

# Cooperation Dynamics on Collaborative Social Networks of Heterogeneous Population

Guiyi Wei, Ping Zhu, Athanasios V. Vasilakos, Yuxin Mao, Jun Luo and Yun Ling

**Abstract**—In collaborative social networks (CSNs), autonomous individuals cooperate for their common reciprocity interests. The intrinsic heterogeneity of individuals’ capability and willingness makes significant impact on the promotion of cooperation rate. In this paper, we propose a two-phase Heterogeneous Public Goods Game (HPGG) model to study the cooperation dynamics in CSNs. We introduce two factors to represent the heterogeneity of individual behaviors and the benefit-to-cost enhancement of population, respectively. Based on HPGG CSN model, we quantitatively investigate the relationship between cooperation rate and individuals’ heterogeneous behaviors from an evolutionary game perspective. Simulations on the population structure of scale-free networks show the evolution of cooperation in CSNs has no-trivial dependence on the individuals’ heterogeneous behaviors. Compared with standard PGG and single-phase heterogeneous PGG, HPGG provides a more precise mechanism to promote cooperation rate of CSNs. Finally, data traces collected from real experiments also demonstrate the preciseness of HPGG in formulating the cooperation dynamics on CSNs.

**Index Terms**—collaborative social network, heterogeneous public goods game, cooperation, evolutionary dynamics, complex network.

## I. INTRODUCTION

A social network [1] is a social structure made of individuals called “nodes” which are linked together by one or more specific types of interdependency called “edges”, such as friendship, kinship, financial exchange, communication linkage, etc. In collaborative social networks (CSNs), autonomous individuals with common interests cooperate to accomplish relatively complicated tasks for their reciprocity targets. As we know, the cooperation rate makes significant impact on the possibility of achieving the public goal of a CSN. Unfortunately, most CSNs, either off-line CSNs or online CSNs, both suffer from the problem of low cooperation rate. That is so called “Social dilemma”<sup>1</sup> Such “Social dilemmas” also

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<sup>1</sup>The Prisoner Dilemma Game model first illustrates the conflict of interest between what is best for the individual and what is best for the group, and creates the social dilemma [2].

happens in most online social networks [3], such as Facebook, Twitter and Weibo<sup>2</sup> [4], [5].

With respect to most CSN applications, the public goods game (PGG) [6] is widely used to model the cooperation dynamics on CSNs. However, the intrinsic heterogeneity of autonomous individuals makes it difficult to design an approach for analyzing the cooperation dynamics in a CSN. “Heterogeneity” is the diversity of nature of human interaction, contexts, preferences and social structures, and individual heterogeneity is shown to play an important role in the promotion of cooperation [7]. In PGG modeled CSNs, a round of game can be divided into two phases: investment phase and payoff phase. In the investment phase, heterogeneity is mainly represented by the diversity of the individual’s capability and willingness to cooperate. Due to different contributions in the investment phase, individuals reasonably obtain different gains in the payoff distribution phase. From the view of evolutionary dynamics, heterogeneity also derives from mutation probability, structured network graph, and dynamic linkage, etc. Recently, just a few work has partially investigated the heterogeneity problem in PGG. Cao et al. [8] investigated the cooperation dynamics with heterogeneous contributions. Zhang et al. [9] proposed an evolutionary PGG model with an unequal payoff allocation mechanism to analyze the cooperative behaviors.

In this paper, we systematically formulate and investigate the quantitative relationship between the cooperation rate and the heterogeneity of individuals’ behaviors. The existed work that exploited game theoretical approaches studied the similar problems either from heterogeneous investment phase [8] or from heterogeneous payoff phase [9]. To the best of our knowledge, this paper is the first work that exploits two-phase heterogeneous public goods game to study the evolution of cooperation on CSNs from both the investment phase and the payoff phase simultaneously.

From the view of evolutionary mechanism, we model the cooperation in CSNs into a two-phase Heterogeneous Public Goods Game (HPGG). Based on HPGG CSN model, we quantitatively investigate the relationship between cooperation rate and individuals’ heterogeneous behaviors from an evolutionary game theoretic perspective. Our contributions in this work are multi-fold:

- We novelly model the problem of cooperation dynamics

<sup>2</sup>Akin to a hybrid of Twitter and Facebook, Weibo refers to microblogging services in China. Weibo helps its users build their own online social networks on which multimedia information, including text messages, graphical emoticons, images, music or video files, can be shared through users’ online social relations.

in CSNs into HPGG model in which the CSN participants (autonomous individuals) are treated as players (or agents). Due to the diversity of players' social relations, players' investments and payoffs are considered heterogeneous in HPGG. The simultaneous analysis on the investment phase and the payoff phase helps us gain a deep insight into the cooperation dynamics on CSNs.

- We propose two factors to describe individuals' heterogeneous behavior, i.e., the heterogeneity factor  $\alpha$  and the benefit-to-cost enhancement factor  $r$ . This configuration makes HPGG more realistic and precise in modeling most CSNs in real world.
- We propose two theorems for promoting cooperation rate of CSNs. The two theorems solve the problem of how to choose proper heterogeneity factor  $\alpha$  and benefit-to-cost enhancement factor  $r$  to promote the cooperation rate in a special CSN. And we theoretically prove the correctness of the proposed theorems.
- Simulations on the population structure of scale-free networks show the evolution of cooperation in CSNs has no-trivial dependence on the individuals' heterogeneous behaviors. We find some useful laws about cooperation dynamics on CSNs of heterogeneous population via comprehensive simulations and real world experiments, including: (1) the cooperation rate monotonically increases with  $r$ ; (2) for an arbitrarily given  $r$ , there are two distinct intervals of  $\alpha$  where the cooperation rate monotonically decreases and increases, respectively; and (3) there is a unique turning point of the cooperation rate for a heterogeneity factor; and the unique turning point in a specific CSN can be obtained by conducting dedicated experiments. We also find the conditions under which HPGG outperforms standard PGG and single-phase heterogeneous PGG in promoting cooperation rate in CSNs of heterogeneous population. By comparative analysis on the simulation results of the three models, we discover the intrinsic mechanism of HPGG model that the co-acted two-phase policy promotes cooperative behavior.

The rest of this paper is organized as follows. HPGG model is proposed in Section II. In Section III, we theoretically analyze the cooperative behavior and evolutionary dynamics in CSNs based on HPGG model. In Section IV, we conduct simulations and experiments to demonstrate and explain our findings. In Section V, we analyze related work comprehensively. And finally, we conclude the work in Section VI.

## II. TWO-PHASE HETEROGENEOUS PGG MODEL

In this section, we model a CSN into a two-phase heterogeneous public goods game to investigate multi-person interactions by focusing on the evolutionary dynamics of cooperation. For the easy understanding of the proposed model and theory parts, we list some important notions in Table I.

### A. Individual behavior

We consider a CSN with  $N$  players. Player  $i$  is represented as a vertex  $v_i$  of a graph  $G(V, E)$ , with  $v_i \in V$ . An interaction between two players  $i$  and  $j$  is represented by an undirected

TABLE I  
SOME IMPORTANT NOTIONS USED IN THE PAPER

Symbol	Meaning
$N$	The number of players in a game.
$N_C$	The number of players who take cooperation strategy.
$k_i$	The degree of player $i$ in BA scale-free network
$\langle k \rangle$	The average degree of BA scale-free network.
$\alpha$	The factor for individual's heterogeneous behavior.
$r$	The benefit-to-cost enhancement factor for the population.
$\lambda$	Threshold used to constrain the range of $\alpha$ .
$\theta$	Threshold used to constrain the range of $r$ .
$\kappa$	The player's evolutionary noise in the game.
$\zeta$	Observation noise of $\alpha$ , $\zeta \sim N(0, 1)$ .
$\xi$	Observation noise of $r$ , $\xi \sim N(0, 1)$ .
$\rho_c$	Cooperation rate of a CSN, $\rho_c = \frac{N_C}{N}$ .
$I_i$	Player $i$ 's investment.
$P_i$	Player $i$ 's payoff.
$P_{iC}$	Cooperator $i$ 's payoff under HPGG.
$P_{jD}$	Defector $j$ 's payoff under HPGG.
$P'_{iC}$	Cooperator $i$ 's payoff under single-phase heterogeneous PGG.
$P'_{jD}$	Defector $j$ 's payoff under single-phase heterogeneous PGG.
$\eta$	The ratio of $P_{iC}$ to $P_{iD}$ .
$\eta'$	The ratio of $P'_{iC}$ to $P'_{iD}$ .
$H_{i \rightarrow j}$	The probability of player $i$ copying $j$ 's strategy.

edge  $e_{ij} \in E$ . The number of neighbors of player  $i$  is the degree  $k_i$  of vertex  $v_i$ . The average degree of the network is denoted as  $\langle k \rangle$ . The terms vertex, individual, participant and player are used interchangeably in this paper; likewise for edge, interaction, and link.

