Harmonic Quorum Systems: Data Management in 2D/3D Wireless Sensor Networks with Holes*

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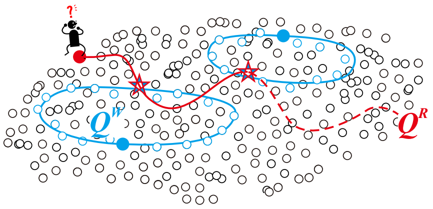
***Abstract*—With the development of ever-expanding *wireless sensor networks* (WSNs) that are meant to connect physical worlds with human societies, gathering sensory data at a single point is becoming less and less practical. Unfortunately, the alternative in-network data management schemes may fail to operate in the face of communication voids (or holes) in WSNs (especially 3D WSNs). In response to this challenge, we propose *harmonic quorum systems* (HQSs) as a light-weight data management system for 2D/3D WSNs. HQSs innovate in exploiting a few scalar fields (constructed using pure localized algorithms) to guide data accesses. This liberates HQSs from depending on any routing mechanisms or location services, hence making HQSs efficient and robust against anomalies in WSN topologies. We implement HQSs in TinyOS, and we perform intensive simulations using TOSSIM to validate the performance of HQSs.**

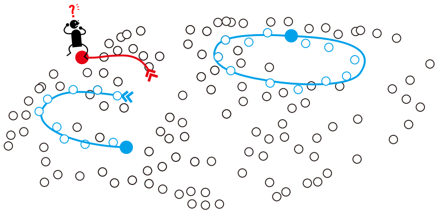
I. INTRODUCTION

As the fundamental functionality of *wireless sensor networks* (WSNs) is to sense the physical world and to deliver sensor data to network users, efficient (sensory) data collection protocols (often to a small set of locations) have been intensively investigated [8], [9], [18]. However, as a result of many new developments in WSNs (e.g., its recent extension to *cyber-physical systems*, or CPSs [1]), new challenges that cannot be fully handled by traditional data collection schemes have emerged. On one hand, while the networked sensors may produce a huge amount of data, the data consumers may not be interested in every piece of data from every sensor. For example, different control units (*actuators*) in a CPS may require data from different sets of sensors at different times. On the other hand, there can be many data consumers spatially distributed (or even moving) in the network region, far more numerous and scattered than the traditional data collectors (or *sinks*). Because these consumers (such as the actuators in a CPS) have to be located where they are needed, relocating them to facilitate data collection is not feasible.

A natural solution to the temporally and spatially distributed sensory data consumption is to use the whole WSN as a data storage and to let a consumer query data in an on-demand manner [3], [25]. In order to manage the data storage and query in an **energy efficient** and **load balancing** way, we have recently applied a well known concept in distributed systems, *quorum systems*, to WSNs [17], [20]. The idea, as illustrated in Fig. 1 (a), is the following. Each node chooses a set of other nodes (termed *write quorum*, or Q^W) to replicate its sensory data, and a data consumer also queries a set of nodes (termed *read quorum*, or Q^R) for any data it needs. If a read quorum intersects a write quorum that stores the data being queried, then the data query returns successfully. In general, a quorum systems is designed such that every read quorum intersects every write quorum, hence a node (resp. data consumer) is free to choose any write (resp. read) quorum to access.



(a) Quorum systems in a WSN without holes



(b) Quorum systems in a WSN with holes

Fig. 1. The quorum systems proposed in [17] use geometric primitives (e.g., curves) to define quorums, in order to (i) improve the design flexibility and (ii) leverage on the existing geographic routing protocol. Whereas this approach works fine in dense WSNs (a), it may fail to operate in WSNs with holes (b).

Due to the lack of dedicated routing mechanisms (as routing protocols for WSNs are mostly designed for data collection purpose, e.g., [8]), our quorum designs presented in [17] rely on a certain geographic routing protocol.[1] The dependence on a stateless geographic routing scheme is also meant to reduce the route maintenance cost in large scale WSNs, it, however, makes our quorum systems sensitive to the geometric property of a WSN field. This is particular the case when there exist *communication voids* (or *holes*) in the field, as shown in Fig. 1 (b). For 2D WSNs, *perimeter routing* [12] can be used to bypass holes but it heavily loads the boundary region of holes [27]. The situation is exacerbated by quorum systems, as a write access also leaves data replications along the routing

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[1] We actually adapted the *trajectory based forwarding* [24] to route data accesses along a certain curve.

path. For 3D WSNs, it has been proven that no stateless protocol can bypass all holes [7]. As a result, data queries to a quorum system can fail when the underlying routing paths are blocked by certain holes.

In this paper, we tackle the problem of constructing quorum systems in WSNs with holes from a very distinct perspective. We take a cross-layer design approach, such that the quorum systems and the data routing mechanisms are jointly constructed. This can be done exactly because the routing mechanism we need is data-centric rather than address-centric: a route visits a specific node set (read or write quorum) instead of certain nodes. The basic idea is to construct several scalar fields (namely *harmonic fields*) such that the WSNs boundaries (including those of holes) bear specific values in respective fields. Our *harmonic quorum system*s (HQSs) use the *level set*s of a field as write quorums, for which a gossip-based routing mechanism can spread data across the whole quorum, and a data query simply follows the *gradient* of different fields to form a read quorum. The intersection between read and write quorum is guaranteed by the fact that tracing the gradient of a scalar field is bounded to pass all level sets. Specifically, we make the following main contributions in this paper:

- We, for the first time, propose the idea of using scalar fields to support data management and the corresponding data-centric routing in WSNs.
- We jointly design quorum systems and the data routing mechanisms to serve on-demand data queries from WSN users, such that the existence of holes in a WSN field cannot hamper any data access.
- Our HQS-based data management is totally location-free, as a by-product of the field-guided routing mechanism. This substantially reduces the overhead of coordinating data accesses in large scale WSNs.
- We implement our HQSs in TinyOS, and we perform intensive TOSSIM simulations to evaluate its performance in terms of energy efficiency and load balancing.

