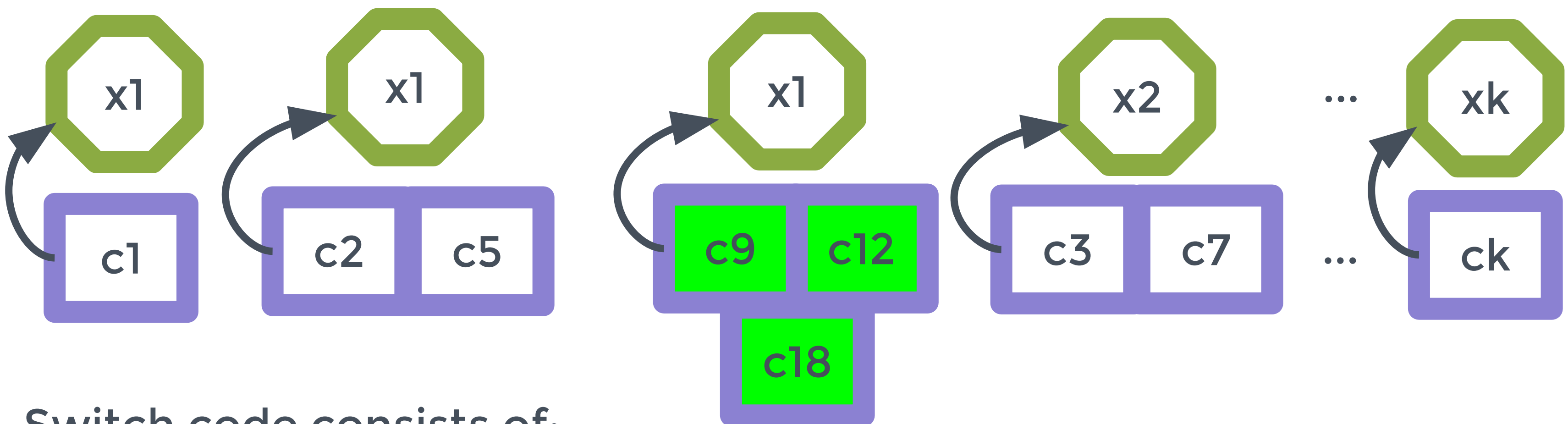
this work: linear encoding and decoding
i.e. decode via simple XOR.

(n,k,R) Switch Code (Wang et al. ISIT'2013)



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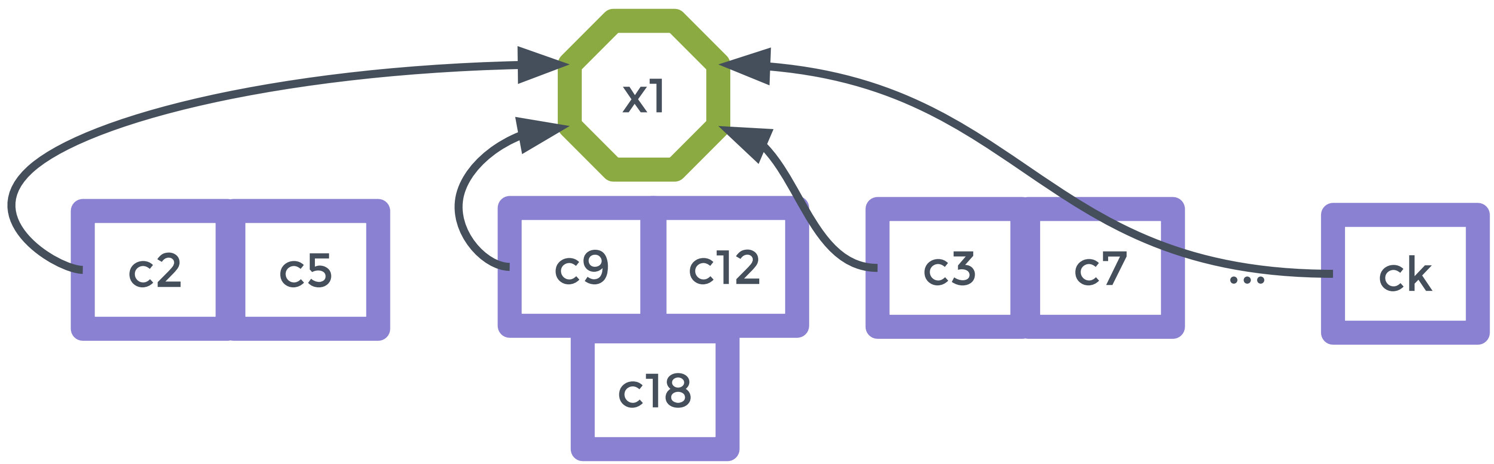
this work: small query size, r.