Each player in a CSN just takes one of the two strategies:  $C$  or  $D$ .  $C$  strategy means a player cooperates with other players, while  $D$  strategy player does nothing at the discrete time step  $t$  (he might cooperate at  $t + 1$  if his strategy changes to  $C$  during the strategy update process at time step  $t$ ).

### B. Population structure

In this paper, we adopt the *BA scale-free network* to represent the population structure of CSNs, which is constructed according to the "growth" and "preferential attachment" mechanisms. Starting from  $m_0$  fully connected nodes, a new node with  $m$  ( $m \leq m_0$ ) edges is added to the system at every step. The new node links to  $m$  different nodes by a "preferential attachment" mechanism. The probability of connecting to an existing node  $i$  is proportional to its degree, i.e.,  $p_i = k_i / \sum_j k_j$ , where  $j$  runs over all existing nodes and  $k_i$  is the degree of node  $i$ . After  $t$  time steps this algorithm produces a graph with  $N = t + m_0$  vertices and  $mt$  edges.

### C. Two phases in HPGG model

The proposed HPGG model derives from an extension upon the traditional (standard) PGG model. We divide the process of a game into two phases: investment contribution phase and payoff distribution phase.

In the investment phase, players contribute into the public pool heterogeneously. To describe heterogeneity of players in this phase, we let player  $i$ 's investment determined by its degree  $k_i$  and the heterogeneity factor  $\alpha$ . Here, we use  $\alpha$  to denote the willingness of a player in the investment phase. The degree is explained as the capability of a player in a

certain society.  $\alpha > 0$  implies the more powerful a player's capability is, the bigger a player makes contribution, and vice versa  $\alpha < 0$ .

In the payoff phase, players obtain gains from the total profit heterogeneously. We let the payoff of  $i$ ,  $P_i$ , jointly determined by the central game player  $i$ , its  $k_i$  neighbors, the heterogeneity factor  $\alpha$  and the benefit-to-cost enhancement factor  $r$ . Detailed investment formulation and payoff calculation are described in Section II-E.

#### D. Population dynamics and strategy update rule

Let  $S(t) = (s_1(t), s_2(t), \dots, s_N(t))$  denote a configuration of the population strategies  $s_i(t) \in \{C, D\}$  at time step  $t$ , the global synchronous system dynamics leads to  $S(t+1)$  by simultaneously updating all the players' strategies according to the chosen rule, such as Fermi update rule [10]. Here, by synchronous, we mean that player's strategy will not change (even if it had already changed during its strategy updating process at time step  $t$ ) until all the other  $N-1$  players complete their strategy updating processes.

The evolution process is the same as the standard evolutionary game. At each step, all nodes are synchronously updated according to a strategy update rule. Note that in realistic CSNs, individuals are rational and there may be environmental noise which influences individuals' decisions (e.g., strategy mutation). So, to deal with it, we adopt the Fermi updating rule which considers individual rationality and environmental noise. When player  $i$  updating its strategy, it will first select a neighbor  $j$  out from all its  $k_i$  neighbors at random, and then adopt  $j$ 's strategy with the probability

$$H_{i \rightarrow j} = \frac{1}{1 + \exp[(P_i - P_j)/\kappa]}. \quad (1)$$

Here,  $\kappa$  characterizes the environmental noise, including bounded rationality, individual trials, errors in decision, etc.  $\kappa \rightarrow \infty$  leads to neutral (random) drift whereas  $\kappa \rightarrow 0$  corresponds to the imitation dynamics where player  $j$ 's strategy replaces player  $i$ 's whenever  $P_j > P_i$ . For finite value of  $\kappa$ , the smaller  $\kappa$  is, the more likely the fitter strategy is to replace to the less fit one, thus the value of  $\kappa$  indicates the intensity of selection.

#### E. Investment and payoff

Consider the heterogeneous investment contribution phase first. Player  $i$  will invest  $I_i = N \cdot k_i^\alpha / \sum_j k_j^\alpha$  if he is a cooperator, where  $j$  runs over all existing  $N$  nodes. If  $\alpha > 0$  (or  $< 0$ ), large-degree (or small-degree) nodes contribute more if they are cooperators. When  $\alpha = 0$ , all cooperative players have the same contribution (i.e.,  $\forall i, I_i = 1$ ) and our model reverts to the standard PGG model. The defectors contribute nothing during the investment phase.

Next, we consider the heterogeneous payoff distribution phase. The payoff of each player is distributed according to its neighbors' degrees, i.e., player  $i$ 's distribution factor is calculated as  $k_i^\alpha / (k_i^\alpha + \sum_l k_l^\alpha)$ . Here,  $l$  runs over all  $i$ 's  $k_i$  neighbors. If  $\alpha > 0$  (or  $< 0$ ), nodes with large degree (or small degree) can obtain a higher sharing payoff, both for

cooperators and defectors. Also,  $\alpha = 0$  denotes the equal distribution mechanism (i.e., distribution factor for player  $i$  is  $\frac{1}{k_i+1}$ ). Note that "equal" here does not mean all the  $N$  players' share are the same, it means within a certain PGG (i.e., composed of  $i$  and its  $k_i$  neighbors), all the  $k_i+1$  players' share are equal.

The payoff of player  $i$  thus can be expressed as:

$$\begin{cases} P_{iC} &= r \times (I_i + \sum_l s_l I_l) \times \frac{k_i^\alpha}{k_i^\alpha + \sum_l k_l^\alpha} - I_i \\ P_{iD} &= r \times (\sum_l s_l I_l) \times \frac{k_i^\alpha}{k_i^\alpha + \sum_l k_l^\alpha} \end{cases} \quad (2)$$

where  $s_l$  is the strategy of neighbor  $l$  ( $l$  runs over all  $i$ 's neighbors,  $s_l = 1$  for cooperate,  $s_l = 0$  for defect).  $r$  is the benefit-to-cost enhancement factor and  $r > 1$  holds to ensure that groups of cooperators can obtain positive benefit.

### III. COOPERATIVE BEHAVIOR IN HPGG MODEL

The key quantity to characterize the cooperative behavior is the cooperation rate, which is defined as the ratio of the cooperator number  $N_C$  to the total number of players  $N$  at the steady state, i.e.,  $\rho_c = \frac{N_C}{N}$ . It is obvious that the cooperation rate ranges from 0 to 1, where 0 corresponds to the case of all defectors and 1 corresponds to the case of all cooperators.

**Theorem 1.** To promote cooperation rate in CSN, I) given heterogeneity factor  $\alpha$ , we should choose benefit-to-cost enhancement factor  $r$  as large as possible; II) given  $r$ , if  $\alpha$  is negative (positive), we should choose  $\alpha$  as small (large) as possible.

*Proof:* For a player  $i$ ,

$$\begin{aligned} \eta &= \frac{P_{iC}}{P_{iD}} \approx \frac{k_i^\alpha + \rho_c k_i \langle k \rangle^\alpha}{\rho_c k_i \langle k \rangle^\alpha} - \frac{k_i^\alpha + k_i \langle k \rangle^\alpha}{r \rho_c k_i \langle k \rangle^\alpha} \\ &= \frac{r-1}{r \rho_c k_i} \left( \frac{k_i}{\langle k \rangle} \right)^\alpha + \frac{r \rho_c - 1}{r \rho_c} \end{aligned} \quad (3)$$

where  $\langle k \rangle$  denotes the ensemble average over all  $N$  nodes.