We organize the rest of our paper as follows. In Sec. II, we explain the basics of quorum systems and also discuss the related work. We describe our system model and assumptions in Sec. III, and we give an overview of our proposed cross-layer design as well. We present detailed protocols and algorithms for HQSs in Sec. IV. We then evaluate the performance of HQSs in Sec. V, before concluding our paper in Sec. VI.

II. Background on Quorum Systems

In this section, we first briefly present the basic ideas of quorum systems. Then we explain the advantage of quorum systems over other data management schemes for WSNs, and we also discuss the recent progress in applying quorum systems to WSNs.

A. Basics of Quorum Systems

We first cite the definition given in [15], [19], which actually defines the quorum system in an asymmetric fashion, differing from its original (symmetric) version [22].

Definition *1 (Quorum System):* A quorum system $\mathcal{Q}$ is defined upon a finite set (or *universe*) $\mathcal{U} = \{u_1, u_2, \cdots, u_n\}$ of nodes. $\mathcal{Q} \subset 2^{\mathcal{U}}$ consists of two disjoint sets, $\mathcal{Q}^R$ and $\mathcal{Q}^W$, of subsets of $\mathcal{U}$, such that each subset in $\mathcal{Q}^R$ intersects every subset in $\mathcal{Q}^W$. Each subset Q^R (resp. Q^W) in $\mathcal{Q}^R$ (resp. $\mathcal{Q}^W$) is called a *read* (resp. *write*) *quorum*.

We refer interested readers to [17] for the definitions of metrics for quorum systems. In general, the metrics are concerning the *total load* (indicating the energy efficiency) and the *maximum load* (indicating the load balancing) that a quorum system imposes upon the underlying networks.

Given a well designed quorum system $\mathcal{Q}$, the data accesses are coordinated exactly as we have explained in Sec. I, no matter what kind of underlying networks on which $\mathcal{Q}$ is based. Therefore, the tricks mostly lie in the design phase, aiming at (i) guaranteeing intersections between $\mathcal{Q}^R$ and $\mathcal{Q}^W$, and (ii) avoiding overloading the underlying network. A typical design is illustrated in Fig. 2, which, by default, guarantees the required **intersections**.

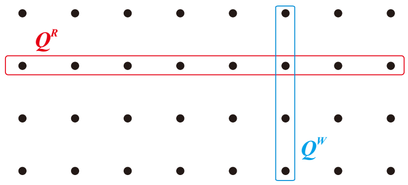

Fig. 2. A typical quorum system design on a grid universe.

As conventional distributed systems are supposed to be built upon wired networks, each node is virtually connected to every other node thanks to the existence of end-to-end routing protocols. Consequently, though the grid topology shown in Fig. 2 appears to be inflexible, tuning the load imposed upon the underlying network can be achieved by arbitrarily shuffling of (the position of) the nodes within the grid topology.

B. Other Data Management Schemes in WSNs

As discussed in Sec. I, there exist other distributed data management schemes for WSNs (e.g., [3], [25]), which also replicate sensory data at nodes other than the sources and allows the queries to originate anywhere and anytime. However, as these data replication locations become new hotspots [25], the load distribution can be very unbalanced [26]. Although spatio-temporal data indexing is applied to improve load balancing in [3], the rather complicated indexing mechanism limits its scalability towards large scale WSNs. In particular, it needs to rely on a stateless end-to-end routing protocol, which may not work well in WSNs with holes. Moreover, all these schemes require a location service to relate sensory data to specific geographic locations; this imposes a significant overhead upon operating a WSN.

C. Recent Progresses on Geometric Quorum Systems

As the bandwidth resource is very scarce in WSNs, maintaining a fully connected routing topology is way too costly. Therefore, the grid-like quorum design shown in Fig. 2 loses its power, as the arbitrary node (position) shuffling does not work anymore. Motivated by the geometric mapping technique discussed in [26], we propose *geometric quorum systems* (GQSs) for 2D WSNs in [17], where quorums are formed by simple geometric primitives (e.g., curves) with tunable parameters. While the intersection can be easily achieved by the geometric properties of those primitives, the network load can be fine-tuned by varying the parameters of the primitives. We also extended this approach to 3D WSNs in [20], following the same principles. Another benefit of using geometric primitives to define quorums is that the *trajectory based forwarding* (TBF) [24] can be adapted to route data accesses. As TBF is governed by geometric primitives and is not address-centric, it works perfectly for geometrically defined quorums. Unfortunately, as we discussed in Sec. I, the existence of holes in a WSN may hamper the applicability of GQSs in WSNs.

III. NETWORK MODEL AND SOLUTION OVERVIEW

In this section, we first introduce our model and assumptions, and we also present the basic principles of our proposal, *harmonic quorum systems* (HQSs). Moreover, we give a brief overview on *harmonic function*, as the field it creates is an essential part of HQSs.

A. Model and Assumptions

A WSN is represented by $\mathcal{U}$, with $u_i \in \mathcal{U}$ being a sensor node. We denote by $\mathcal{N}(u_i)$ the set of nodes that share direct communication links with u_i. We make the following assumptions on the network:

- Nodes on the network boundaries are aware of their status through a boundary detection mechanism, e.g., [14], [28].
- There exist m holes (or communication voids, as shown in Fig. 1 (b) in the network, mainly due to irregular node distribution and the existence of obstacles.
- If we neglect all the holes, the network region is topologically equivalent to a 2D disk or a 3D ball.

We assume that the holes all have diameters comparable to that of the WSN, otherwise simple techniques (such as *expanding ring search* [11]) can overcome small holes with diameter of a couple of hops even in 3D WSNs. We denote by γ_i the boundary of the i-th hole and by γ_0 the external boundary. We do not require location awareness, as maintaining a location service in a WSN is very costly and hence may not always be preferable. However, this does not prevent our proposal from delivering a location-sensitive data service, relying on limited location or proximity information (e.g. [21]).

