Importance of Symbol Equity in Coded Modulation for Power Line Communications

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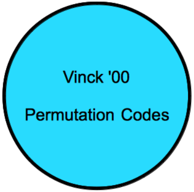
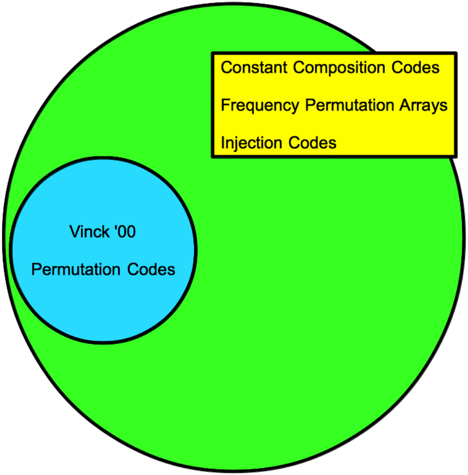
Outline

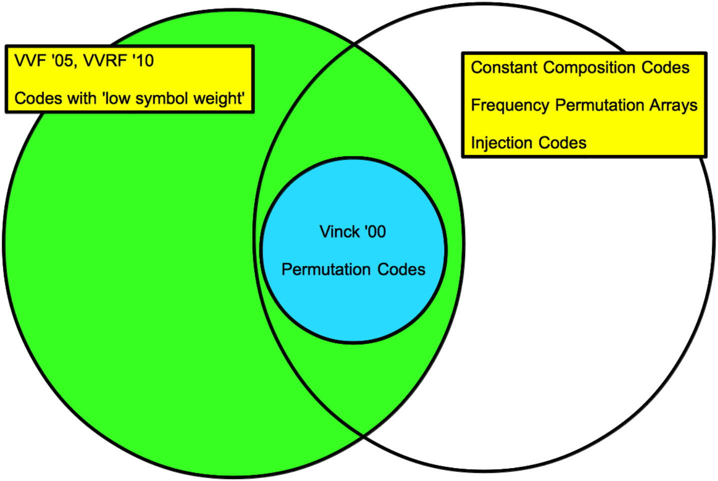
Coding for PLC



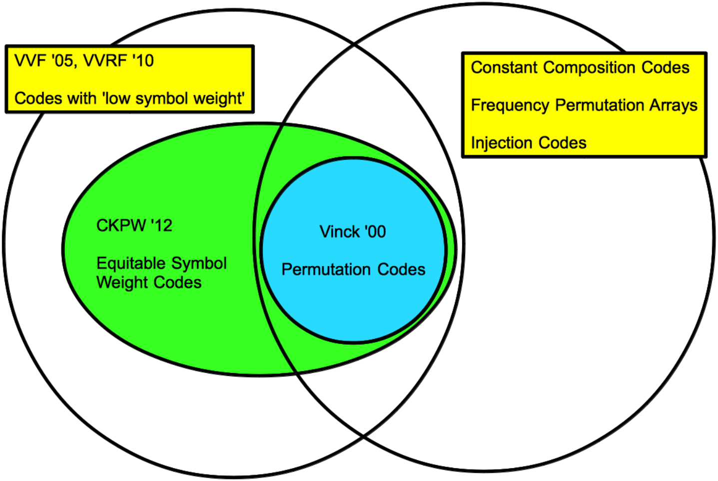
Coding for PLC



Coding for PLC

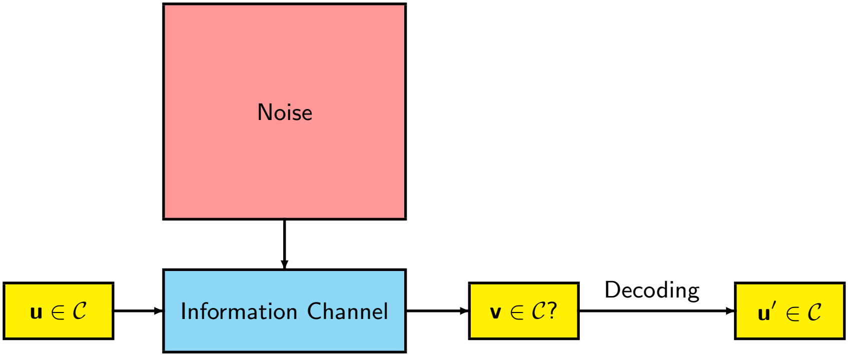


Coding for PLC

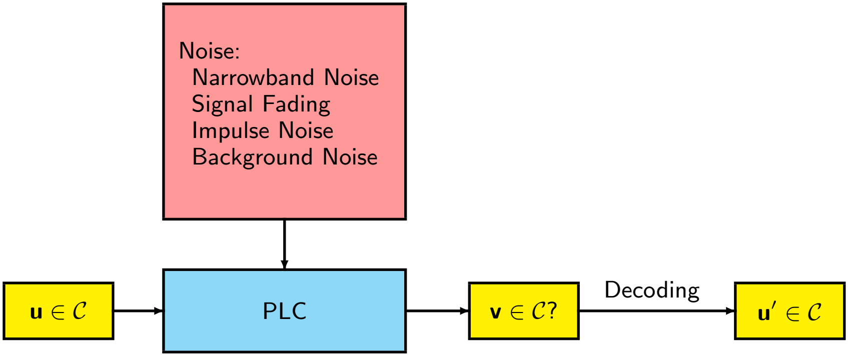


Outline

Transmission in general



Coded Modulation for PLC



A Coded Modulation Scheme

Let $\mathcal{C}_1$ consist of the following words:

$$
\begin{array}{lll}
(0,2,4,2,1,4,5,5,3,6) & (0,5,3,1,6,3,4,4,1,2) & (0,3,6,6,5,1,2,2,4,1) \\
(6,1,3,5,3,2,5,6,4,0) & (5,1,6,4,2,0,4,5,2,3) & (3,1,4,0,0,6,2,3,5,2) \\
(6,0,2,4,6,4,3,0,5,1) & (5,6,2,0,5,3,1,6,3,4) & (3,4,2,5,1,1,0,4,6,3) \\
(4,0,1,3,5,0,5,1,6,2) & (2,6,0,3,1,6,4,0,4,5) & (1,4,5,3,6,2,2,5,0,4) \\
(6,5,1,2,4,6,1,2,0,3) & (5,3,0,1,4,2,0,1,5,6) & (3,2,5,6,4,0,3,6,1,5) \\
(2,0,6,2,3,5,0,3,1,4) & (1,6,4,1,2,5,3,2,6,0) & (4,4,3,6,0,5,1,0,2,6) \\
(1,3,1,0,3,4,6,4,2,5) & (4,2,0,5,2,3,6,3,0,1) & (2,5,5,4,0,1,6,1,3,0)
\end{array}
$$