As Eq.(3) indicates,  $\eta$  is the ratio of  $P_{iC}$  to  $P_{iD}$ . So, the larger  $\eta$  is, the more likely player  $i$  will choose to cooperate, and thus results in a larger  $\rho_c$ . We denote this relationship as

$$\rho_c \sim \chi(\eta) \quad (4)$$

We next calculate the first derivative of  $\eta$  by differentiating  $\eta$  with  $\alpha$

$$\delta_\eta = \frac{\partial \eta}{\partial \alpha} = \ln \left( \frac{k_i}{\langle k \rangle} \right) \frac{r-1}{r \rho_c k_i} \left( \frac{k_i}{\langle k \rangle} \right)^\alpha \quad (5)$$

According to the function theory, if  $\delta_\eta > 0$  (or  $< 0$ ), then  $\eta$  is a monotonic increasing (or decreasing) function of  $\alpha$ . However, in Eq. (5), the sign of  $\delta_\eta$  is closely related to player  $i$ 's degree  $k_i$ . If  $k_i > \langle k \rangle$  (or  $< \langle k \rangle$ ), then  $\delta_\eta > 0$  (or  $< 0$ ). Using mean field theory, integrate from 1 to  $+\infty$  on  $\delta_\eta dk_i$ , we can approximate  $\delta_\eta$  as

$$\begin{aligned} \delta_\eta &\approx \frac{\int_1^{+\infty} \delta_\eta dk_i}{\langle k \rangle} = \frac{(r-1)(1 + \alpha \ln \langle k \rangle)}{\alpha^2 \langle k \rangle^3 r \rho_c} \\ &\Rightarrow \begin{cases} < 0 & \text{if } \alpha < \lambda \\ > 0 & \text{if } \alpha > \lambda \end{cases} \quad \text{where } \lambda = -\frac{1}{\ln \langle k \rangle} \end{aligned} \quad (6)$$

Thus, we get

$$\eta(\alpha) \text{ is } \begin{cases} \text{monotonic increasing with } \alpha, & \text{if } \alpha > \lambda \\ \text{monotonic decreasing with } \alpha, & \text{if } \alpha < \lambda \end{cases} \quad (7)$$

Combine Eq.(4,7), we get

$$\rho(\alpha) \text{ is } \begin{cases} \text{monotonic increasing with } \alpha, & \text{if } \alpha > \lambda \\ \text{monotonic decreasing with } \alpha, & \text{if } \alpha < \lambda \end{cases} \quad (8)$$

According to BA scale-free network construction,  $\langle k \rangle = 2m \geq 2$ , so  $\frac{-1}{\ln 2} \approx -1.44 \leq \lambda < 0$ . So, under a fixed  $r$ , when  $\alpha < \lambda$  (or  $> \lambda$ ), the cooperation rate  $\rho_c$  monotonically decreases (or increases) with  $\alpha$ . Clearly, It is thus indicated that by setting  $\alpha$  as small (or large) as possible (if  $\alpha < \lambda$  (or  $> \lambda$ )), our HPGG will facilitate the emergence of cooperation. Thus, the II) part of Theorem 1 is theoretically explained.

The proof of the I) part is similar and simpler. According to Eq.(3), we can easily derive that  $\eta(r)$  is monotonic increasing with  $r$ . Combined this with Eq.(4), we judge that the cooperation rate  $\rho_c$  monotonically increases with benefit-to-cost enhancement factor  $r$ . ■

**Theorem 2.** Given benefit-to-cost enhancement factor  $r$ , under condition of  $(k_i < r-1 \cap k_i < \langle k \rangle) \cup (k_i > r-1 \cap k_i > \langle k \rangle)$ , HPGG outperforms (i.e., having a larger cooperation rate) single-phase heterogeneous PGG (i.e., heterogeneous payoff distribution PGG) in the region of  $\alpha < 0 \cup \alpha > \theta$ , where  $\theta = \ln \frac{k_i}{r-1} / \ln \frac{k_i}{\langle k \rangle}$ .

*Proof:* We define  $\eta'$  for PGGs that consider only the heterogeneous payoff distribution phase as:

$$\begin{aligned} \eta' &= \frac{P'_{iC}}{P'_{iD}} \approx \frac{r(\rho_c k_i + 1)k_i^\alpha - (k_i^\alpha + k_i \langle k \rangle^\alpha)}{r\rho_c k_i \times k_i^\alpha} \\ &= \frac{r\rho_c k_i + r - 1}{r\rho_c k_i} - \frac{1}{r\rho_c} \left( \frac{\langle k \rangle}{k_i} \right)^\alpha \end{aligned} \quad (9)$$

where  $P'_{iC}, P'_{iD}$  are payoffs of player  $i$  in PGGs that consider only heterogeneous payoff distribution phase. Consider relationship (4), we have  $\eta \sim \chi^{-1}(\rho_c)$ . Defining the relationship of cooperation rate and heterogeneity factor as  $\rho_c \sim \tau(\alpha)$ , we can then redefine  $\eta \sim \chi^{-1}(\rho_c)$  as

$$\eta \sim \chi^{-1}(\tau(\alpha)) \quad (10)$$

Therefore, the cooperation rate difference can be defined as:

$$\begin{aligned} \Delta_\eta(\alpha) &= \eta - \eta' \\ &= \frac{r-1}{r\rho_c k_i} \left( \frac{k_i}{\langle k \rangle} \right)^\alpha + \frac{1}{r\rho_c} \left( \frac{\langle k \rangle}{k_i} \right)^\alpha - \frac{k_i + r - 1}{r\rho_c k_i} \end{aligned} \quad (11)$$

Consider  $\Delta_\eta(\alpha) > 0$ , we solve Eq. (11) as

$$\begin{cases} \left( \frac{k_i}{\langle k \rangle} \right)^\alpha < \frac{k_i}{r-1} \text{ or } \left( \frac{k_i}{\langle k \rangle} \right)^\alpha > 1 & \text{if } k_i < r-1 \\ \left( \frac{k_i}{\langle k \rangle} \right)^\alpha < 1 \text{ or } \left( \frac{k_i}{\langle k \rangle} \right)^\alpha > \frac{k_i}{r-1} & \text{if } k_i > r-1 \end{cases} \quad (12)$$

Solving Eq. (12), we get the following conditional solution:

$$\alpha < 0 \cup \alpha > \theta$$

under condition of:

$$(k_i < r-1 \cap k_i < \langle k \rangle) \cup (k_i > r-1 \cap k_i > \langle k \rangle)$$

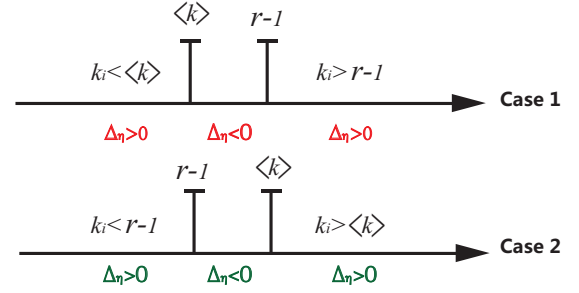


Fig. 1. (Color Online) Conditional solution for  $\Delta_\eta(\alpha) > 0$  with two cases of  $\langle k \rangle < r-1$  and  $\langle k \rangle > r-1$ .

where

$$\theta = \ln \frac{k_i}{r-1} / \ln \frac{k_i}{\langle k \rangle} \quad (13)$$

According to Eq. (13), in the region of  $0 < \alpha < \theta$ ,  $\Delta_\eta(\alpha) < 0$  will always hold. In case of  $\Delta_\eta(\alpha) > 0$ , the solution is much more complicated. If  $\langle k \rangle < r-1$  (or  $> r-1$ ), to guarantee  $\Delta_\eta > 0$ , the value of  $\alpha$  should be set within  $\alpha < 0 \cup \alpha > \theta$  under a condition of  $k_i < \langle k \rangle \cup k_i > r-1$  (or  $k_i < r-1 \cup k_i > \langle k \rangle$ ). Figure 1 summarizes the conditional results for  $\Delta_\eta(\alpha) > 0$ , where **Case 1** means  $\langle k \rangle < r-1$  and **Case 2** corresponds to  $\langle k \rangle > r-1$ . However, if  $\langle k \rangle < k_i < r-1$  (**Case 1**) or  $r-1 < k_i < \langle k \rangle$  (**Case 2**), in the region of  $\alpha < 0 \cup \alpha > \theta$ ,  $\Delta_\eta(\alpha)$  is still negative. However, as Theorem 2 indicates, our aim is to prove that in the entire domain of  $k_i$  (i.e.,  $[1, +\infty]$ ), when  $\alpha < 0$  or  $\alpha > \theta$ ,  $\Delta_\eta(\alpha) > 0$  will always hold. Thus, the proof is based on the following idea: if the number of players with degree  $k_i$  (which results in a negative  $\Delta_\eta(\alpha)$ ) is less than number of players with degree  $k_j$  (which results in a positive  $\Delta_\eta(\alpha)$ ), then  $\Delta_\eta(\alpha) > 0$  holds. According to [26], the BA scale-free network is power law dependent of the degree distribution,  $d(k) \sim k^{-d}$ , with the exponent  $d$  typically satisfying  $d = 2.9 \pm 0.1$ . We then have

$$\begin{cases} \int_1^{\langle k \rangle} k_i^{-d} dk_i + \int_{r-1}^{+\infty} k_i^{-d} dk_i - \int_{\langle k \rangle}^{r-1} k_i^{-d} dk_i > 0 & \text{if } \langle k \rangle < r-1 \\ \int_1^{r-1} k_i^{-d} dk_i + \int_{\langle k \rangle}^{+\infty} k_i^{-d} dk_i - \int_{r-1}^{\langle k \rangle} k_i^{-d} dk_i < 0 & \text{if } \langle k \rangle > r-1 \end{cases} \quad (14)$$