Related Work - Locally Recoverable Codes with Multiple Alternatives

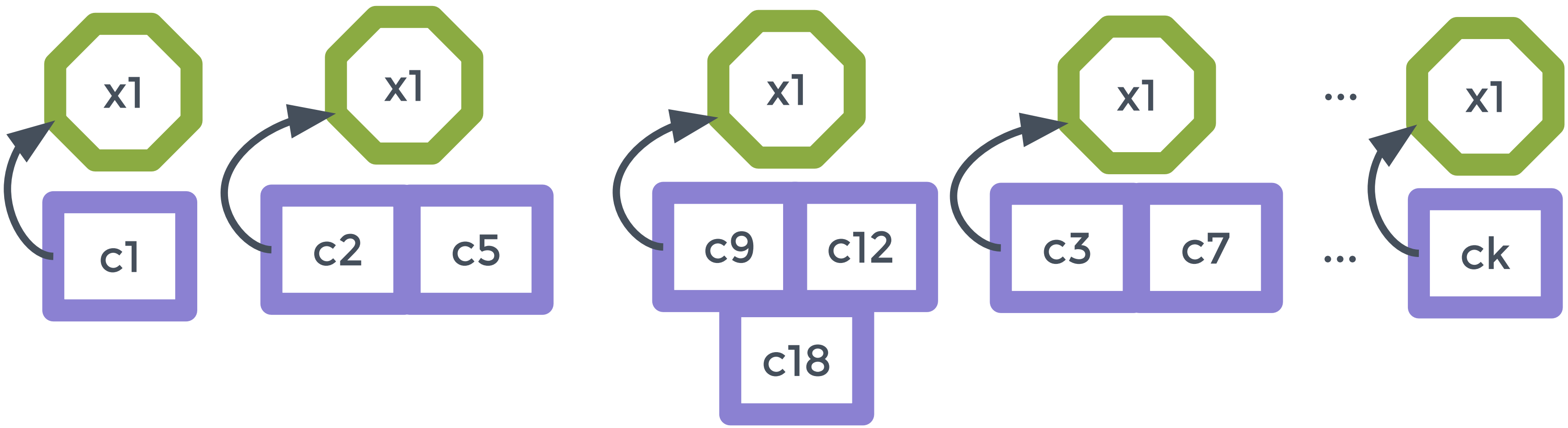


When a node fails, want:

- repair it by accessing a small number of other nodes
- many alternative repair sets

(Oggier and Datta 2011; Pamies-Juarez et al. 2013; Rawat et al. 2014; Tamo and A. Barg 2014)

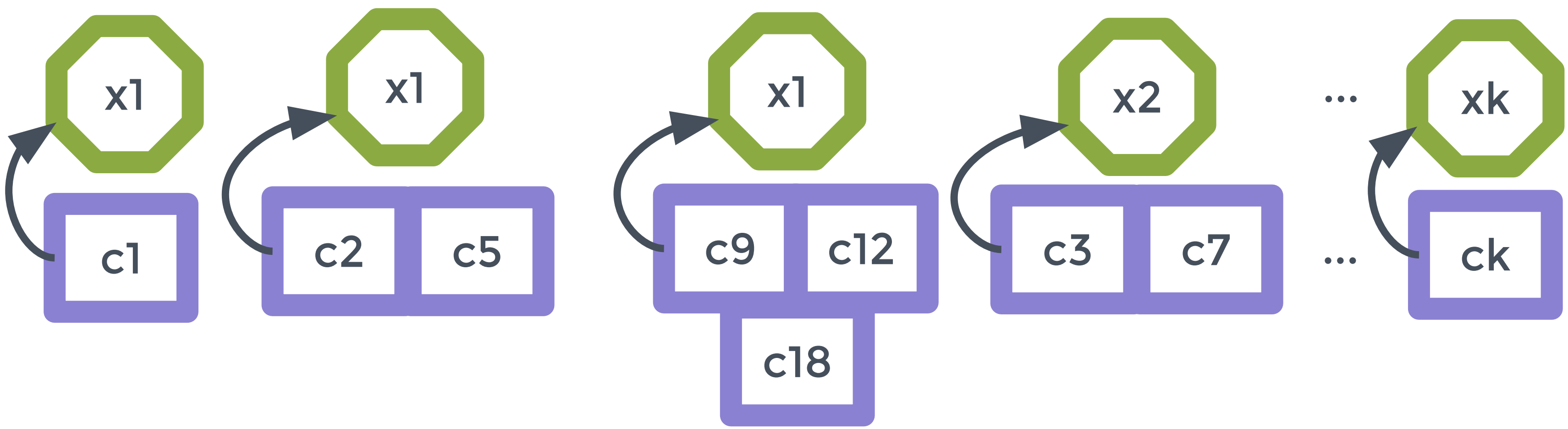
Related Work - Locally Recoverable Codes with Multiple Alternatives



Differences. A switch code

- Requests with different bits
- Interested only in the information bits

Related Work - Primitive Multiset Batch Codes (Ishai et al. STOC'2004)



Switch codes is a specialization of primitive multiset batch codes.

- Ishai et al.: positive rates (so, R<k).
- Switch codes: k is close to R.

Simplex Code as a Switch Code

x1	x2	x3	x1+x2	x1+x3	x2+x3	x1+x2 +x3

A simplex code of dimension k has length 2^k-1.

Example: k=3. So, n=7.

Simplex Code as a Switch Code

x1	x2	x3	x1+x2	x1+x3	x2+x3	x1+x2 +x3

A simplex code of dimension k has length 2^k-1.

Example: k=3. So, n=7 and R=4.