B. Principles of Harmonic Quorum Systems

We first consider a WSN with one hole, as shown in Fig. 3 (a). Given the size of the hole, it is highly probable that many quorums will (at least partially) fall into the hole, if we design quorum systems by considering the WSN region as a topology disk [17]. Therefore, how to route data accesses to these quorums may become a problem, especially when the WSN occupies a 3D volume (e.g., for atmospherical or underwater sensing).[2]

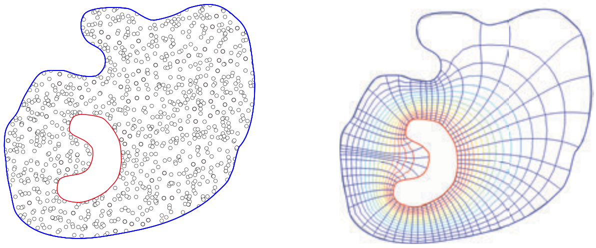
(a) WSN with one hole (b) Field with level sets and gradients

Fig. 3. A WSN with one hole (a) and an imaginary field constructed upon it (b). The level sets of the field are colored to to represent respective values.

Our alternative solution is illustrated in Fig. 3 (b). Assume we can build a *field* in the network region, in which the internal and external network boundaries take distinct values (typically 1 and 0). Consequently, nodes within the network region may assume different values according to the respective *level sets* (of the field) it belongs to. Here the "belonging" relation can be defined by proximity. Now the quorum design is immediate: use level sets as write quorums and trace the gradient to form read quorums. As tracing gradient is bounded to pass all values of the scalar field and as all the level sets are connected for this one-hole WSN, the intersection between read and write quorums are guaranteed. In order to apply this idea to construct a full-fledged quorum systems in WSNs, we still need to handle the following four problems:

P1: What if multiple holes exist in a WSN region?
P2: What scalar field(s) should be used and how to construct and maintain them in a WSN?
P3: How to fine-tune the load imposed to the sensor nodes?
P4: How to accommodate location sensitive sensory data?

We briefly discuss the intuition for solving P1 here, and we introduce *harmonic function* that we use to generate the fields in Sec. III-C. Details about protocols and algorithms to address these problems are presented in Sec. IV. It is important to note that **the idea behind the solution to P4 also allows HQSs to work in WSNs without holes** (Sec. IV-C2).

The problem with multiple holes is that the level sets may not be connected (in topological sense) anymore, especially for those close to the internal boundaries, as shown in Fig. 4 (a). On one hand, if we still use a disconnected level set as a write quorum, the data need to be replicated to all the disjoint components of this set. However, as we do not assume the reliance on any underlying routing protocol, there is simply no way to get to any other components. On the other hand, if we simply replicate data to one component of a level set to which the data source belongs, an arbitrary read quorum may not intersect this component, shown by Fig. 4 (a).

The solution is still based on scalar fields and their respective gradients. Suppose we construct a specific field for each

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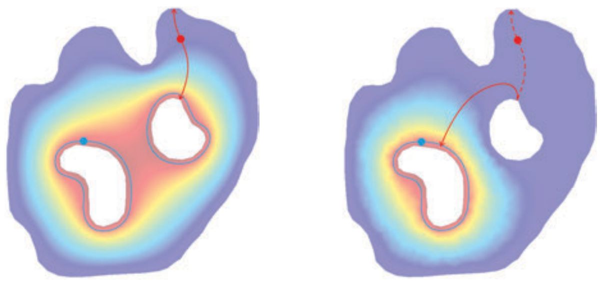
[2] Although we are concerned with both 2D and 3D WSNs, we have to provide examples in 2D to facilitate visual illustration. However, our quorum design applies to both 2D and 3D WSNs.



(a) Disconnected level set (b) "Jump" from one hole to another

Fig. 4. A WSN with two holes is bounded to produce a field with disconnected level sets (a), which compromises intersection. "Jumping" from one hole to another can be achieved by constructing another field and tracing its gradient (b).

hole, and the i-th field is such that only the i-th hole boundary has value 1 while other boundaries all have value 0. The 0-th field is the one we have constructed: all hole boundaries have value 1 and only the external boundary has value 0, and it is used to define write quorums. The write quorums only use connected components, and the read quorum first trace the gradient of the 0-th field. If no intersection is found, then the read quorum switches to an arbitrary field it has not chosen yet and traces its gradient. In the worse case, a WSN with m holes needs to have $m + 1$ fields and a read quorum may trace up to m fields in order to intersect the required write quorum. We show a case with 2 holes in Fig. 4 (b). Actually, if we properly index the holes, only $\mathcal{O}(\log m)$ fields need to be traced (see Sec. IV-C1).

C. *Harmonic Function Overview*

The essence of our field construction is the notion of *harmonic function*. On a domain $\mathcal{D} \subseteq \mathbb{R}^d$ ($d \geq 2$), a harmonic function $f : \mathcal{D} \to \mathbb{R}$ is a twice-differentiable function, whose second partial derivative satisfies Laplace's equation [4]:

$$\triangle f(u) = 0, \qquad \forall\ u \notin \partial\mathcal{D}, \tag{1}$$

where $\partial\mathcal{D}$ is the boundary of $\mathcal{D}$. The function values are specified on all boundaries, referred to as Dirichlet boundary conditions. With all boundary values being set, the solution to the Laplace's equation is **unique**. A solution to Laplace's equation has the following properties:

- Mean-Value Property: the average value over any spherical surface in $\mathcal{D}$ is equal to the value at the center of the sphere.
- Maximum Principle: harmonic functions are free of local minima or maxima within $\mathcal{D}$ and always assume global maximum and minimum on $\partial\mathcal{D}$.