The first inequality of Eq. (14) will always hold; the second inequality of Eq. (14) holds under the condition of  $r > 1 + \frac{1}{2} + \langle k \rangle^{1-d} \frac{1}{1-d}$ . Thus, we have proved that in the region of  $\alpha < 0 \cup \alpha > \theta$ ,  $\Delta_\eta(\alpha) > 0$  is hold under condition provided in Eq.(13). Combining Eqs.(4,10,14), one can always have a positive (negative) cooperation rate improvement when  $\alpha < 0 \cup \alpha > \theta$  ( $0 < \alpha < \theta$ ). ■

#### IV. EXPERIMENTS AND DISCUSSIONS

In this section, we validate two theorems provided in Section III. We then compare the performance of HPGG with standard PGG and single-phase heterogeneous PGG, respectively. We also discuss the intrinsic mechanism of HPGG model and evaluate cooperation dynamics when global parameters  $\alpha$  and



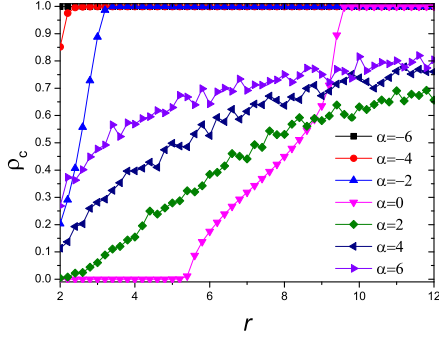


Fig. 2. (Color Online) The cooperation rate  $\rho_c$  vs.  $r$  for different  $\alpha$ .

$r$  have observation noise. In addition, to collect real world data traces, we design and emulate a Weibo-like system and carry out experiments with human participation. We use the real world data traces verify the preciseness of our theoretical results and simulation results.

All the simulations were carried out on a BA scale-free network with network size  $N = 1000$  and  $m = m_0 = 4$ ; therefore, the average degree  $\langle k \rangle = 8$ . Initially, cooperators ( $C$ ) and defectors ( $D$ ) are randomly distributed among the population with equal probability (50%). The equilibrium frequencies of cooperators are obtained by averaging over 3000 generations after a transient time of 10,000 generations. Each piece of data is averaged over 1000 runs on 1000 different networks<sup>3</sup>.

#### A. Cooperative behavior of HPGG model

Figure 2 reports the relationship of cooperation rate  $\rho_c$  and benefit-to-cost enhancement factor  $r$  for different  $\alpha$ . One can see that the cooperation rate monotonically increases with  $r$  for all  $\alpha$  ( $-6 \leq \alpha \leq 6$ ). For a fixed  $r$ , when  $\alpha \leq 0$ , the cooperation rate  $\rho_c$  monotonically decreases with  $\alpha$  (see Figure 2); when  $\alpha \geq 2$ , the cooperation rate  $\rho_c$  monotonically increases with  $\alpha$  (see Figure 2). Remember that  $\alpha > 0$  represents realistic CSN where people with complex social relationships, strong powers, high positions will invest and receive more in a project. Our result (Figure 2) reveals the fact that powerful people (i.e., large-degree nodes) always cooperate to keep their positions, because cooperation (communication) can preserve their social ties and thus get a larger payoff. Once they defect (isolate), they might lose their social ties, become a small-degree node and thus get a smaller payoff. Accordingly, if  $\alpha < 0$ , Figure 2 reveals the same fact that cooperation is the only right way to survive. Besides, in Figure 2, we observe that for any  $\alpha$ , to promote cooperation, one should choose  $r$  as large as possible. This can be explained by stimulating theory [11]. Large stimulation (i.e., the benefit-to-cost enhancement factor  $r$  in our paper) will certainly arouse an interest in

<sup>3</sup>According to the pioneering work [23], we have tried to average over 50 different networks in the first place. However, the average cooperation rate is not stable in our HPGG. To be concrete, if  $\alpha < 0$ , averaging over 50 different networks can get a stable cooperation rate; however when  $\alpha > 0$ , the cooperation rate is not stable until averaging over 1000 different networks.

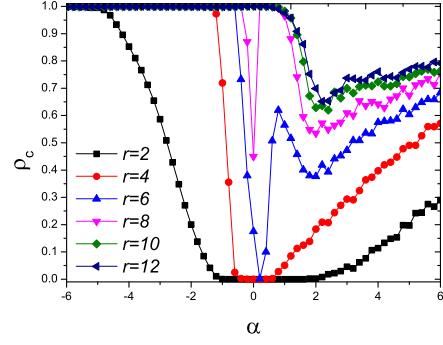


Fig. 3. (Color Online) Cooperation rate  $\rho_c$ , as a function of the heterogeneity factor  $\alpha$  for different  $r$ .

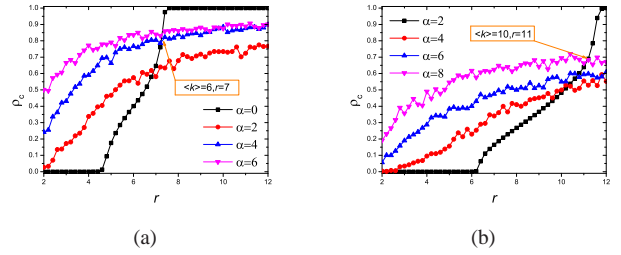


Fig. 4. (Color Online) The turning point value of  $r$  for different  $\langle k \rangle$  when considering  $\rho_c$  vs.  $r$  relationship in HPGG model. (a) The cooperation rate  $\rho_c$  vs.  $r$  for different  $\alpha$  when  $\langle k \rangle = 6$ ; (b) The cooperation rate  $\rho_c$  vs.  $r$  for different  $\alpha$  when  $\langle k \rangle = 10$ .

cooperation. The same technology can always be found in knowledge sharing management of virtual community.

In Figure 3, we investigate the relationship of cooperation rate  $\rho_c$  and heterogeneity factor  $\alpha$  in detail for different  $r$ . As Figure 3 shows, for small  $r$  ( $r \leq 4$ ), the cooperation rate  $\rho_c$  first decreases ( $\alpha \leq 0$ ) and then increases ( $\alpha \geq 0$ ). For moderate  $r$  ( $6 \leq r \leq 8$ ), the cooperation rate  $\rho_c$  first decreases and then increases. After that, it will again decrease first and then increase. For a large  $r$  ( $r \geq 10$ ), the cooperation rate  $\rho_c$  again first decreases and then increases.

The simulation results shown in Figures 2 and 3 coincide with suggestions in Theorem 1, indicating that our HPGG model is correct and efficient.

#### B. HPGG vs. standard PGG

In Figure 2, We find an interesting phenomenon that for  $0 \leq \alpha \leq 6$ , given  $r$  ( $r \leq \langle k \rangle$ ),  $\rho_c$  monotonically increases with  $\alpha$ . However, if  $r \geq \langle k \rangle + 1$ , the  $\alpha = 0$  curve exceeds other curves and becomes the most cooperative curve. We redo our simulations by setting  $\langle k \rangle = 6$  and 10 respectively, the results are the same (see Figure 4), indicating that  $r = \langle k \rangle + 1$  is a turning point. Recall that  $\alpha = 0$  denotes the mechanism of equal contribution and distribution. Our results suggest that if the benefit-to-cost enhancement factor  $r$  is large enough (i.e., larger than network average degree  $\langle k \rangle$ ), then HPGG model has no advantage over the standard PGG model.