Then $\mathcal{C}_1$ is a code of length 10 over alphabet $\Sigma = \{0,1,2,3,4,5,6\}$ with minimum distance 9.

A Coded Modulation Scheme

Consider a codeword

$$\mathbf{u} = (0, 2, 4, 2, 1, 4, 5, 5, 3, 6)$$

Time →

Frequency ↓

0	*	-	-	-	-	-	-	-	-	-
1	-	-	-	-	*	-	-	-	-	-
2	-	*	-	*	-	-	-	-	-	-
3	-	-	-	-	-	-	-	-	*	-
4	-	-	*	-	-	*	-	-	-	-
5	-	-	-	-	-	-	*	*	-	-
6	-	-	-	-	-	-	-	-	-	*

Narrowband Noise

Narrowband noise results in certain frequencies being received at all timeslots. For example, narrowband noise occurs at frequency 2.

Time →

Frequency ↓

0	*	-	-	-	-	-	-	-	-	-
1	-	-	-	-	*	-	-	-	-	-
2	*	*	*	*	*	*	*	*	*	*
3	-	-	-	-	-	-	-	-	*	-
4	-	-	*	-	-	*	-	-	-	-
5	-	-	-	-	-	-	*	*	-	-
6	-	-	-	-	-	-	-	-	-	*

Signal Fading

Signal fading results in certain frequencies **not** being received at all timeslots. For example, signal fading occurs at frequency 5.

Time →

Frequency ↓

0	*	-	-	-	-	-	-	-	-	-
1	-	-	-	-	*	-	-	-	-	-
2	-	*	-	*	-	-	-	-	-	-
3	-	-	-	-	-	-	-	-	*	-
4	-	-	*	-	-	*	-	-	-	-
5	-	-	-	-	-	-	-	-	-	-
6	-	-	-	-	-	-	-	-	-	*

Impulse Noise

Impulse noise results in all frequencies being received at certain timeslots. For example, impulse noise occurs at the last timeslot.