As a WSN $\mathcal{U}$ is a **finite set** of nodes in $\mathbb{R}^2$ or $\mathbb{R}^3$, the Laplace's equation needs to take a discrete form

$$f(u) = \frac{1}{|\mathcal{N}(u)|} \sum_{v \in \mathcal{N}(u)} f(v), \qquad \forall\ u \notin \partial\mathcal{U}, \tag{2}$$

with Dirichlet condition that boundary nodes assume value 1 or 0. This equation is indeed a discrete form of the mean-value property. In addition, whereas tracing the gradient of a field from one boundary (minimum) to another boundary (maximum) is not always possible in a continuous domain $\mathcal{D}$ due to the existence of saddle points, this property is guaranteed for a discrete set, as saddle point (being an infinitesimal notion) does not exist in a discrete form. **This makes the gradients of harmonic fields ideal choices for supporting data routing in quorum systems**.

IV. PROTOCOLS FOR CONSTRUCTING AND ACCESSING HARMONIC QUORUMS

We first explain how to construct multiple harmonic fields for a WSN in Sec. IV-A. Then we present the distributed data access protocols built upon these fields in Sec IV-B. Finally, we discuss in Sec. IV-C how to optimize the system performance by taking into account, for example, data access rates or location sensitivity. We focus on 3D WSNs in this section, as the protocols for 3D are also feasible in 2D.

A. *Constructing Harmonic Fields through Diffusion*

1) Computing A Single Harmonic Field: A standard method to compute the harmonic function is to solve the Laplace equation (1), which is a sparse linear system in discrete form. Although many efficient sparse linear solvers are available, e.g., Cholesky factorization, such methods are centralized and not suitable for WSNs. Observing (1) is equivalent to the following heat diffusion equation

$$\frac{\partial f}{\partial t} = -\triangle f, \tag{3}$$

as the stationary condition (3) is indeed (1). This differential equation can be solved by the Euler method, which has a potential to be implemented in a distributed manner.

Given the finite set $\mathcal{U}$, the discrete Laplace operator of an interior node $u \notin \partial\mathcal{U}$ is defined as $\triangle f(u) = f(u) - \sum_{v\in\mathcal{N}(u)} \omega_{uv} f(v)$, where ω_{uv} is the weight of directed communication link (u, v). The choice of the weight is usually application dependent. We choose a straightforward weight determined by vertex degree, i.e., $\omega_{uv} = \frac{1}{\sqrt{deg(u), deg(v)}}$, with $deg(u)$ being the degree of node u. Now the discrete version of (3) becomes

$$f(\mathcal{U}\backslash\partial\mathcal{U}) = [I - \delta L]\, f(\mathcal{U}\backslash\partial\mathcal{U}), \tag{4}$$

where δ is the step size of the Euler method and L is the *Laplacian* [6] of the graph $G_{\mathcal{U}}$ induced by $\mathcal{U}$ and the neighborhood relations $\mathcal{N}(u), \forall u \in \mathcal{U}$. The distributed implementation of (4) is shown by **Algorithm 1**. Here the boundary conditions $f_i, i = 0, 1, \cdots, m$ depends on which field is being computed. For example, the conditions for the 0-th field are $f_0 = 0$ and $f_i = 1, i = 1, \cdots, m$.

Because (4) is obviously a *fixed-point iteration*, the convergence of this algorithm follows immediately from that the linear function $g(\vec{x}) = (I - \delta L)\vec{x}$ has a Lipschitz constant (actually the second largest eigenvalue of $I - \delta L$) smaller than 1, given the property of L [6]. It can be also shown that the time complexity (hence message complexity, as they are in proportion) is $\mathcal{O}(\log^{-1}(\varepsilon^{-1}))$ [5].

Algorithm 1: Computing harmonic function

```
Input: The Dirichlet boundary condition $f_i$. For each
       $u \in \mathcal{U}$, the step size $\delta$, the stopping tolerance $\varepsilon$
Output: A harmonic function $f : \mathcal{U} \to \mathbb{R}$
1 INITIALIZE $f(u) \leftarrow f_i$, $u \in \gamma_i$; otherwise $f(u) \leftarrow 0$
2 For every interior vertex $u$ periodically (every $\pi$ ms):
3 if $| \triangle f(u)| > \varepsilon$ then
4     $\triangle f(u) \leftarrow f(u) - \sum_{v \in \mathcal{N}(u)} \omega_{uv} f(v)$
5     $f(u) \leftarrow f(u) - \delta \triangle f(u)$
6     BROADCAST $f(u)$ to all $v \in \mathcal{N}(u)$
7 end
```

2) Computing Multiple Harmonic Fields: Remember that we need to compute multiple fields with different boundary conditions. For example, the conditions for the i-th field ($i \neq 0$) are $f_i = 1$ and $f_j = 0, j \neq i$. A naive solution is to run **Algorithm 1** $m + 1$ times to compute these fields separately. Fortunately, all these fields can be computed altogether, as the message broadcast in line 6 of **Algorithm 1** may piggyback several field values, provided that each field has its value tagged by a unique field identifer.

A convenient way of generate unique field identifers is through randomization. Without loss of generality, we may take $id_0 = 0$. To generate $id_i, i = 1, \cdots, m$, each hole boundary node comes up with a random number. As each hole boundary is a connected 2-manifold, we can run a gossip-based consensus protocol (e.g., [5]) to let all nodes belonging to the same hole boundary agree on one value (e.g., the maximum one), and this value is taken as the field identifier. Since the diffusion process of **Algorithm 1** for computing each field may start from the corresponding hole boundary, this identifier will be conveyed to every other node. Although the field values along with the identifiers are stacked in one broadcast (line 6), the computations (lines 3–5) are done separately for respective fields.