The same conclusion can be found in the  $\alpha - \rho$  curve. Returning to Figure 3, we observe that  $r = \langle k \rangle + 1$  is a turning

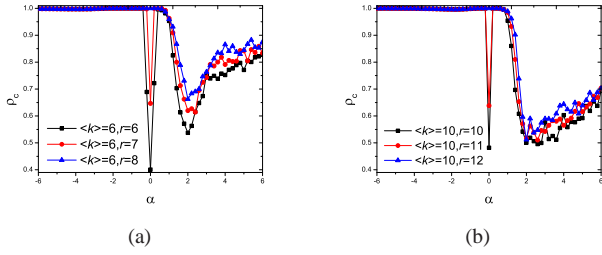


Fig. 5. (Color Online) The turning point value of  $r$  for different  $\langle k \rangle$  when considering  $\rho_c$  vs.  $\alpha$  relationship in HPGG model. (a) Cooperation rate  $\rho_c$  as a function of the heterogeneity factor  $\alpha$  for different  $r$  when  $\langle k \rangle = 6$ ; (b) Cooperation rate  $\rho_c$  as a function of the heterogeneity factor  $\alpha$  for different  $r$  when  $\langle k \rangle = 10$ .

point. The curve mutates when  $\alpha = 0$  (the mechanism of equal contribution and distribution) and  $r > 8$ . Figure 5 show the corresponding result by setting  $\langle k \rangle = 6$  and 10 respectively. Our result again shows that if the benefit-to-cost enhancement factor  $r$  is large enough (e.g., larger than average network degree  $\langle k \rangle$ ), then HPGG model has no advantage over the standard PGG model. Combining Figures 2-5, we have:

**Guidance:** The introduction of heterogeneity into standard PGG can always promote cooperation in CSNs. However, when the benefit-to-cost enhancement factor  $r$  (caused by environment changes) exceeds a certain threshold (i.e.,  $\langle k \rangle + 1$ ), we should go back to use the standard PGG to model CSNs.

### C. HPGG vs. single-phase heterogeneous PGG

Here, we simulate the cooperative behavior of heterogeneous payoff distribution PGG. Again, the BA scale-free network is also set to  $N = 1000$  and  $m = m_0 = 4$ . The equilibrium frequencies of cooperators are obtained by averaging over 3000 generations after a transient time of 10,000 generations. Each piece of data is averaged over 1000 runs on 1000 different networks. Figure 6 reports the performance improvement of HPGG over single-phase heterogeneous PGG. Here, given  $\alpha$  and  $r$ ,  $imp_{\rho_c} = \rho_{HPGG} - \rho_S$ , where  $\rho_{HPGG}$  means the cooperation rate in HPGG model and  $\rho_S$  represents the cooperation rate in the PGG model only having the heterogeneous payoff distribution phase. As Figure 6 shows, when  $\alpha < 0$  or  $\alpha > \theta$  (see Eq. (13)), we can always have a positive cooperation rate improvement  $imp_{\rho_c}$ ; and when  $0 < \alpha < \theta$ , we can always have a negative cooperation rate improvement  $imp_{\rho_c}$ .

In simulation, we can calculate condition in Theory 2 as  $r > 2.4$ . According to theoretical analysis,  $r = 2$  will diverge from simulation results (as Figure 6 shows, the  $r = 2$  curve intersects with  $imp_{\rho_c} = 0$  curve further away from the origin ( $\alpha = 0$ ), and this coincides with our theoretical analysis in Theorem 2).

### D. Observation noise of $\alpha$ and $r$

In this subsection, we discuss the cooperative behavior with observation noise on  $\alpha$  and  $r$ . As we know in network science that when users observe global factors there is always observation noise unless the factors are publicly accessible

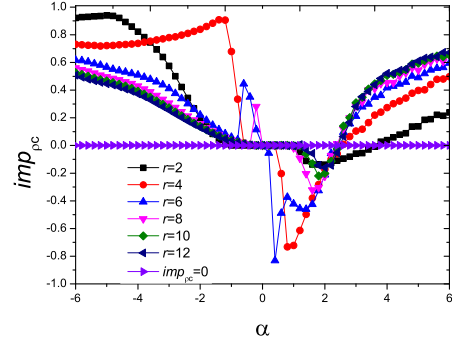


Fig. 6. (Color Online) Improvement of cooperation rate  $imp_{\rho_c}$ , as a function of the heterogeneity factor  $\alpha$  for different  $r$ .

and used. Considering heterogeneity of observation noise in reality, we redefine  $\alpha$  and  $r$  as:

$$\begin{aligned}\alpha_i &= \langle \alpha \rangle + \zeta_i \\ r_i &= \langle r \rangle + \xi_i\end{aligned}\quad (15)$$

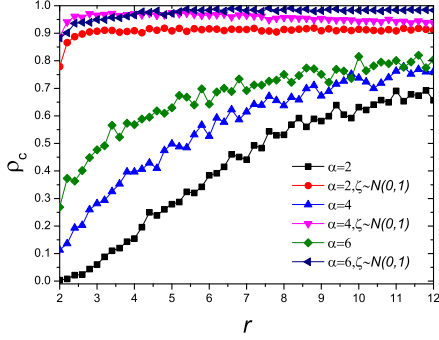
where  $\alpha_i$  is used to represent heterogeneity factor with observation noise.  $\zeta_i$  is a normal distribution variable with zero mean (i.e.,  $\int_{-\infty}^{+\infty} \zeta d\zeta = 0$ ) and unit variance whose probability density function satisfies  $P(\zeta) = \frac{1}{\sqrt{2\pi}} e^{-\zeta^2/2}$ . The same definition is also for  $r_i$ .

The experiments are described as following: in Figure 7(a), we investigate the relationship of the cooperation rate  $\rho_c$  and the heterogeneity factor  $\alpha$  in detail for different  $r$ . Here each player  $i$  is assigned with  $\alpha_i$  individually.  $\langle \alpha \rangle$  is set as 2.0, 4.0 and 6.0 respectively. Note that for each network realization (1000 total), all  $\alpha_i$  are regenerated and during 10,000 evolutionary steps of a specified networks, the value of  $\alpha_i$  are fixed. In Figure 7(b), we explore the relationship of cooperation rate  $\rho_c$  and benefit-to-cost enhancement factor  $r$  in detail for different  $\alpha$  ( $\alpha > 0$ ). Here each player  $i$  is assigned with  $r_i$  individually.  $\langle r \rangle$  is set as 4.0, 8.0 and 12.0 respectively. For each network realization, all  $r_i$  are regenerated and during 10,000 evolutionary steps of a specified networks, the value of  $r_i$  are fixed. Our results show that with observation noise on  $\alpha$ , the cooperation level of HPGG is significantly increased. The result accords with our findings in Section IV-A, i.e., in HPGG, individual's heterogeneity can promote cooperation. As Figure 7 indicated, the noise of  $r$  has little influence on cooperation rate.

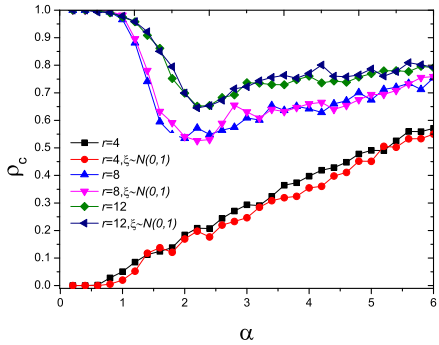
### E. Intrinsic mechanism of HPGG

In this subsection, we try to explain the intrinsic mechanism of HPGG model in the effectiveness of promoting cooperative behavior via detecting the co-action of the two phases.