Time →

Frequency ↓

0	*	-	-	-	-	-	-	-	-	*
1	-	-	-	-	*	-	-	-	-	*
2	-	*	-	*	-	-	-	-	-	*
3	-	-	-	-	-	-	-	-	*	*
4	-	-	*	-	-	*	-	-	-	*
5	-	-	-	-	-	-	*	*	-	*
6	-	-	-	-	-	-	-	-	-	*

Background Noise

Insertion noise results in certain frequencies being received at certain timeslots. *Deletion noise* results in certain frequencies **not** being received at certain timeslots.

Time →

Frequency ↓

0	*	-	-	-	-	-	-	-	-	-
1	-	-	-	-	*	-	-	-	-	-
2	-	*	-	*	-	-	-	-	-	-
3	-	-	-	-	-	-	-	-	*	-
4	-	-	*	-	-	-	-	-	-	-
5	-	-	-	-	-	-	*	*	-	-
6	-	*	-	-	-	-	-	-	-	*

Demodulator Output

Time →

Frequency ↓

0	*	-	-	-	-	-	-	-	-	*
1	-	-	-	-	*	-	-	-	-	*
2	*	*	*	*	*	*	*	*	*	*
3	-	-	-	-	-	-	-	-	*	*
4	-	-	*	-	-	-	-	-	-	*
5	-	-	-	-	-	-	-	-	-	*
6	-	*	-	-	-	-	-	-	-	*

Represent this output as

$$\mathbf{v} = (\{0,2\},\{2,6\},\{2,4\},\{2\},\{1,2\},\{2\},\{2\},\{2\},\{2,3\},\{0,1,2,3,4,5,6\})$$

Minimum distance decoding

Objective of Decoder

Output the **unique** codeword in $\mathcal{C}$ with the least distance from $\mathbf{v}$

For any $\mathbf{c} \in \Sigma^n$ and $\mathbf{v} \in (2^\Sigma)^n$, let

$$d(\mathbf{c}, \mathbf{v}) = |\{i : \mathbf{c}_i \notin \mathbf{v}_i\}|$$

So, when $\mathbf{u}$ and $\mathbf{v}$ are respectively,

$$\begin{aligned}\mathbf{u} &= (\quad 0, \quad 2, \quad 4, \quad 2, \quad 1, \quad 4, \quad 5, \quad 5, \quad 3, \quad 6 \quad)\\ \mathbf{v} &= (\{0,2\},\{2,6\},\{2,4\},\{2\},\{1,2\},\{2\},\{2\},\{2\},\{2,3\},\{0,1,2,3,4,5,6\})\end{aligned}$$

$$d(\mathbf{u}, \mathbf{v}) = 3$$

Minimum distance decoding

Compute $d(\mathbf{c}, \mathbf{v})$ for all $\mathbf{c} \in \mathcal{C}_1$.

$(0,2,4,2,1,4,5,5,3,6) \leftarrow 3$	$(0,5,3,1,6,3,4,4,1,2) \leftarrow 8$	$(0,3,6,6,5,1,2,2,4,1) \leftarrow 6$
$(6,1,3,5,3,2,5,6,4,0) \leftarrow 8$	$(5,1,6,4,2,0,4,5,2,3) \leftarrow 7$	$(3,1,4,0,0,6,2,3,5,2) \leftarrow 7$
$(6,0,2,4,6,4,3,0,5,1) \leftarrow 8$	$(5,6,2,0,5,3,1,6,3,4) \leftarrow 6$	$(3,4,2,5,1,1,0,4,6,3) \leftarrow 7$
$(4,0,1,3,5,0,5,1,6,2) \leftarrow 9$	$(2,6,0,3,1,6,4,0,4,5) \leftarrow 6$	$(1,4,5,3,6,2,2,5,0,4) \leftarrow 7$
$(6,5,1,2,4,6,1,2,0,3) \leftarrow 7$	$(5,3,0,1,4,2,0,1,5,6) \leftarrow 8$	$(3,2,5,6,4,0,3,6,1,5) \leftarrow 8$
$(2,0,6,2,3,5,0,3,1,4) \leftarrow 7$	$(1,6,4,1,2,5,3,2,6,0) \leftarrow 5$	$(4,4,3,6,0,5,1,0,2,6) \leftarrow 8$
$(1,3,1,0,3,4,6,4,2,5) \leftarrow 8$	$(4,2,0,5,2,3,6,3,0,1) \leftarrow 7$	$(2,5,5,4,0,1,6,1,3,0) \leftarrow 7$

Choose a codeword $\mathbf{u}'$ which attains the minimum distance and decode $\mathbf{v}$ to $\mathbf{u}'$. In our example, $\mathbf{v}$ is correctly decoded to $\mathbf{u}$.