B. Quorum Access though Gradient Field Tracing

1) Field States and Their Maintenance: At the convergence of the diffusion processes presented in Sec. IV-A, each node will have a *state table* storing its field values in different harmonic fields. We illustrate such a table in Table I, where field identifiers are assumed to be 16-bit values and they are ascendingly ordered. Although the existence of this table makes the later data access protocols **stateful**, maintaining this table is much more light-weight than maintaining a routing tree (for data collection) or many end-to-end routing paths. Firstly, the field values remain intact even if some links temporarily disappear due to channel quality fluctuation. As these field values represent the geometry of the network region, they need to be changed only when nodes are removed or relocated. Secondly, even upon node removals or relocations, the cost of recovering the harmonic fields is very low: only nodes within a couple of hops from the removed or relocated nodes will be affected. Finally, given the quorum systems running on top of these fields, maintaining them is almost free, as the diffusion messages can be piggybacked along with data traffics.

TABLE I
STATE TABLE

i	Field Id	Field Value $f_i(u)$
0	0000000000000000	0.8243
1	0001010111000101	0.1324
2	0011010011011001	0.2457
···	··· ··· ··· ···	···

2) Write Quorum and Gossip Dissemination: As we briefly explained in Sec. III-B, a write quorum is composed of level sets of the 0-th field. Typically, when a source node wants to replicate its data, it chooses the level set it belongs to or other neighboring (in terms of field value) level sets if needed. As each level set (or one of its connected components if the set is disconnected) can be considered as a 2-manifold, the data replication within a write quorum can be performed in a similar way to data dissemination in a planar WSN. The most straightforward way is flooding, but we propose to used a *gossip-based* dissemination protocol shown in **Algorithm 2**. Note that, for a source node, the event that triggers this procedure is a reception of a message sent from some upper layer application; otherwise the triggering message comes from a lower layer communication protocol.

Algorithm 2: Gossip-based write quorum access

```
Parameters: Replication probability $p$; dissemination
            probability $q$, depth $\psi$, and range $ttl$
upon RECEIVE($u_0, f(u_0), \tau, \boldsymbol{data}$) do:
if $u = u_0$ then
    $\tau \leftarrow 0$
    BROADCAST ($u_0, f(u_0), \tau, \boldsymbol{data}$) to all $v \in \mathcal{N}(u)$
else
    if receive $\boldsymbol{data}$ for the first time then
        $\tau \leftarrow \tau + 1$; REPLICATE($\boldsymbol{data}$) with probability $p$
        if $|f(u) - f(u_0)| < \psi$ AND $\tau < ttl$ then
            BROADCAST ($u_0, f(u_0), \tau, \boldsymbol{data}$) to all
            $v \in \mathcal{N}(u)$ with probability $q$
        end
    else return
end
```

The replication probability p and dissemination range ttl are used to reduce the storage usage of sensor nodes and to accommodate location sensitive applications, we will discuss their usage in Sec. IV-C. By default, we set $p = 1$ and $ttl = \infty$. The dissemination probability q is what makes this protocol a gossip-based one; it is well known that, with a properly set q, the gossip dissemination can significantly reduce the overhead compared with flooding (due to the reduced packet collisions), whereas still maintain the same level of reliability (i.e., the data replication reaches every node belonging to the quorum) [10], [16]. The dissemination depth ψ is needed for two reasons. Firstly, as the field is constructed upon a discrete set, an error bound around a specific value is necessary to define a meaningful level set. Secondly, it also gives us a leverage to fine-tune the shape of the each quorum (discussed in Sec. IV-C).

3) Read Quorum and Gradient Tracing: We hereby formalize the gradient tracing idea explained in Sec. III-B. For any node $u \in \mathcal{U}$ with a location vector $\vec{u} = [x_u, y_u, z_u]$, its gradient in the field $f(\mathcal{U})$ is given by $\Delta f(u) = \left[\frac{\partial f}{\partial x_u}, \frac{\partial f}{\partial y_u}, \frac{\partial f}{\partial z_u}\right]$. The discrete approximation of the gradient is the so called *steepest ascending direction* $\vec{d}_u^{\mathrm{sa}} = \vec{v} - \vec{u} : \arg\max_{v \in \mathcal{N}(u)} f(v) - f(u)$, i.e, the direction towards the farthest (in a field f) neighbor of u. Similarly, the negative gradient is approximated by *steepest descending direction* $\vec{d}_u^{\mathrm{sd}} = \vec{v} - \vec{u} : \arg\min_{v \in \mathcal{N}(u)} f(v) - f(u)$. We now present our gradient tracing protocol in **Algorithm 3**. Note that this protocol shares parameter with **Algorithm 2**.

```
Algorithm 3: Gradient tracing read quorum access
   Parameters: Replication probability $p$; dissemination
               probability $q$, depth $\psi$, and range $ttl$
1  upon RECEIVE($u_0, f(u_0), sign, \varrho, \boldsymbol{query}$) do:
2  if $u = u_0$ then
3      if LOCALQUERY($p, q, \psi, ttl, \boldsymbol{query}$) = fail then
4          $\varrho \leftarrow$ enqueue$\{1, \cdots, m\}$
5          FORWARD ($u_0, f(u_0), sign, \varrho, \boldsymbol{query}$) according
           to both $\vec{d}_u^{\mathrm{sa}}$ ($\Rightarrow sign = 1$) and $\vec{d}_u^{\mathrm{sd}}$ ($\Rightarrow sign = -1$)
6      else return
7  else
8      if LOCALQUERY($p, q, \psi, ttl, \boldsymbol{query}$) = fail then
9          if $f(u) = 1$ then
10             $i \leftarrow$ dequeue($\varrho$); switch to the $i$-th field
11         else if $f(u) = 0$ OR $\varrho = \emptyset$ then return
           FORWARD ($u_0, f(u_0), sign, \varrho, \boldsymbol{query}$) according
           to either $\vec{d}_u^{\mathrm{sa}}$ (if $sign = 1$) or $\vec{d}_u^{\mathrm{sd}}$ (if $sign = -1$)
12     else return
13 end
```

This means that data sources and potential data consumers have to agree on these parameters in advance, in order to preserve the intersection property of HQSs.