Figure 8 shows the  $\alpha - \rho_c$  curves for different  $r$  in single-phase heterogeneous PGGs (Figure 8(a) is the heterogeneous investment contribution PGG model ( $M_1$ ), Figure 8(b) is the heterogeneous payoff distribution PGG model ( $M_2$ ), and we use  $M_3$  to represent our proposed HPGG model). In the region



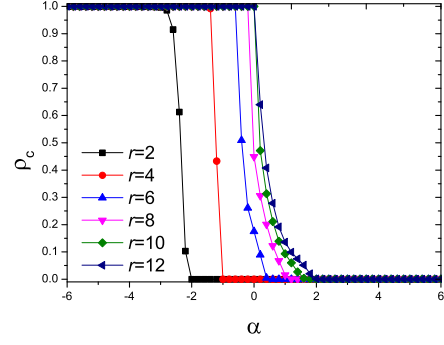
(a) Cooperation rate  $\rho_c$ , as a function of benefit-to-cost enhancement factor  $r$  for different heterogeneity factor  $\alpha$  ( $\alpha = 2.0, 4.0$  and  $6.0$  respectively) with standard normal distribution observation noise  $\zeta$ .



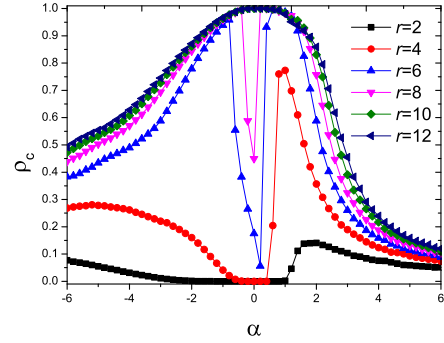
(b) Cooperation rate  $\rho_c$ , as a function of heterogeneity factor  $\alpha$  for different benefit-to-cost enhancement factor  $r$  ( $r = 4.0, 8.0$  and  $12.0$  respectively) with standard normal distribution observation noise  $\xi$ .

Fig. 7. (Color Online) Comparison of cooperation rate with observation noise.

of  $\alpha < 0 \cup \alpha > \theta$ , the cooperation rate enhancement  $imp_{\rho_c} > 0$  is straightforward. Both  $M_1$  and  $M_2$  are  $C$  and approaching  $D$ . Here,  $X \in \{M_1, M_2\}$  is  $C$  (or  $D$ ) means that in model  $X$ , the cooperator (or defector) dominates the defector (or cooperator) in the social system. When  $M_1$  knows  $M_2$  is going to decrease his cooperation rate, he shows  $M_2$  the sincerity of cooperation by increasing his cooperation rate. On the other hand,  $M_2$  observes  $M_1$ 's sincerity and also agrees to cooperate. It is indicated that the intrinsic power of  $M_3$  is that it combines  $M_1$  and  $M_2$ , makes them stimulate each other during the evolutionary game steps, and thus departs from the social dilemma. We can infer that HPGG model has a built-in function of reputation. However, in the region of  $0 < \alpha < \theta$ , the cooperation rate is decreased ( $imp_{\rho_c} < 0$ ). Here  $M_1$  is  $C$  and going to be  $D$ ;  $M_2$  is  $D$  and going to be  $C$ .  $M_1$  observes that  $M_2$  prefers to cooperate, so  $M_1$  chooses  $D$  to maximize its payoff opportunistically.  $M_2$  observes  $M_1$ 's behavior ( $D$ ) and chooses  $D$  as the response, thus they run into the social dilemma.



(a) Heterogeneous investment contribution PGG



(b) Heterogeneous payoff distribution PGG

Fig. 8. (Color online) The cooperation rate  $\rho_c$  vs.  $\alpha$  for different  $r$  in single-phase heterogeneous PGG.

### F. Experiments on an emulated Weibo-like system

The Weibo system can be considered as a Chinese version of Twitter. Sina Weibo [4], used by over 30% of Chinese Internet users, is one of the most popular online social network systems in China. Most Weibo systems, including Sina Weibo [4] and Tencent Weibo [5] in China, adopt a similar marketing strategy: users publish, re-publish or quote interesting information (e.g., messages, images, music and video files) to attract their networked users and instant access followers. The Weibo system operator shares a part of advertisement income with users according to their contributions in Web page advertisement distribution (advertisement coverage contribution). This marketing strategy encourages Weibo users to invest in building big social networks and posting original attractive messages, especially for some celebrity users.

In this section, we design an emulated Weibo-like system to investigate the cooperation dynamics in an online CSN. The emulated system has 100 users. The social relationship among the 100 users forms a scale-free topology, in which each node is a user and each edge represents social relationship between two users. 100 students participated our experiments in which each student acted as a user (a player of HPGG). Each student was encouraged to try their best to maximize her/his payoff.

In our multiple rounds HPGG model, each player has two strategies,  $C$  and  $D$ . Here, a player choosing  $C$  strategy means she/he invests in publishing original messages in the current

round and a player choosing  $D$  means she/he just re-publishes or quotes her/his neighbors' messages with no investment. In each round, a user must choose one strategy,  $C$  or  $D$ . In the first round, all users' strategies are randomly set by the system, 50%  $C$  strategy and 50%  $D$  strategy. From the second round, before decision making, each user can see her/his payoff from the previous round and some information about her/his neighborhood users, including strategies, reference investments, real investments and payoffs from the previous round. The system will give a reference investment  $I_i = N \cdot k_i^\alpha / \sum_j k_j^\alpha$  in the current round if she/he chooses  $C$  strategy. The  $C$  strategy user can use the reference investment or give her/his preferred investment. Here, the deviation from the reference investment is the observation noise of  $\alpha$ . Note that heterogeneity of HPGG model is hereby presented in our emulated system. When all users have chosen their strategies, the system uses Eq.(2) to compute each user's payoff from the current round.

After each round, we collect all users' data traces and compute the cooperation rate of the round. Then, the game goes to the next round. When the standard deviation of the cooperation rates of ten recently continuous rounds is smaller than 0.05, we consider the mean cooperation rate to be stable and then end the game.

As described in Section II, the factors  $\alpha$  and  $r$  play important roles in formulating heterogeneity of HPGG CSNs. In Weibo-like application scenarios, the realistic meaning of  $\alpha$  is the amplifying factor of the cost for a Weibo user in publishing attractive information and maintaining her/his social relationships and the amount of her/his page visitors.  $\alpha$  can be considered as the average willingness of Weibo users to build and boost their social networks. The realistic meaning of  $r$  indicates the rate of Return On Investment (ROI) for users in a Weibo system, i.e., the ratio of obtained advertisement income share from the Weibo system operator to a user's investment. Different CSN applications have their different  $\alpha$  and  $r$ . In Weibo-like application scenarios, it is easy to know that, for a user, bigger  $k$  (social connections) and more attractive information publishing lead to more investment and bigger  $\alpha$ . It is reasonable that more investment leads to more payoff. In our experiments,  $\alpha$  and  $r$  are set by the emulated system at server side. The observation noise of  $\alpha$  is determined by users' investment inputs and the noise of  $r$  is stochastically set at server side according to a normal distribution (See Eq.(15)).

We evaluate the cooperative behavior of the theoretical model in real world application, and the results are shown in Figure 9. As we can see that the human's cooperative behavior approximately accords with our theoretical results, e.g., the inset graph of Figure 9 is a comparison among  $\rho_c$  of theoretical model, theoretical model with noise and real world results for  $\alpha = 6$ . We can see that the real world results are consistent with theoretical and theoretical-noise results in the sense that the evolutionary tendencies of cooperative behavior are consistent, and the real world results always lie between theoretical and theoretical-noise results. We argue that this result is acceptable and anticipated, and the reason can be multi-fold. Firstly, in theoretical-noise model, we assumed a normal distribution with zero mean and unit variance noise,

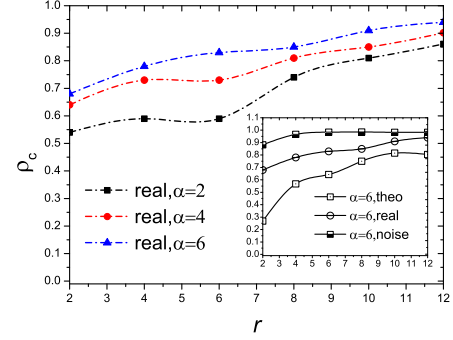
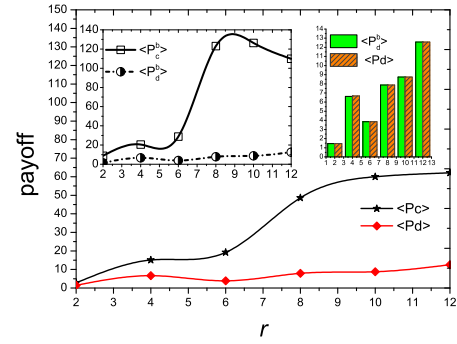
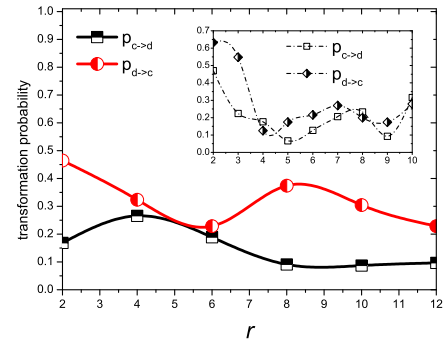


Fig. 9. (Color online) Real world results of cooperation rate  $\rho_c$ , as a function of the benefit-to-cost enhancement factor  $r$  for different heterogeneity factor  $\alpha$  ( $\alpha = 2.0, 4.0$  and  $6.0$  respectively). The inset graph is a comparison among  $\rho_c$  of theoretical, theoretical with noise and real world results for  $\alpha = 6.0$ .