<u>Theorem</u>

A simplex code of dimension k is an

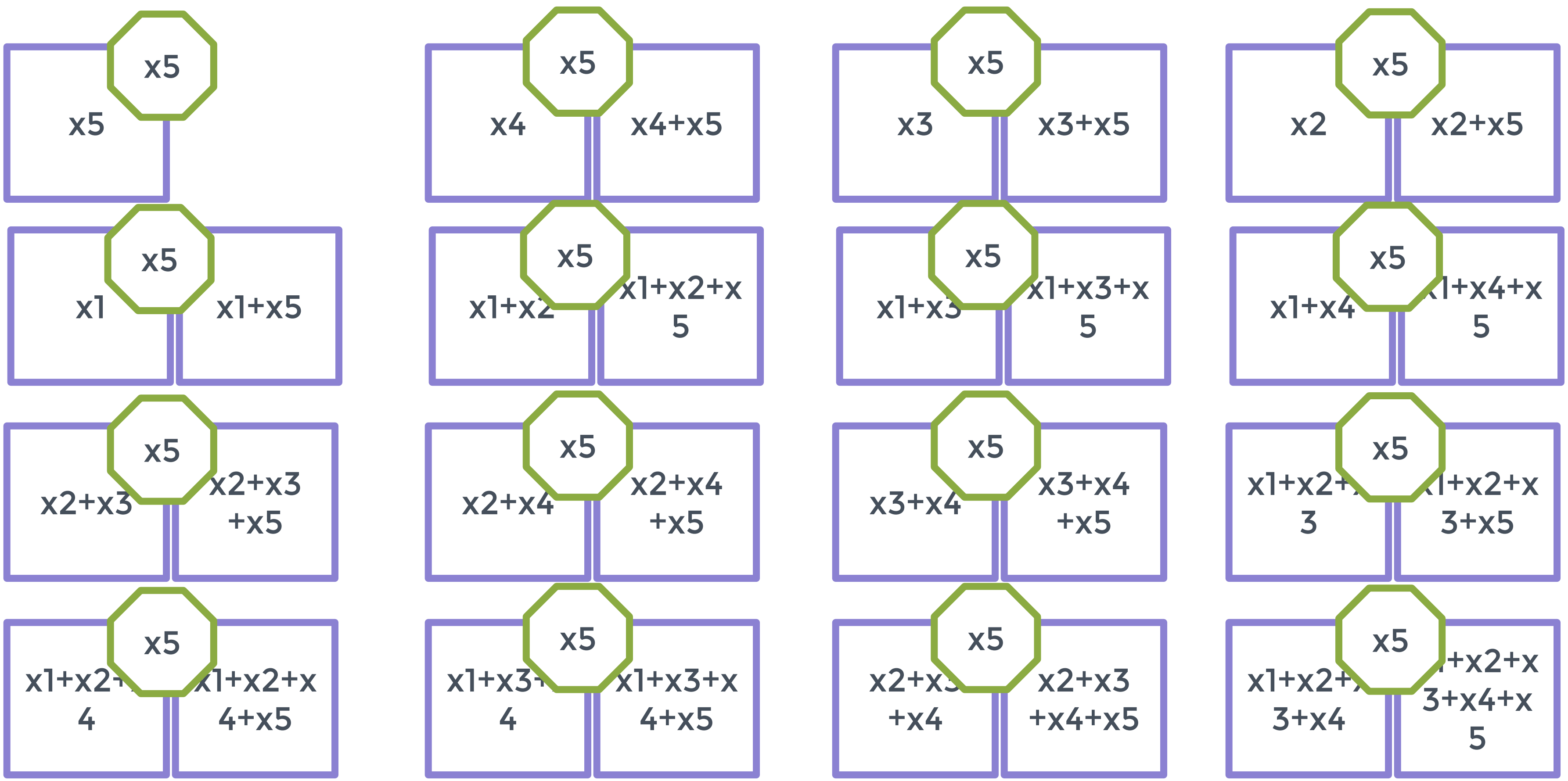
$(n=2^k-1, k, R=2^{k-1})$ switch code

with query size at most two.

Simplex Code with k=5, n=31

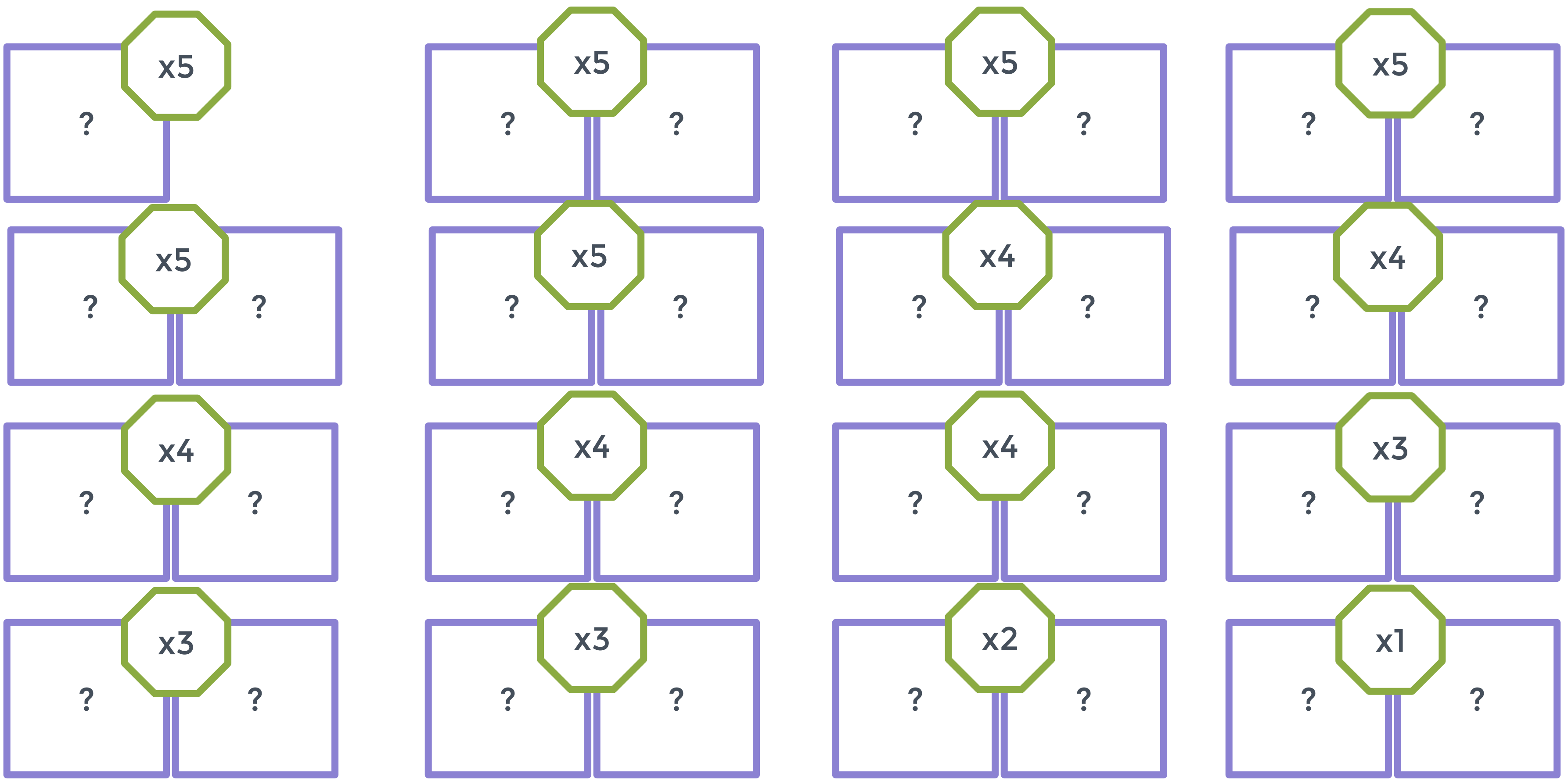
x5		x4	x4+x5	x3	x3+x5	x2	x2+x5
x1	x1+x5	x1+x2	x1+x2+x5	x1+x3	x1+x3+x5	x1+x4	x1+x4+x5
x2+x3	x2+x3+x5	x2+x4	x2+x4+x5	x3+x4	x3+x4+x5	x1+x2+x3	x1+x2+x3+x5
x1+x2+x4	x1+x2+x4+x5	x1+x3+x4	x1+x3+x4+x5	x2+x3+x4	x2+x3+x4+x5	x1+x2+x3+x4	x1+x2+x3+x4+x5