By default, the procedure LOCALQUERY$(\cdots)$ only queries the local database at a node u. However, depending on the application requirements, it may perform an expanding ring search [11] around u, which is necessary to adapt to the tunability of write quorums (see Sec. IV-C for details). The procedure FORWARD$(\cdots)$ calls the underlying MAC protocol to relay the query message to the next hop in a greedy manner; it carries a flag *sign* to indicate the forwarding direction. If the query reaches a hole boundary without being successful (lines 9–11), the protocol switches to the next field in the stack ϱ. The following proposition states the correctness of the gradient tracing under an idea situation.

Proposition 1: Assume that the gossip-based dissemination is reliable and $p = 1$, a read quorum generated by **Algorithm 3**, upon emptying its queue (ϱ), does intersect all write quorums generated by **Algorithm 2**.

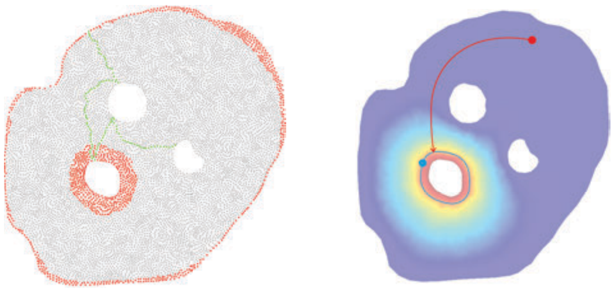
The proof is omitted; it follows directly from the basic properties of level sets and gradients. In reality, gossiping may not be fully reliable (e.g., due to packet loss) and $p < 1$ is possible, but these can be compensated by expanding the range of LOCALQUERY$(\cdots)$ accordingly. We will give further discussions on this issue in Sec. IV-C.

*Remark: The harmonic quorum system can also be defined in a **reversed** way: level sets as read quorums and gradient tracing for write quorums.*

C. System Optimizations

We hereby discuss how to optimize the performance of HQSs to handle three practical issues: namely, tuning the load of HQSs, accommodating location sensitive data accesses, and coping with unreliable replications.

1) Load Fine-Tune: For a node u whose harmonic value $f(u)$ is close to zero (i.e., u is close to the external boundary, see Fig. 4), the level set it belongs to is larger than those close to the internal boundaries. Whereas this has a positive contribution to load balancing (as the network center and hole boundaries are often hotspots due to other networking activities [27]), this phenomenon may not favor the total energy consumption. Furthermore, data sources may have different rates of accessing their respective write quorums, so it is important that we can fine-tune the "shape" of these quorums. This is exactly the reason for the parameter ψ in **Algorithm 2**, tuning it has the effect of changing the load profile of a write quorum. We illustrate the idea of using ψ to fine-tune write quorums in Fig. 5 (a). In general, we can set $\psi_u = g(f(u))$ where $g(\cdot)$ is an increasing function in $[0, 1]$.



(a) Fine-tuning quorum load profiles

(b) Accessing location sensitive data

Fig. 5. The load profile of a write quorum in HQSs can be fine-tuned by the parameter ψ (a), where we use red and green to identify write and read quorums, respectively. Using holes (virtual or real) to identify locations, HQSs can also accommodate location sensitive data (b).

2) What If Data Are Location Sensitive: An intriguing aspect of HQSs is that, although they are proposed to avoid the dependence on any location services, they may readily accommodate location sensitive data should such a need be ever raised. Of course, a location service will be needed under such a circumstance (otherwise location sensitivity becomes meaningless), but our HQSs are still light-weight in the sense that they do not need an exact geographic location to pinpoint where certain location sensitive data are.

To simply illustrate the idea, we assume that certain location sensitive data are generated by some nodes close to a hole.[3] According to **Algorithm 2**, the data will be replicated around the hole (locality preserved), and the location service only need to make the hole identifier available to all potential data consumers. Upon a need to access such data, a consumer simply needs to trace the corresponding field. We show an example in Fig. 5 (b). Moreover, even if there is no hole close to the data sources of certain location sensitive data, *virtual holes* may be created at or around these sources. Creating a virtual hole is very easy: a set of nodes recognize themselves as the virtual hole and fix their harmonic value to 1, then they agree on a common identifier (again using a consensus protocol [5]), they finally initiate a field construction process following **Algorithm 1**.

Remark: The fact that virtual holes can be used for HQSs justifies our claim in Sec. III-B that HQSs also work for WSNs without holes.

In addition, the parameter *ttl* in **Algorithm 2** can be used to further confine the data replication to a sub-area on a level set. As HQSs only guarantee a read quorum to intersect a level set at any arbitrary point, confining data replication to a sub-area on a level set will require the read quorum to further search within the level set. While such a search is trivial in 2D (where a level set is a topological circle), it needs additional supports in 3D (where a level set is a 2-manifold). We refer interested readers to our previous work on GQSs [17] for a searching technique on 2-manifold. Although using *ttl* further reduces the energy consumption of data replication in a write quorum, it incurs high overhead in accessing a read quorum, especially due to the need for additional supports. Therefore, one has to be careful in using this feature.

3) Masking Unreliability in Replication: Several reasons may lead to unreliable replications in a write quorum; these include (i) the gossip-based quorum access (even with $q = 1$, i.e., flooding) is not fully reliable (packet losses can happen), (ii) a lazy replication with $p < 1$ may be invoked to save storage, and (iii) the inherent unreliability in sensor nodes.

HQSs have three main leverages to mask the unreliability. The first leverage is the dissemination depth ψ we have discussed in Sec. IV-C1. Increasing ψ allows the replication process to take place in more neighboring level sets, and hence compensating random packet losses or node failures. The second one is LOCALQUERY$(\cdots)$: we may expand the query range up to a few hops. The third one is to increase the number of rounds a read quorum travel among the holes (by default, only one round is specified by **Algorithm 3**). As all other system designs, HQSs can mask the unreliability to any extent at the cost of increasing overhead.