(a) The mean payoffs of cooperators  $\langle P_c \rangle$  and defectors  $\langle P_d \rangle$  in the population as well as the ones of cooperators and defectors lying around the boundary, i.e.,  $\langle P_c^b \rangle$  and  $\langle P_d^b \rangle$ .



(b) The transformation probabilities of cooperators and defectors

Fig. 10. (Color online) Qualitative explanation to the real world experiment results from mean payoffs of cooperators and defectors, mean payoffs of boundary cooperators and defectors, as well as transformation probability between cooperators and defectors. The heterogeneity factor  $\alpha$  is set to 2.0.

which can not very precisely describe the observed noise inevitably. Secondly, human's strategy is also noised in real world experimental environment, i.e., the process of human decision of  $C$  or  $D$  strategy is much more complicated than strategy selection and updating rules specified in most theoretical models. Besides, during the process of experiments,



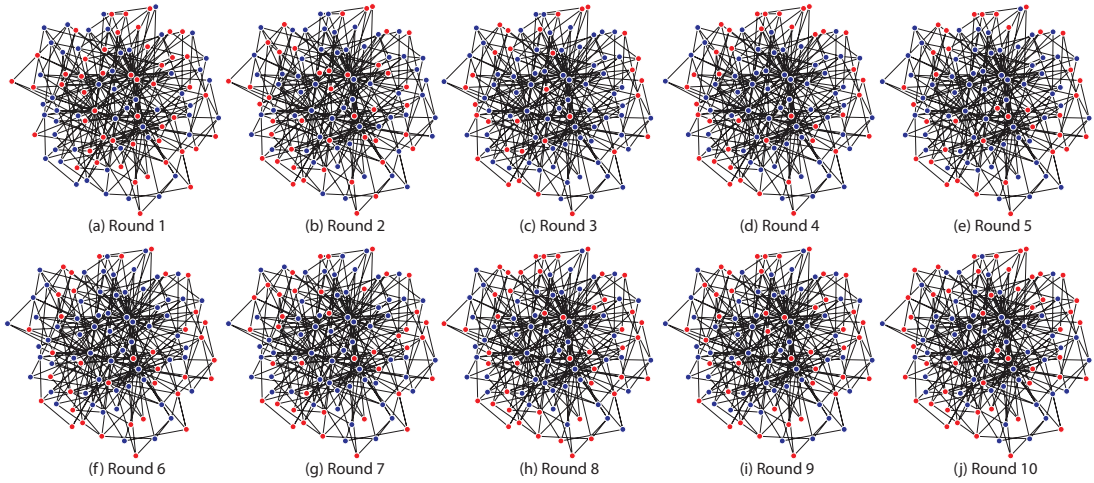


Fig. 11. (Color online) Snapshots of cooperators (blue) and defectors (red) on a 100 nodes scale-free network for  $\alpha=2.0$  and  $r = 6.0$  of the first 10 evolutionary rounds. At the beginning (Round 1), cooperators and defectors are randomly distributed with equal probability. Through dynamic transformation between cooperators and defectors (i.e., Rounds 2-10, controlled by  $P_{c \rightarrow d}$  and  $P_{d \rightarrow c}$ ), the network evolves into Evolutionary Stable Strategy.

we also collect feedback from students that players always deviates from his/her own history investment and payoff, while the neighbors information (e.g., strategy, investment, payoff, etc.) are not fully taken into consideration most of the time. To name but a few, these factors jointly conclude that the real world noise can be conjectured as an approximative skew normal distribution. However, the formulation of this “deviation” is always ethological, which is hard to be explicit expressed mathematically in online CSNs.

To qualitatively explain the real world results in Figure 9, we next evaluate the mean payoffs of cooperators and defectors in the population as well as the ones of cooperators and defectors lying around the boundary. As Figure 10(a) shown, we can find that the cooperators always have a larger payoff over defectors, i.e.,  $\langle P_c \rangle > \langle P_d \rangle$ . This indicates that players will favor to cooperate because the  $C$  strategy results in a bigger income, and thus the cooperation rate  $\rho_c > 50\%$  holds as Figure 9 shown. The up-left inset graph shows the relationship between cooperators and defectors lying around the boundary. As one can see  $\langle P_c^b \rangle > \langle P_d^b \rangle$ , which indicates that boundary cooperators will invade their defective neighbors and results in an increase of cooperators. However, the defectors do not disappear in the system, that’s because defectors can form small compact clusters (as up-right inset graph of Figure 10(a) shows,  $\langle P_d \rangle \approx \langle P_d^b \rangle$ , which indicates that the defectors are placed isolatedly in the ocean of cooperators, i.e., most of defectors are boundary defectors) to compete with cooperators and maintain a dynamic steady state.

We also evaluate the transformation probabilities of cooperators and defectors. Here we use  $P_{c \rightarrow d}$  denoting the transformation probability from cooperators to defectors and  $P_{d \rightarrow c}$  denoting the opposite transformation probability. From Figure 10(b), we can see that  $P_{c \rightarrow d} < P_{d \rightarrow c}$  always holds, which indicates that more defectors turn to play as cooperators than cooperators turning to be defectors, and thus the cooperation rate increases. The inset graph in Figure 10(b) describes the evolution of transformation probability in the first 10 rounds for  $\alpha = 2.0, r = 6.0$  (i.e., Rounds 2-10, note that the first

round has no transformation probability).

Finally, we investigate the dynamic cluster formation process at different evolutionary time steps, i.e.,  $t = 1, 2, \dots, 10$  for  $\alpha=2.0$  and  $r = 6.0$ , the results are shown in Figure 11. At the beginning (Round 1), cooperators and defectors are randomly distributed with equal probability. Then the cooperators invade their defective neighbors quickly and reach to a high cooperation rate of  $\rho_c \approx 0.68$ . After that, through dynamic transformation between cooperators and defectors (i.e., the behavior shown by the inset graph of Figure 10(b)), the cooperation rate gradually decreases and preserve to a steady level of  $\rho_c \approx 0.59$  for the subsequent rounds.

## V. RELATED WORK

Traditionally, in an infinite well-mixed population, cooperation cannot emerge under replicator dynamics [12]. However, observations in the real world usually show the opposite, the players are always altruistic. To explain the emergence and maintenance of cooperative behavior, several mechanisms have been invoked, such as kin selection [13], direct or indirect reciprocity [14], group selection [15], voluntary participation [16], punishment [17], and so on.

Among these work, by introducing spatial structure, the pioneering work done by Nowak and May [18] has increasingly attracted interest from different fields [19] as significant extensions of traditional evolutionary game theory focusing on well-mixed populations. In spatial evolutionary games (SEGs), individuals are situated on the vertices of a graph, and the edges indicate interactions among individuals. The evolutionary success of an individual is determined by its payoff accrued in pairwise interactions with its neighbors [20]. In this context, the network topology plays a key role in the evolution of cooperation, which has been widely studied over the years, e.g., regular networks [21], small-world networks [22] and scale-free networks [23]. Quite recently, the adaptive networks [24] and the mobility of players [25] have also been studied which consider the mutual interaction between network topology and evolution of strategies.

Recently, a prestigious work [26] found that most social networks' degree observe power law distribution. Based on this finding, heterogeneity (diversity) is becoming one of the hottest research focuses on social networks. In [6], the authors studied the cooperation dynamics on heterogeneous graphs by introducing social heterogeneity. In [27], Yang et al. analyzed individual heterogeneity in scale free structure. Work [28]–[30], [32] explored the heterogeneity of game timescale. The work [31] discussed the heterogeneity of mobility in promotion of cooperation in spatial games.