A Locally Recoverable Code with r=2 and 15 alternatives (Kuijper and Napp 2014)



Simplex Code with k=5 is an (n=31,5,R=16) Switch Code

six x5, five x4, three x3, one x2, one x1



Proof by Induction

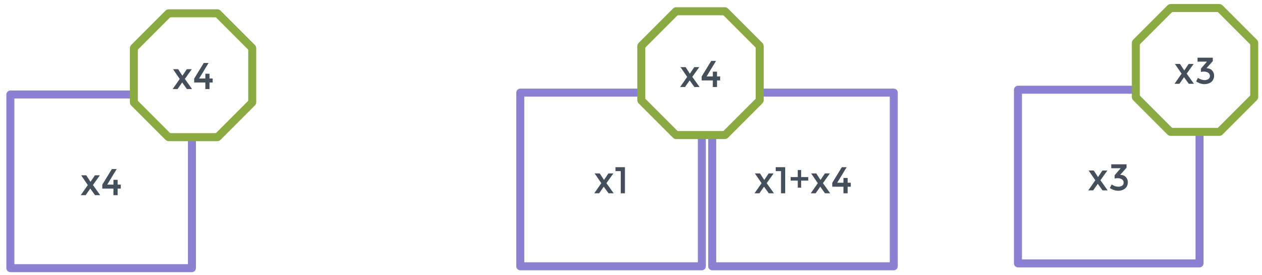
- Induction on k
- k=4 to k=5

<u>Strategy</u>: satisfy requests of x1, x2, x3, x4 first using simplex code with k=4.

	x5	x4	x4+x5	x3	x3+x5	x2	x2+x5
x1	x1+x5	x1+x2	x1+x2+x5	x1+x3	x1+x3+x5	x1+x4	x1+x4+x5
x2+x3	x2+x3+x5	x2+x4	x2+x4+x5	x3+x4	x3+x4+x5	x1+x2+x3	x1+x2+x3+x5
x1+x2+x4	x1+x2+x4+x5	x1+x3+x4	x1+x3+x4+x5	x2+x3+x4	x2+x3+x4+x5	x1+x2+x3+x4	x1+x2+x3+x4+x5