Observation

The code $\mathcal{C}_1$ was able to correct

- **one** narrowband noise error,
- **one** signal fading error,
- **one** impulse noise error,
- **two** background noise errors.

Another code with same parameters

Let $\mathcal{C}_2$ consist of the following words:

$$\begin{array}{ccc} (0,0,0,0,0,0,0,0,0,6) & (1,1,1,1,1,1,1,1,1,6) & (2,2,2,2,2,2,2,2,2,6) \\ (3,3,3,3,3,3,3,3,3,6) & (4,4,4,4,4,4,4,4,4,6) & (5,5,5,5,5,5,5,5,5,6) \end{array}$$

Then $\mathcal{C}_2$ is a code of length 10 over alphabet $\Sigma = \{0,1,2,3,4,5,6\}$ with minimum distance 9.

Another code with same parameters

Let $\mathcal{C}_2$ consist of the following words:

$$\begin{array}{ccc} (0,0,0,0,0,0,0,0,0,6) & (1,1,1,1,1,1,1,1,1,6) & (2,2,2,2,2,2,2,2,2,6) \\ (3,3,3,3,3,3,3,3,3,6) & (4,4,4,4,4,4,4,4,4,6) & (5,5,5,5,5,5,5,5,5,6) \end{array}$$

Then $\mathcal{C}_2$ is a code of length 10 over alphabet $\Sigma = \{0,1,2,3,4,5,6\}$ with minimum distance 9.

But...

The correctness of the minimum distance decoder with code $\mathcal{C}_2$ cannot be guaranteed if one narrowband noise error occurs.

Another code with same parameters

Suppose we transmit

$$(0,0,0,0,0,0,0,0,0,6)$$

and narrowband noise occurs at frequency 1. So, we receive

$$\mathbf{v} = (\{0,1\},\{0,1\},\{0,1\},\{0,1\},\{0,1\},\{0,1\},\{0,1\},\{0,1\},\{0,1\},\{0,1\},\{1,6\}).$$

But

$$d((0,0,0,0,0,0,0,0,0,6),\mathbf{v}) = 0$$
$$d((1,1,1,1,1,1,1,1,1,6),\mathbf{v}) = 0$$

and the minimum distance decoder is unable to give an output.

Conclusion

An additional parameter is required to determine the error-correcting capability of a code with respect to narrowband noise and signal fading.

Outline

Error-correction for narrowband noise

Let $\mathcal{C}$ be a q-ary code of length n over Σ.

Write a codeword $\mathbf{u}$ as $(\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_n)$.
Let $w_\sigma(\mathbf{u}) = |\{i : \mathbf{u}_i = \sigma\}|$ for $\sigma \in \Sigma$.

Definition

$E_{\mathcal{C}}$ is a function $E_{\mathcal{C}} : \{1, \ldots, q\} \to \{1, 2, \ldots, n\}$, such that

$$E_{\mathcal{C}}(e) = \max_{\substack{|\Gamma|=e \\ \Gamma \subset \Sigma}} \left(\max_{\mathbf{c} \in \mathcal{C}} \left\{ \sum_{\sigma \in \Gamma} w_\sigma(\mathbf{c}) \right\} \right)$$

Roughly speaking, $E_{\mathcal{C}}(e)$ measures the maximum number of coordinates, over all codewords in $\mathcal{C}$, affected by e narrowband noise and/or fading errors.