V. PERFORMANCE EVALUATION

The performance evaluation for HQSs is based on two sets of implementations, one for TinyOS [2] and TOSSIM [13], and another one for a high-level simulator programmed by C++. We use the second set of implementation only if a simulated WSN is too large to be handled by TOSSIM. We first report the complexity results for constructing harmonic fields in Sec. V-A. Then we evaluate HQSs' performance, with respect to both energy efficiency and load balancing, for 2D and 3D WSNs in Sec. V-B and V-C, respectively.

A. Harmonic Fields Construction

As the complexity in terms of network size and the error tolerance ϵ is well established in the literature (e.g., [5]), we only investigate the the impact of the message exchange period π on the convergence time. As the time complexity is rather insensitive to the network size and the number of fields being constructed concurrently, we focus on one field construction in WSNs of around 1000 nodes. As the algorithm termination is determined by $\Delta f(u)$ for a given node u, we look at the maximum error $\max_{u \in \mathcal{U}} \Delta f(u)$ as a function of the number of message exchange rounds, and we report the mean values of these maximum errors for 100 traces in Fig. 6.

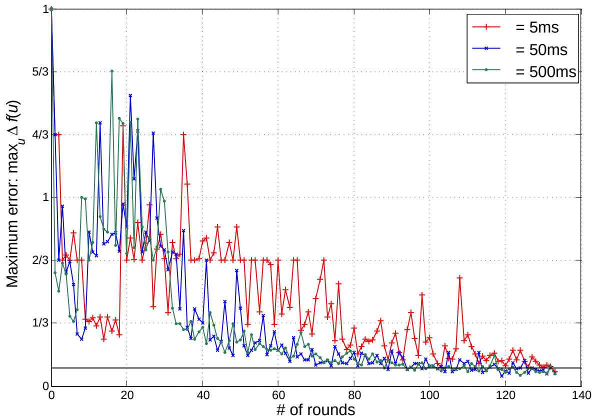


Fig. 6. Convergence of LBS under different values of π.

It is clear that, whereas the total number of rounds to reach a certain error tolerance (indicated by the horizontal line) decreases with π (70, 90, 135 for π = 500ms 50ms, and 5ms, respectively), the absolute time to convergence increases monotonically in π (35s, 4.5s, and 0.675s for π = 500ms 50ms, and 5ms, respectively). As the number of messages sent is proportional to the number of rounds, the algorithm becomes more energy efficient but less time efficient with large values of π. The intuitive explanation is the following. When π gets small, the potential (broadcast) packet collisions become intensive. Consequently, the information each node collects about its neighbors decreases, which in turns requires more rounds to terminate the diffusion process. Therefore, we have a clear tradeoff between the time complexity and the energy efficiency of LBS, and we may tune the value of π to obtain a required (by a certain application) balance between reducing latency and saving energy.

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[3] Holes in a WSN often indicate regions of potential interest, for example, a lake within a terrestrial WSN, so identifying locations by holes does make sense in practice.

B. HQSs for 2D WSNs

We first evaluate the performance of HQSs by comparing with 2D-GQSs [17]. The basic idea of 2D-GQSs, sketched by Fig. 1 (a), is to use simple geometric curves (e.g., circles) for write quorums and another specifically designed curve (spiral if the write quorums are circles) for read quorums. We have shown in [17], 2D-GQSs outperform the conventional data management schemes (e.g., [25]) and another quorum-like approach [26]. However, as we have discussed in Sec. I, 2D-GQSs may lead to unbalanced load distribution when facing holes in the network region. Our simulations in this section aim at confirming this statement and also demonstrate the superior ability of HQSs in handling holes.

We have performed intensive simulations in ten WSNs, four of them shown in Fig. 7. The size of these WSNs vary from 500 to 5000. For each network, we run ten simulations, each with 400 write nodes and 200 read nodes (all randomly chosen), hence 400 write quorums and 200 read quorums. The load (or energy consumption) of individual nodes is set as 1 unit for both sending and receive a message. We use two performance metrics: *maximum load* (i.e., the maximum node energy consumption within the whole WSN, indicating load balancing) and *average load* (indicating energy consumption). Whereas maximum load was used in quorum systems' literature (e.g. [17], [23]), average load (a variance of the total load defined in [17]) is specific for WSN-based quorum systems.

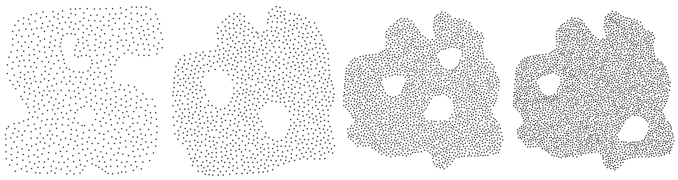
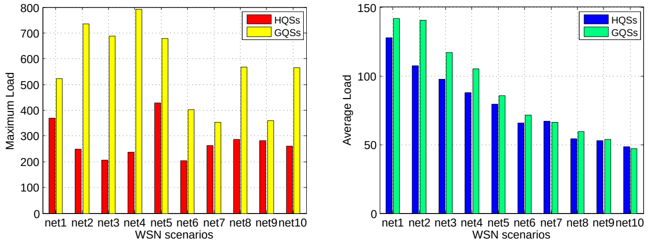

(a) 693 nodes (b) 978 nodes (c) 1194 nodes (d) 2255 nodes

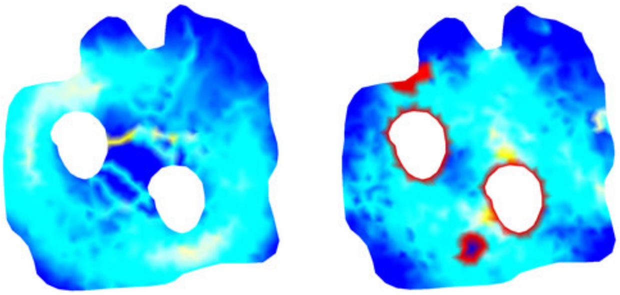
Fig. 7. Four examples of the simulated 2D WSNs.