Proof by Induction
- A Naive "Doubling" Approach

- Induction on k
- k=4 to k=5



Proof by Induction
- A Naive "Doubling" Approach

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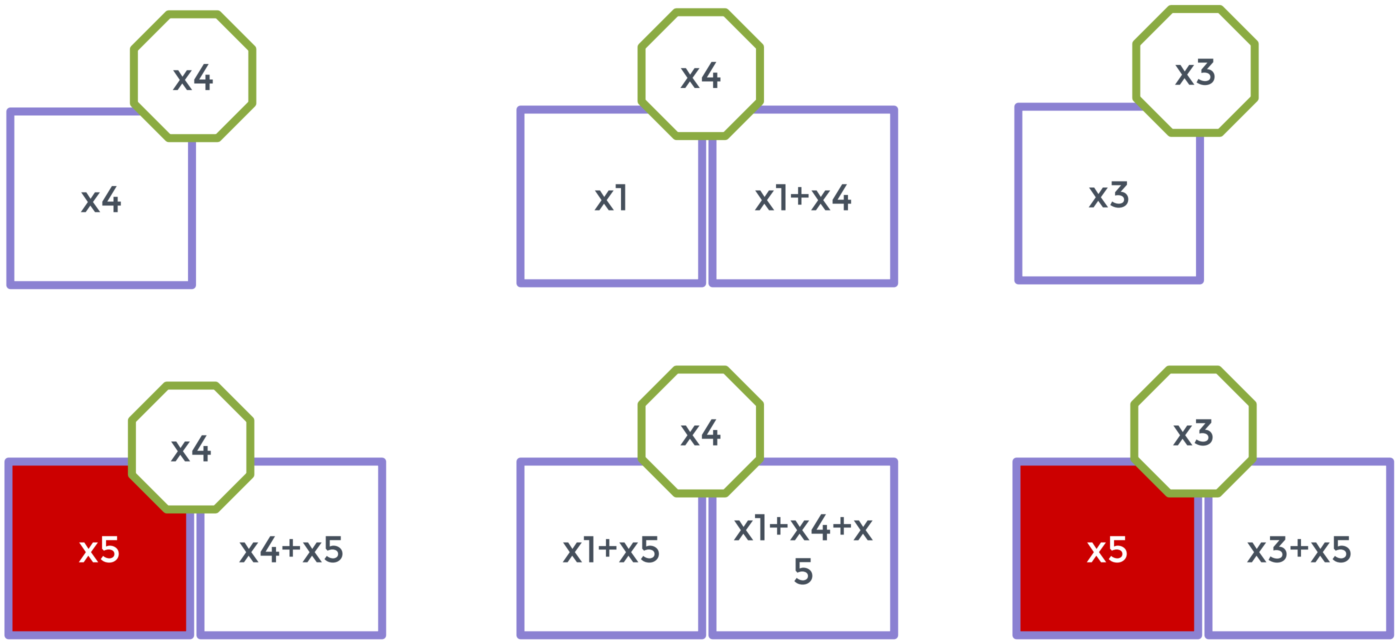
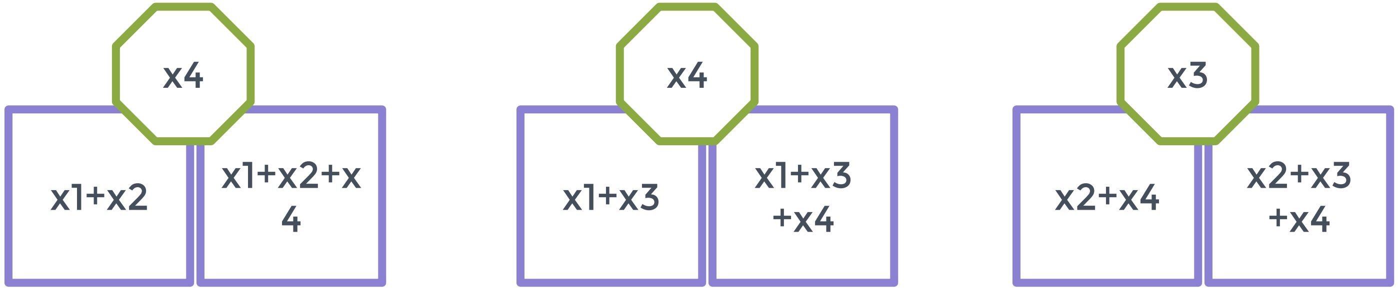


Proof by Induction - Type I Solution

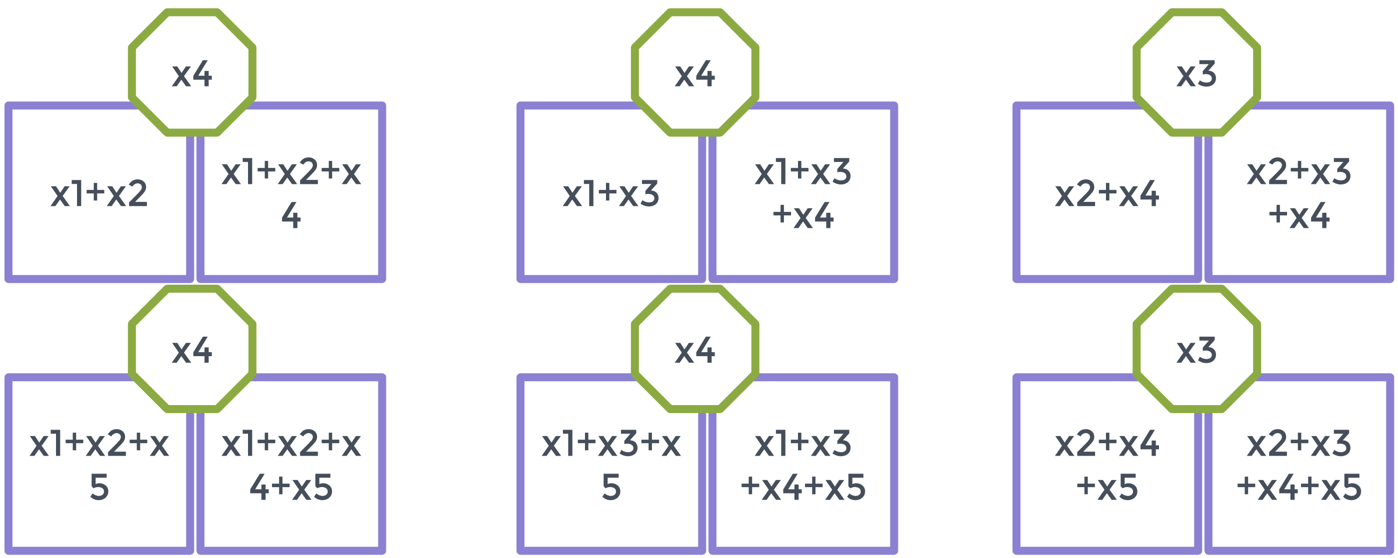
- Induction on k
- k=4 to k=5



Type I solution: All query sets do not contain "singletons".