Determining $E_{\mathcal{C}}$

Example

Consider $\mathcal{C}_2$:

$$\begin{array}{ccc} (0,0,0,0,0,0,0,0,0,6) & (1,1,1,1,1,1,1,1,1,6) & (2,2,2,2,2,2,2,2,2,6) \\ (3,3,3,3,3,3,3,3,3,6) & (4,4,4,4,4,4,4,4,4,6) & (5,5,5,5,5,5,5,5,5,6) \end{array}$$

Then

$$E_{\mathcal{C}_2}(1) = 9, \text{ and } E_{\mathcal{C}_2}(e) = 10 \text{ if } e \geq 2.$$

Definition

$E_{\mathcal{C}}$ is a function $E_{\mathcal{C}} : \{1, \ldots, q\} \to \{1, 2, \ldots, n\}$, such that

$$E_{\mathcal{C}}(e) = \max_{\substack{|\Gamma|=e \\ \Gamma \subset \Sigma}} \left(\max_{\mathbf{c} \in \mathcal{C}} \left\{ \sum_{\sigma \in \Gamma} w_\sigma(\mathbf{c}) \right\} \right)$$

Determining $E_{\mathcal{C}}$

Example

Consider $\mathcal{C}_1$:

$$\begin{array}{ccc}
(0,2,4,2,1,4,5,5,3,6) & (0,5,3,1,6,3,4,4,1,2) & (0,3,6,6,5,1,2,2,4,1) \\
(6,1,3,5,3,2,5,6,4,0) & (5,1,6,4,2,0,4,5,2,3) & (3,1,4,0,0,6,2,3,5,2) \\
(6,0,2,4,6,4,3,0,5,1) & (5,6,2,0,5,3,1,6,3,4) & (3,4,2,5,1,1,0,4,6,3) \\
(4,0,1,3,5,0,5,1,6,2) & (2,6,0,3,1,6,4,0,4,5) & (1,4,5,3,6,2,2,5,0,4) \\
(6,5,1,2,4,6,1,2,0,3) & (5,3,0,1,4,2,0,1,5,6) & (3,2,5,6,4,0,3,6,1,5) \\
(2,0,6,2,3,5,0,3,1,4) & (1,6,4,1,2,5,3,2,6,0) & (4,4,3,6,0,5,1,0,2,6) \\
(1,3,1,0,3,4,6,4,2,5) & (4,2,0,5,2,3,6,3,0,1) & (2,5,5,4,0,1,6,1,3,0)
\end{array}$$

Then

$$E_{\mathcal{C}_1}(1) = 2, E_{\mathcal{C}_1}(2) = 4, \ldots, E_{\mathcal{C}_1}(6) = 9, E_{\mathcal{C}_1}(7) = 10$$

Definition

$E_{\mathcal{C}}$ is a function $E_{\mathcal{C}} : \{1, \ldots, q\} \to \{1, 2, \ldots, n\}$, such that

$$E_{\mathcal{C}}(e) = \max_{\substack{|\Gamma|=e \\ \Gamma \subset \Sigma}} \left(\max_{\mathbf{c} \in \mathcal{C}} \left\{ \sum_{\sigma \in \Gamma} w_\sigma(\mathbf{c}) \right\} \right)$$

Error-correction for PLC

Proposition

Let $\mathcal{C}$ be a code of length n over alphabet Σ of size q with distance d. On transmission of any codeword, the code $\mathcal{C}$ is able to correct e_{NBD} narrowband noise errors e_{SFD} signal fading errors, e_{IMP} impulse errors noise, e_{INS} insertion errors and e_{DEL} deletion errors, if and only if the following holds:

$$e_{\text{DEL}} + e_{\text{IMP}} + e_{\text{INS}} + E_{\mathcal{C}}(e_{\text{SFD}}) + E_{\mathcal{C}}(e_{\text{NBD}}) < d.$$

Error-correction for PLC

Example

$\mathcal{C}_1$ was able to correct **one** narrowband noise error, **one** signal fading error, **one** impulse noise error, **two** background noise errors, because

$$\begin{aligned} & e_{\mathrm{DEL}} + e_{\mathrm{IMP}} + e_{\mathrm{INS}} + E_{\mathcal{C}_1}(e_{\mathrm{SFD}}) + E_{\mathcal{C}_1}(e_{\mathrm{NBD}}) \\ & = 1 + 1 + 1 + E_{\mathcal{C}_1}(1) + E_{\mathcal{C}_1}(1) \\ & = 7 \\ & < 9 = d \end{aligned}$$

$\mathcal{C}_2$ was **unable** to correct **one** narrowband noise error

$$\begin{aligned} & e_{\mathrm{DEL}} + e_{\mathrm{IMP}} + e_{\mathrm{INS}} + E_{\mathcal{C}_2}(e_{\mathrm{SFD}}) + E_{\mathcal{C}_2}(e_{\mathrm{NBD}}) \\ & = E_{\mathcal{C}_2}(1) \\ & = 9 = d \end{aligned}$$

Determining $E_{\mathcal{C}}$ for Permutation Codes

Example

When $n = q$, $\mathcal{C}$ is a *permutation code* if $w_\sigma(\mathbf{u}) = 1$ for all $\mathbf{u} \in \mathcal{C}$ and $\sigma \in \Sigma$.

$$E_{\mathcal{C}}(e) = e$$

if and only if $\mathcal{C}$ is a permutation code.