In Fig. 8, we compare HQSs with 2D-GQSs in terms of these two metrics; only average values for the ten simulations are presented, and the variances are negligible. It is clear that HQSs performs better than GQSs in almost all cases, in particular with respect to the maximum load (GQSs sometimes have a tripled maximum load compared with that of HQSs). As main reason, as we have expected (see Sec. I) is that many quorums of GQSs need to take detours around the holes, hence increasing the total energy consumption where imposing a very high load on the hole boundaries. Note that the maximum load of HQSs is mostly only twice of the average load, indicating an almost perfect load distribution. To better illustrate this, we render the load distribution for two WSNs in Fig. 9 using a color spectrum, where red color represents a load larger than 400. It is easy to discern the hotspots generated by GQSs around the hole boundaries. HQSs, on the contrary, totally annihilate the load concentrations on these boundaries, this stems from the ability of the harmonic field to adapt to the geometry of a WSN region. It is interesting to note that HQSs may also create minor hotspots. The reason is that if a set of quorum accessing nodes belong to the same or neighboring level sets (i.e., having similar field values), their gradient tracing may converge to one path. We currently handle this issue by some slight randomization in the gradient tracing, but we are working on a more systematic solution to this problem.



(a) Load balancing (b) Energy efficiency

Fig. 8. Comparing HQSs with GQSs against the two metrics.


(a) HQSs in Net2 (b) GQSs in Net2

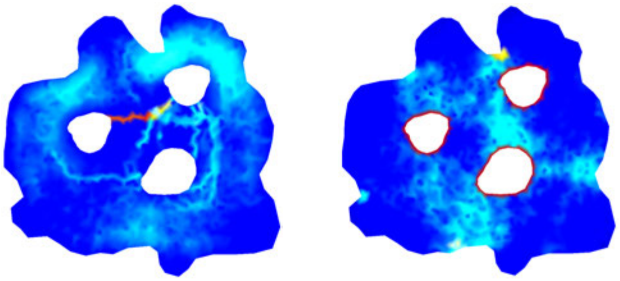
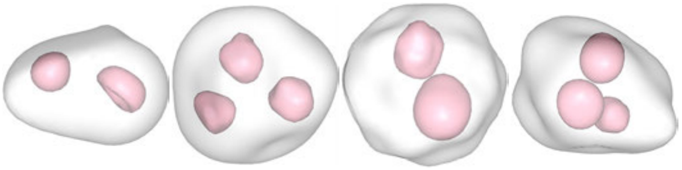

(c) HQSs in Net5 (d) GQSs in Net5

Fig. 9. Color spectrum for visualizing the load distributions.

C. HQSs for 3D WSNs

For 3D WSNs, we consider five arbitrarily shaped network with 2 to 3 holes inside; four of these WSNs are shown in Fig. 10 (only 3D volumes representing the envelopes of individual WSNs are drawn, otherwise the holes are not discernable).

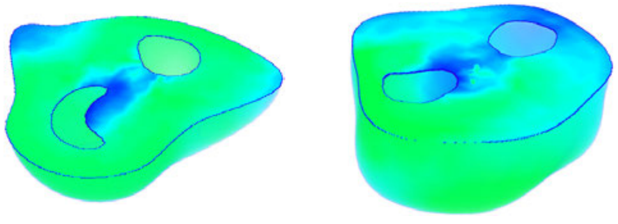
As we are the first to build a routing-independent data management systems for 3D WSNs, we do not have counterparts to compare with. Our previous proposal for 3D-GQSs [20] and also other data management systems based on geographic routing (e.g., [3]) may fail to deliver message to a quorum when facing holes. Therefore, the goal of this set of simulations are mainly meant to verify the feasibility of HQSs in 3D WSNs. To make the simulation results more



(a) 2389 nodes (b) 2658 nodes (c) 3896 nodes (d) 4265 nodes

Fig. 10. Four examples of the simulated 3D WSNs.

tangible, we render the WSN 3D volumes by a color spectrum to illustrate the load distribution. As shown in Fig. 11, the load balancing effect of HQSs is very remarkable; there is in general no hotspot being created in the network region.



(a) WSN in Fig. 10 (a) (b) WSN in Fig. 10 (c)

Fig. 11. Color spectrum for cut views of the load distributions in 3D WSNs.

VI. CONCLUSION

In this paper, we have investigated the problem of distributed data management in 2D/3D WSNs, bearing in mind the existence of communication voids (or holes) in a network. As geographic routing protocols used by most of the existing data management systems may fail in the face of holes, we propose to construct quorum systems for distributed data management and create an innovative idea of using harmonic fields to guide the data access. This results in the so called harmonic quorum systems (HQSs). The benefits of marrying quorum systems with harmonic fields are many. First, the harmonic fields can be constructed efficiently and maintained easily, which significantly reduces the overhead of data replications and queries. Second, as HQSs are independent of routing protocols, they do not inherit from a routing protocol the impossibility of guaranteeing delivery in the face of holes. Finally, because harmonic fields adapt well to the geometry of a network region, using them to organize data replications and to guide data queries leads to very balanced load distribution. We have implemented HQS in TinyOS and our simulations confirm all the aforementioned advantages of HQSs.

Due to space limitations, we have not been able to present many other simulation results, including those concerning load fine-tuning and location sensitive data accesses. Moreover, we are also investigating a systematic way to randomize the gradient tracing, such that data accesses based on it do not coincide with each other with high probability. These extensions will be conveyed in our future work. Finally, given the advantage of harmonic fields, we are planning to extend their usage to other networking aspects.

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