Proof by Induction
- Doubling a Type I Solution

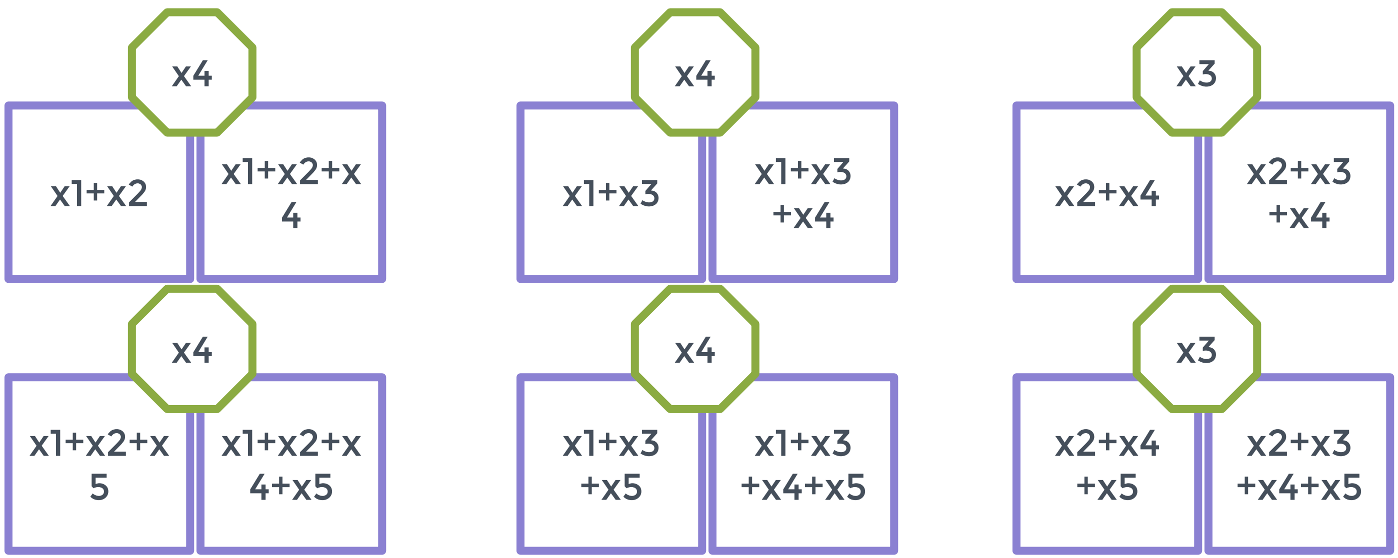
- Induction on k
- k=4 to k=5



zero x5, four x4, two x3, zero x2, zero x1

Proof by Induction
- Doubling a Type I Solution

- Induction on k
- k=4 to k=5



zero x5, four x4, two x3, zero x2, zero x1

goal: six x5, five x4, three x3, one x2, one x1

Proof by Induction
- Completing the Solution

goal: six x5, five x4, three x3, one x2, one x1

current: zero x5, four x4, two x3, zero x2, zero x1

	x5	x4	x4+x5	x3	x3+x5	x2	x2+x5
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x2+x3	x2+x3 +x5	x2+x4	x2+x4 +x5	x3+x4	x3+x4 +x5	x1+x2+x 3	x1+x2+x 3+x5
x1+x2+x 4	x1+x2+x 4+x5	x1+x3 +x4	x1+x3 +x4+x5	x2+x3 +x4	x2+x3 +x4+x5	x1+x2+x 3+x4	x1+x2+x 3+x4+x 5

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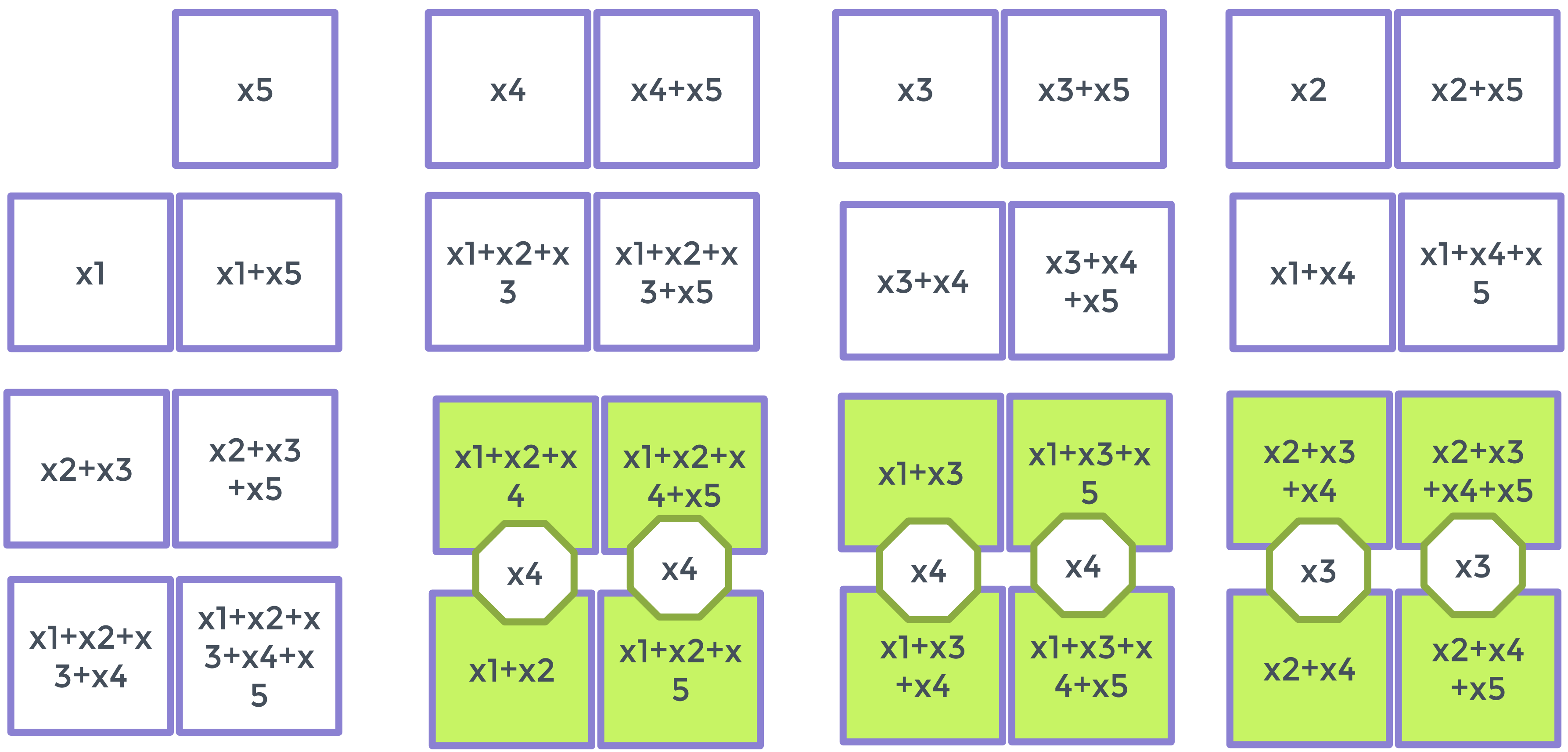