So, $\mathcal{C}$ is able to correct ... if and only if

$$e_{\mathrm{DEL}} + e_{\mathrm{IMP}} + e_{\mathrm{INS}} + e_{\mathrm{SFD}} + e_{\mathrm{NBD}} < d.$$

Determining $E_{\mathcal{C}}$ for certain classes of codes

Example

When $n \leq q$, $\mathcal{C}$ is an *injection code* if $w_\sigma(\mathbf{u}) \leq 1$ for all $\mathbf{u} \in \mathcal{C}$ and $\sigma \in \Sigma$.

$$E_{\mathcal{C}}(e) = \begin{cases} e, & \text{if } e \leq n \\ n, & \text{otherwise} \end{cases}$$

if and only if $\mathcal{C}$ is an injection code.
In particular, when $q = n$, this gives $E_{\mathcal{C}}(e) = e$ for all e if and only if $\mathcal{C}$ is a *permutation code*, that is, $w_\sigma(\mathbf{u}) = 1$ for all $\mathbf{u} \in \mathcal{C}$ and $\sigma \in \Sigma$.

Example

When $q|n$, $\mathcal{C}$ is a *frequency permutation array* if $w_\sigma(\mathbf{u}) = n/q$ for all $\mathbf{u} \in \mathcal{C}$ and $\sigma \in \Sigma$.

$$E_{\mathcal{C}}(e) = ne/q$$

if and only if $\mathcal{C}$ is a FPA.

Determining $E_{\mathcal{C}}$ for certain classes of codes

Example

Let $\Sigma = \{1, \ldots, q\}$. Consider a vector $(c_1, c_2, \ldots, c_q)$ with $c_1 \geq c_2 \geq \cdots \geq c_q$ such that $\sum_{j=1}^{q} c_j = n$. $\mathcal{C}$ is a *constant-composition code* if $w_j(\mathbf{u}) = c_j$ for all $\mathbf{u} \in \mathcal{C}$ and $1 \leq j \leq q$.
If $\mathcal{C}$ is constant-composition code, then

$$E_{\mathcal{C}}(e) = \sum_{i=1}^{e} c_i.$$

Consider $\mathcal{C}_3$ consist of the following words:

$$(0, 0, 0, 0, 1, 2, 3, 4, 5, 6)$$
$$(1, 2, 3, 4, 0, 0, 0, 0, 5, 6)$$

Then,

$$E_{\mathcal{C}_3}(e) = e + 3$$

Determining $E_\mathcal{C}$ for certain classes of codes

Example

A codeword $\mathbf{u}$ has *symbol weight* r, where $r = \max_{\sigma\in\Sigma} w_\sigma(\mathbf{u})$.
A code has *bounded symbol weight* r if all its codewords have symbol weight at most r.
If $\mathcal{C}$ has bounded symbol weight r, then

$$E_\mathcal{C}(1) = r, \quad E_\mathcal{C}(e) \geq \min\{n, r+e-1\}.$$

Consider $\mathcal{C}_4$ consist of the following words:

$$\begin{array}{lll} (0,0,1,1,2,2,3,3,4,4) & (1,1,2,2,3,3,4,4,5,5) & (2,2,3,3,4,4,5,5,6,6) \\ (3,3,4,4,5,5,6,6,0,0) & (4,4,5,5,6,6,0,0,1,1) & (5,5,6,6,0,0,1,1,2,2) \\ (6,6,0,0,1,1,2,2,3,3) & & \end{array}$$

Then,

$$E_{\mathcal{C}_4}(e) = \begin{cases} 2e, & \text{if } e \leq 5 \\ 10, & \text{otherwise} \end{cases}$$

Determining $E_{\mathcal{C}}$ for certain classes of codes

Definition

A codeword $\mathbf{u}$ has *equitable symbol weight* if $w_\sigma(\mathbf{u}) \in \{\lfloor n/q \rfloor, \lceil n/q \rceil\}$ for any $\sigma \in \Sigma$. A code is an *equitable symbol weight code* if all the codewords have equitable symbol weight.