As for the evolutionary cooperation of PGG, much has been done on the effort of a substantial and persistent cooperation level [6], [16], [27], [33]. However, most of these work assumed that participants equally make investment contributions and obtain payoffs, i.e., heterogeneity of PGG was not fully explored. In realistic social networks, participants presented large heterogeneity in both the contribution phase and the payoff phase due to their social structures. Besides work [8] and [9], Sirakoulis and Karafyllidis also discussed the heterogeneity of benefit-to-cost enhancement factor in PGG in a power-aware embedded-system [34].

## VI. CONCLUSION

In this paper, we have proposed a novel two-phase heterogeneous PGG model (HPGG) to study the evolution of cooperation in CSNs. In HPGG, a round of game is divided into two phases: an investment phase and a payoff phase. In the two phases, the individuals of a CSN invest and obtain payoff in a heterogeneous mechanism. Theoretical analysis and results from simulations and real world experiments demonstrated that the proposed HPGG model is precise for formulating the quantitative relationship between the cooperation rate and the individuals' heterogeneous behaviors. Simulations on CSNs of scale-free network population structure show that the cooperation rate has non-trivial dependence on individuals' heterogeneous behaviors represented by the introduced heterogeneity factor  $\alpha$  and benefit-to-cost enhancement factor  $r$ . For any  $\alpha$ , the cooperation rate monotonically increases with  $r$ . For a given  $r$ , if  $\alpha$  is smaller (or larger) than the threshold  $\lambda$ , the cooperation rate  $\rho_c$  monotonically decreases (or increases). Furthermore, we observed that the cooperation rate increases in the range of  $\alpha < 0 \cup \alpha > \theta$  due to HPGG model's intrinsic mechanism. In addition, we also found that  $r = \langle k \rangle + 1$  is the unique turning point of the cooperation rate in HPGG model. When  $r$  is larger than the network's average degree  $\langle k \rangle$ , HPGG model has no advantage over the standard PGG model.

## REFERENCES

- [1] S. Wasserman and K. Faust, *Social Network Analysis: Methods and Applications*. Cambridge Univ. Press, 1994, Cambridge.
- [2] A. Rapoport and A. M. Chammah, *Prisoners Dilemma*. University of Michigan Press, 1965, Ann Arbor.
- [3] K. Lewis, M. Gonzalez and J. Kaufman, Social selection and peer influence in an online social network. *Proc. Natl. Acad. Sci. USA*, 109(1), 68-72, 2012.
- [4] Sina Weibo, <http://weibo.com/>
- [5] Tencent Weibo, <http://t.qq.com/>
- [6] F. C. Santos, M. D. Santos and J. M. Pacheco, Social diversity promotes the emergence of cooperation in public goods games. *Nature* 454, 213-216, 2008.
- [7] F. C. Santos, F. L. Pinheiro, T. Lenaerts and J. M. Pacheco, The role of diversity in the evolution of cooperation. *Journal of Theoretical Biology*, 299, 88-96, 2012.
- [8] X.-B. Cao, W.-B. Du, and Z.-H. Rong, The evolutionary public goods game on scale-free networks with heterogeneous investment. *Physica A*, 389(6), 1273-1280, 2010.
- [9] H.-F. Zhang, H.-X. Yang, W.-B. Du, B.-H. Wang and X.-B. Cao, Evolutionary public goods games on scale-free networks with unequal payoff allocation mechanism. *Physica A*, 389(5), 1099-1104, 2010.
- [10] G. Szabó and C. Tóke, Evolutionary prisoners dilemma game on a square lattice. *Phys. Rev. E*, 58(1), 69-73, 1998.
- [11] J. Koh and Y.-G. Kim, Knowledge sharing in virtual communities: an e-business perspective. *Expert Systems with Applications*, 26(2), 155-166, 2004.
- [12] J. Hofbauer and K. Sigmund, *Evolutionary Games and Population Dynamics*. Cambridge University Press, 1998, Cambridge.
- [13] W. D. Hamilton, The genetical evolution of social behaviour. *Journal of Theoretical Biology*, 7(1), 1-52, 1964.
- [14] M. A. Nowak and K. Sigmund, Evolution of indirect reciprocity. *Nature*, 437, 1291-1298, 2005.
- [15] A. Traulsen and M. A. Nowak, Evolution of cooperation by multilevel selection. *Proc. Natl. Acad. Sci. USA*, 103(29), 10952-10955, 2006.
- [16] C. Hauert, S. D. Monte, J. Hofbauer and K. Sigmund, Volunteering as Red Queen Mechanism for Cooperation in Public Goods Game. *Science*, 296(5570), 1129-1132, 2002.
- [17] C. Hauert, A. Traulsen, H. Brandt, M. A. Nowak and K. Sigmund, Via freedom to coercion: The emergence of costly punishment. *Science*, 316(5833), 1905-1907, 2007.
- [18] M. A. Nowak and R. M. May, Evolutionary games and spatial chaos. *Nature*, 359, 826-829, 1992.
- [19] G. Szabó and G. Fáth, Evolutionary games on graphs. *Physics Reports*, 446(4-6), 97-216, 2007.
- [20] H. Ohtsuki, C. Hauert, E. Lieberman and M. A. Nowak, A simple rule for the evolution of cooperation on graphs and social networks. *Nature* 441, 502-505, 2006.
- [21] M. Doebeli and N. Knowlton, The evolution of interspecific mutualisms. *Proc. Natl. Acad. Sci. USA*, 95(15), 8676C8680, 1998.
- [22] Z.-X. Wu, X.-J. Xu, Y. Chen and Y.-H. Wang, Spatial prisoners dilemma game with volunteering in Newman-Watts small-world networks. *Phys. Rev. E*, 71(3), 037103, 2005.
- [23] F. C. Santos and J. M. Pacheco, Scale-Free Networks Provide a Unifying Framework for the Emergence of Cooperation. *Physical Review Letters*, 95(9), 098104, 2005.
- [24] M. G. Zimmermann, V. M. Eguíluz and M. S. Miguel, Coevolution of dynamical states and interactions in dynamic networks. *Phys. Rev. E*, 69(6), 065102(R), 2004.
- [25] J. Zhang, W.-Y. Wang, W.-B. Du and X.-B. Cao, Evolution of cooperation among mobile agents with heterogeneous view radii. *Physica A*, 390(12), 2251-2257, 2011.
- [26] A. L. Barabási and R. Albert, Emergence of Scaling in Random Networks. *Science*, 286(5439), 509-512, 1999.
- [27] H.-X. Yang, W.-X. Wang, Z.-X. Wu, Y.-C. Lai and B.-H. Wang, Diversity-optimized cooperation on complex networks. *Phys. Rev. E*, 79(5), 056107, 2009.
- [28] Z.-X. Wu, Z.-H. Rong and P.-Holme, Diversity of reproduction time scale promotes cooperation in spatial prisoners dilemma games. *Phys. Rev. E*, 80(3), 036106, 2009.
- [29] A. Traulsen, M. A. Nowak and J. M. Pacheco, Stochastic payoff evaluation increases the temperature of selection. *Journal of Theoretical Biology*, 244(2), 349-356, 2007.
- [30] X.-J. Chen, F. Fu and L. Wang, Interaction stochasticity supports cooperation in spatial Prisoners dilemma. *Phys. Rev. E*, 78(5), 051120, 2008.
- [31] M. H. Vainstein, A. T. C. Silval and J. J. Arenzon, Does mobility decrease cooperation? *Journal of Theoretical Biology*, 244(4), 722-728, 2007.
- [32] C. P. Roca, J. A. Cuesta and A. Sánchez, Time Scales in Evolutionary Dynamics. *Physical Review Letters*, 97(15), 158701, 2006.
- [33] H. Brandt, C. Hauert and K. Sigmund, Punishing and abstaining for public goods. *Proc. Natl. Acad. Sci. USA*, 103(2), 495-497, 2006.
- [34] G. C. Sirakoulis and I. G. Karafyllidis, Cooperation in a Power-Aware Embedded-System Changing Environment: Public Goods Games With Variable Multiplication Factors. *IEEE Transactions on Systems, Man and Cybernetics, Part A: Systems and Humans*, 42(3), 596-603, 2012.



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