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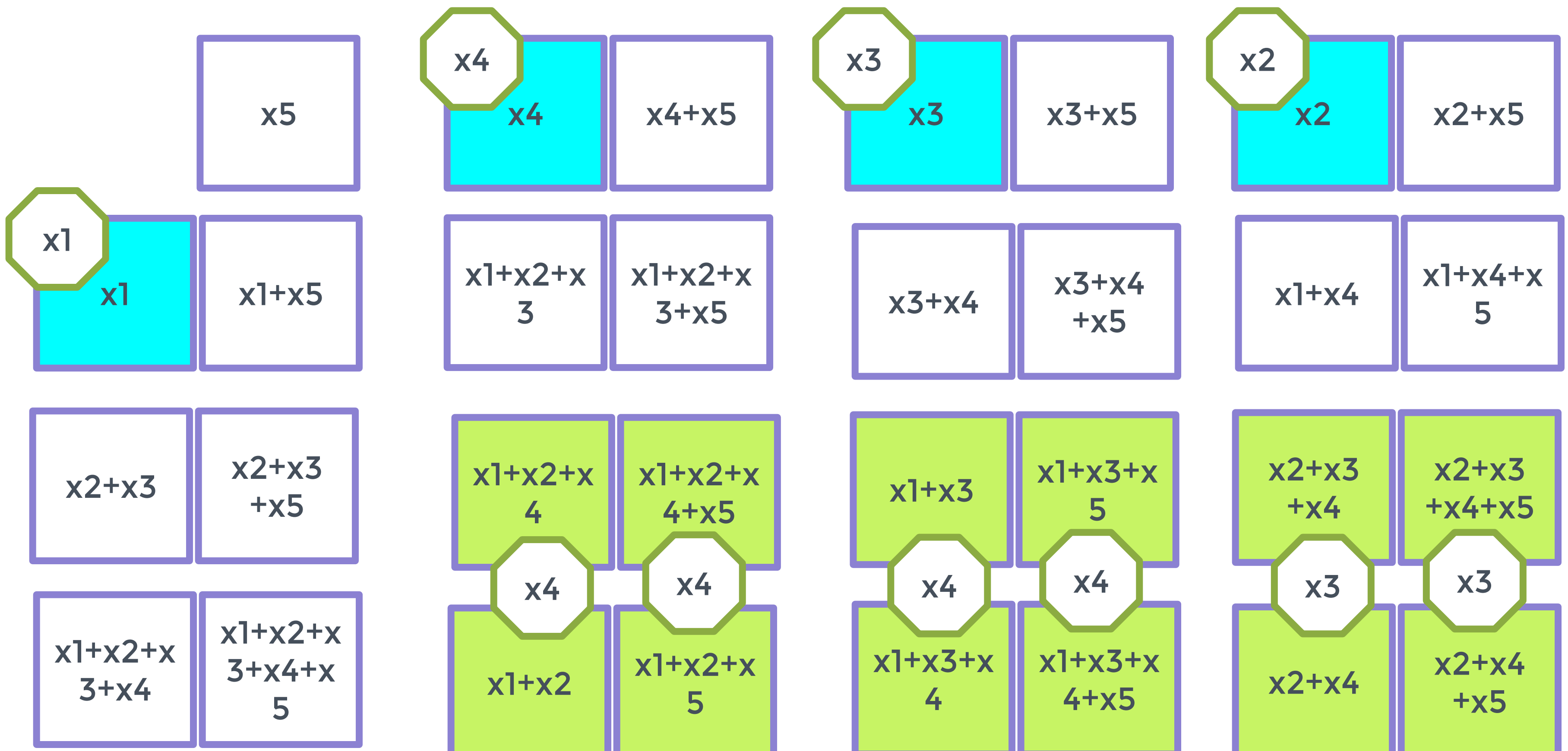