Example

If $\mathcal{C}$ is an equitable symbol weight code,

$$E_{\mathcal{C}}(e) = \begin{cases} re, & \text{if } e \leq q - t \\ r(q-t) + (e-q+t)(r-1), & \text{otherwise,} \end{cases}$$

where $r = \lceil n/q \rceil$ and $t = qr - n$.

Determining $E_{\mathcal{C}}$ for certain classes of codes

Recall $\mathcal{C}_1$:

(0, 2, 4, 2, 1, 4, 5, 5, 3, 6)	(0, 5, 3, 1, 6, 3, 4, 4, 1, 2)	(0, 3, 6, 6, 5, 1, 2, 2, 4, 1)
(6, 1, 3, 5, 3, 2, 5, 6, 4, 0)	(5, 1, 6, 4, 2, 0, 4, 5, 2, 3)	(3, 1, 4, 0, 0, 6, 2, 3, 5, 2)
(6, 0, 2, 4, 6, 4, 3, 0, 5, 1)	(5, 6, 2, 0, 5, 3, 1, 6, 3, 4)	(3, 4, 2, 5, 1, 1, 0, 4, 6, 3)
(4, 0, 1, 3, 5, 0, 5, 1, 6, 2)	(2, 6, 0, 3, 1, 6, 4, 0, 4, 5)	(1, 4, 5, 3, 6, 2, 2, 5, 0, 4)
(6, 5, 1, 2, 4, 6, 1, 2, 0, 3)	(5, 3, 0, 1, 4, 2, 0, 1, 5, 6)	(3, 2, 5, 6, 4, 0, 3, 6, 1, 5)
(2, 0, 6, 2, 3, 5, 0, 3, 1, 4)	(1, 6, 4, 1, 2, 5, 3, 2, 6, 0)	(4, 4, 3, 6, 0, 5, 1, 0, 2, 6)
(1, 3, 1, 0, 3, 4, 6, 4, 2, 5)	(4, 2, 0, 5, 2, 3, 6, 3, 0, 1)	(2, 5, 5, 4, 0, 1, 6, 1, 3, 0)

Example

Then $\mathcal{C}_1$ is an equitable symbol weight code of length 10 over alphabet $\Sigma = \{0, 1, 2, 3, 4, 5, 6\}$ with minimum distance 9 and

$$E_{\mathcal{C}_1}(e) = \begin{cases} 2e, & \text{if } e \leq 3 \\ e + 3, & \text{otherwise.} \end{cases}$$

Outline

1. Vinck's Coded Modulation Scheme
2. An Additional Parameter $E_{\mathcal{C}}$
3. Optimality of Equitable Symbol Weight Codes wrt $E_{\mathcal{C}}$
4. Summary

Comparing $E_{\mathcal{C}}$

Fix n and q. Let $\mathcal{F}_{n,q}$ be the following family of functions,

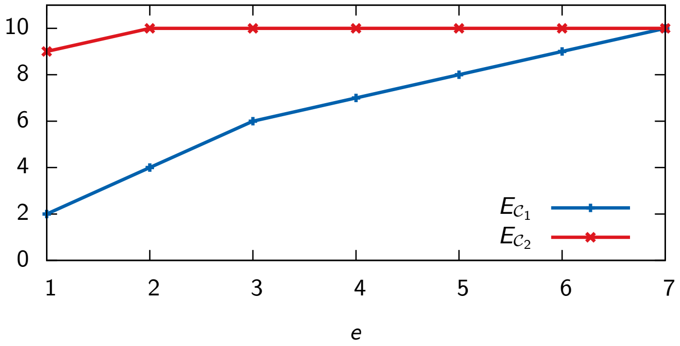
$$\mathcal{F}_{n,q} := \{E_{\mathcal{C}} : \mathcal{C} \text{ is a } q\text{-ary code of length } n\}$$

Definition

Define an ordering relation on the finite family $\mathcal{F}_{n,q}$ as follows: $f \preceq g$ if either $f(e) = g(e)$ for all $1 \leq e \leq q$, or there exists an $1 \leq e' \leq q$ such that $f(e) = g(e)$ for all $e \leq e' - 1$ and $f(e') < g(e')$.