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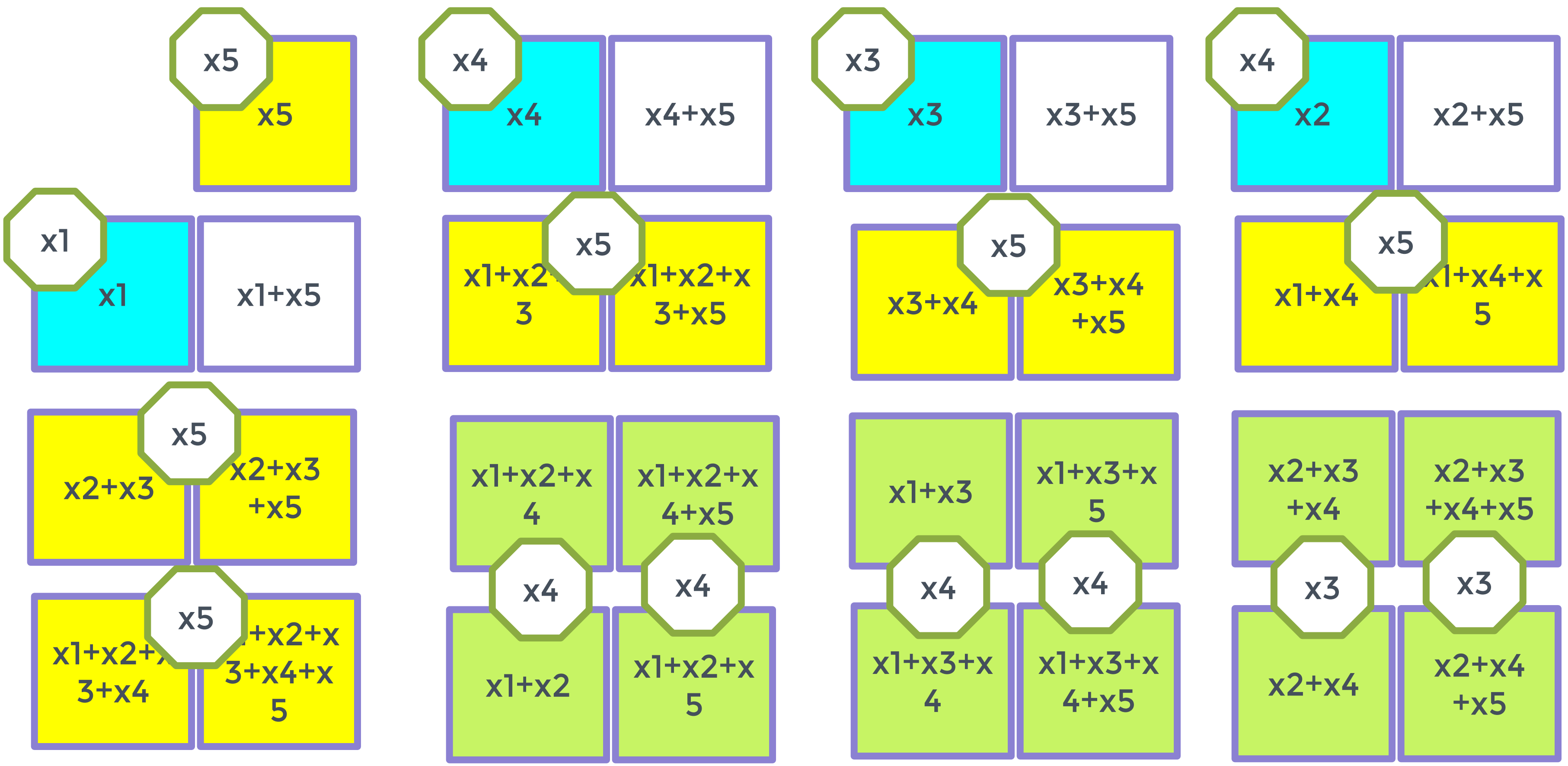

Concatenating Simplex Codes

x1	x2	x3	x1+x2	x1+x3	x2+x3	x1+x2 +x3
y1	y2	y3	y1+y2	y1+y3	y2+y3	y1+y2 +y3

<u>Construction I</u>

Concatenating m copies of $(2^k-1, k, 2^{k-1})$ simplex codes yields an

$(m(2^k-1), mk, 2^{k-1})$ switch code.

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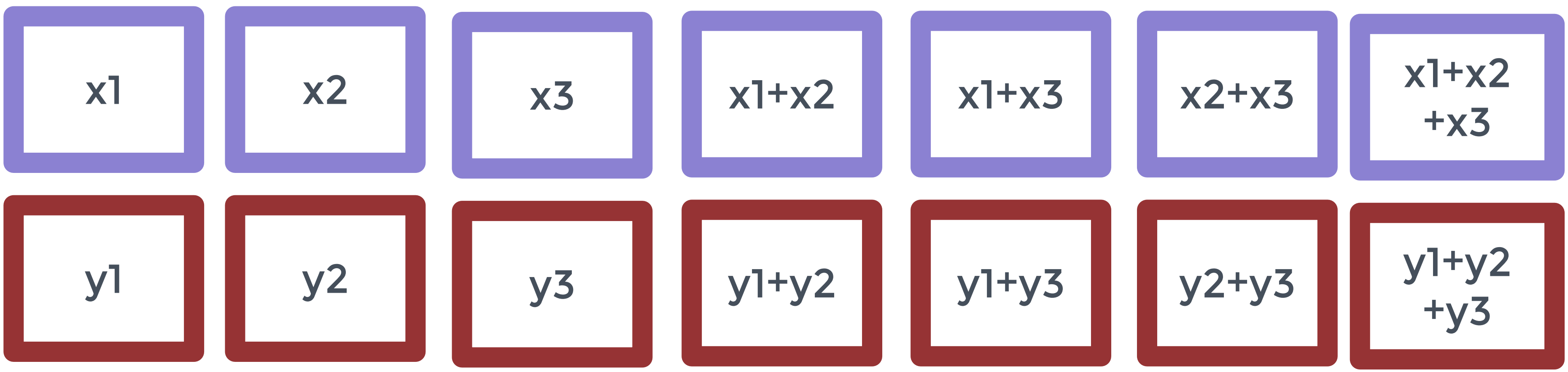
obtain “k close to R”

Properties of Construction I



- binary alphabet
- small query size
- explicit decoding algorithm
- optimal with respect to encoding degree

Work in Progress

Generalizing the proof - query size at most three (or bigger)

- Simplex codes is in fact a special class of ‘subset codes’ (Ishai et al. STOC’2004)
  - Randomized decoding algorithm with no guarantee of success
  - Our work: deterministic and provable decodability
  - Extend our decoding to the general class of subset codes
- Simplex code is a shortened first order Reed Muller code

Questions?

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Credits

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