Intuitively, given two codes $\mathcal{C}$ and $\mathcal{C}'$, $E_{\mathcal{C}} \preceq E_{\mathcal{C}'}$ means that $\mathcal{C}$ has better error-correcting capability than $\mathcal{C}'$ in PLC.

Comparing $E_{\mathcal{C}_1}$ and $E_{\mathcal{C}_2}$



Conclusion

Therefore, $\mathcal{C}_1$ has better error-correcting capability than $\mathcal{C}_2$ in PLC.

Importance of symbol equity

Proposition

Let $f^*_{n,q}$ be defined as

$$f^*_{n,q}(e) = \begin{cases} re, & \text{if } e \le q - t, \\ r(q-t) + (e-q+t)(r-1), & \text{otherwise,} \end{cases}$$

where $r = \lceil n/q \rceil$ and $t = qr - n$. Then $f^*_{n,q}$ is the unique least element in $\mathcal{F}_{n,q}$ with respect to the total order $\preceq$.

Corollary

$\mathcal{C}$ is a q-ary equitable symbol weight code of length n if and only if $E_{\mathcal{C}} = f^*_{n,q}$.

Importance of symbol equity

Definition

Let $\mathcal{C}$ be a code of distance d. The *narrowband noise and signal fading error-correcting capability* of $\mathcal{C}$ is

$$c(\mathcal{C}) = \min\{e : E_{\mathcal{C}}(e) \geq d\}.$$

A code $\mathcal{C}$ can correct up to $c(\mathcal{C}) - 1$ narrowband noise and signal fading errors.

Corollary

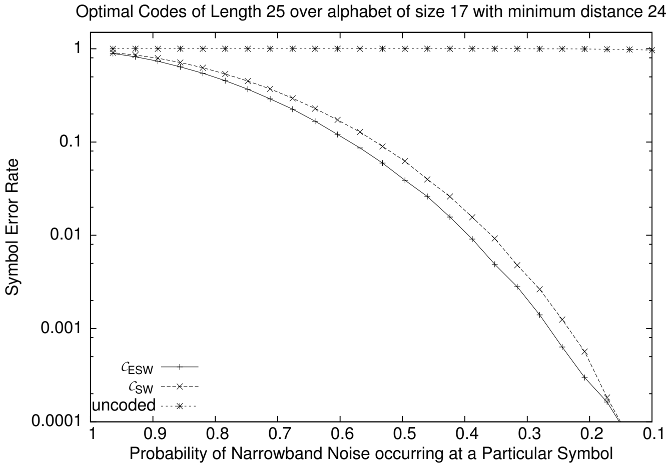
Let $\mathcal{C}$ be an q-ary code of length n and distance d. Then

$$c(\mathcal{C}) \leq \min\{e : f^*_{n,q}(e) \geq d\},$$

and equality is achieved when $\mathcal{C}$ is an equitable symbol weight code.

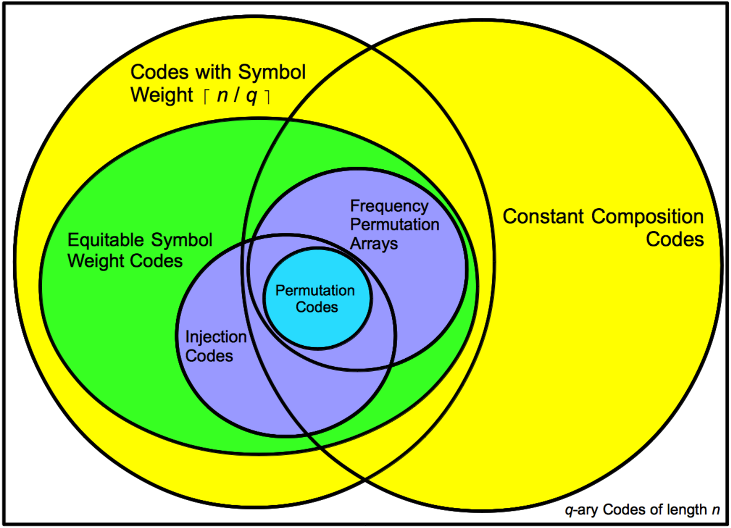
So, an equitable symbol weight code has the best narrowband noise and signal fading error-correcting capability, among codes of the same distance and symbol weight.

Simulation Results



Outline

Codes for PLC



Thank you for your attention!