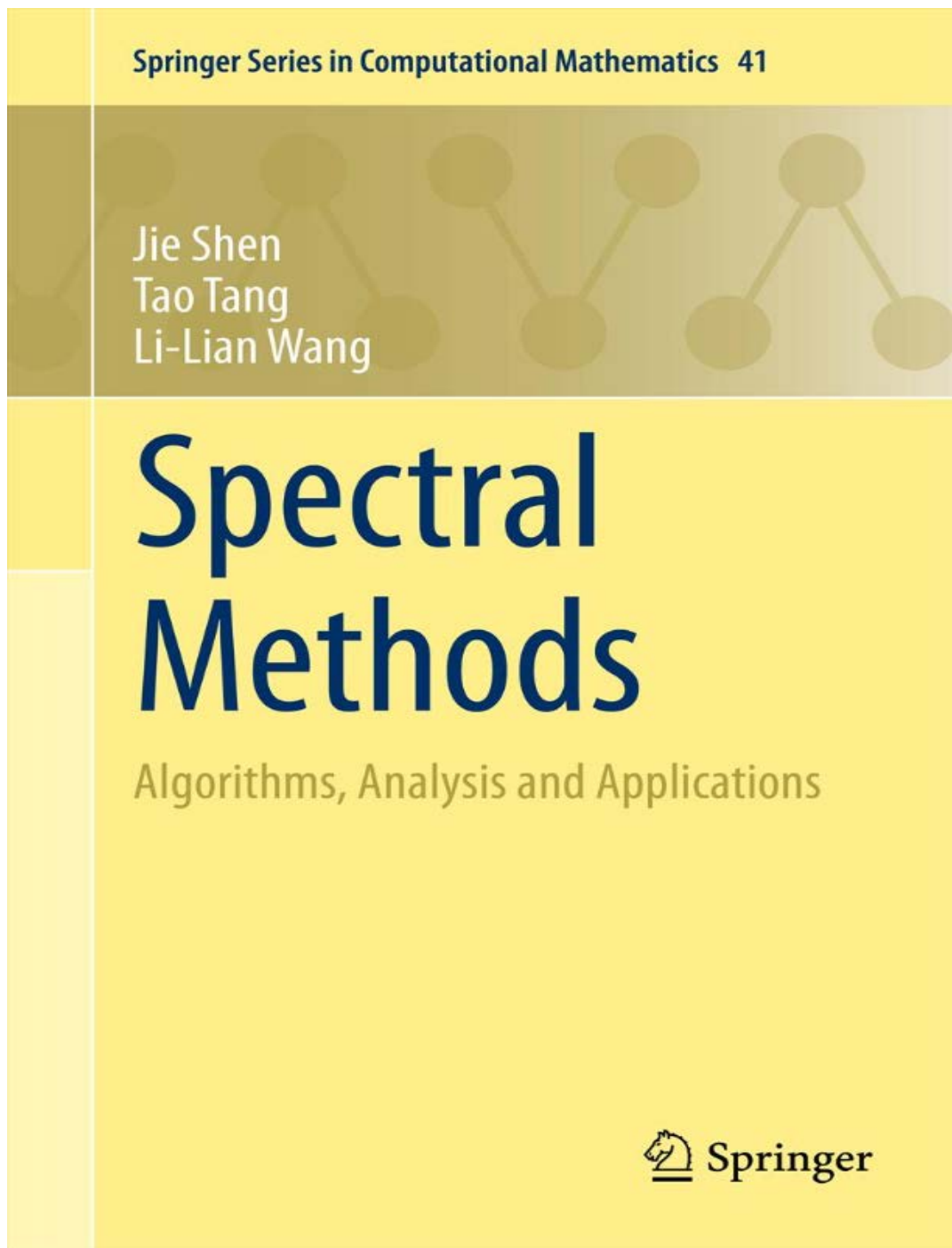


INSTRUCTIONS FOR MATLAB ROUTINES



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A. Introduction

This note is devoted to some instructions to the `Matlab` routines for the fundamental spectral algorithms presented in the book:

Jie Shen, Tao Tang and Li-Lian Wang. *Spectral Methods: Algorithms, Analysis and Applications*. Springer Series in Computational Mathematics, 41, Springer, 2011.

This package consists of a collection of `Matlab` functions which are indispensable for spectral-Galerkin/collocation approximations of partial differential equations. These include routines for

- Evaluation of Legendre polynomials, Jacobi polynomials, (generalized) Laguerre polynomials/functions and Hermite polynomials/functions.
- Computation of the Gaussian-type quadrature nodes and weights associated with these orthogonal polynomials/functions.
- Differentiation in the physical space via differentiation matrix.
- Implementation of pseudospectral techniques: discrete transforms and differentiation in frequency spaces.
- Applications of these basic algorithms for solving PDEs.

We attempt to provide one-to-one correspondence between the codes and the equations, formulas and algorithms in the book, and have made significant efforts to test these functions. However, the users are advised to choose the underlying degree of modes and parameters from the range of standard setting. You are more than welcome to email us if you find bugs or have comments.

We also refer to the readers to the following online software in `Fortran` or `Matlab` on spectral methods:

- **Fortran Routines for Spectral Methods** by D. Funaro, which are available at <http://cdm.unimo.it/home/matematica/funaro.daniele/rout.htm>
- **A Matlab Differentiation Matrix Suite** by J. Weideman and S. Reddy, which is downloadable at <http://dip.sun.ac.za/weideman/research/differ.html>
- **PseudoPack** by W.S. Don and B. Costa, which can be found at <http://www.math.hkbu.edu.hk/~wsdon/>
- Some codes for **Spectral Methods in Matlab** by L. Trefethen, which can be downloaded from <http://people.maths.ox.ac.uk/trefethen/spectral.html>

B. Instructions

1. Legendre Spectral Methods

- (1) **[dy, y]=lepoly(n,x);**
Evaluate the Legendre polynomial and its first-order derivative of degree n at a vector-valued x .
- (2) **[DY, Y]=lepolym(n,x);**
Evaluate the Legendre polynomials and their first-order derivatives of degree $\leq n-1$ at a vector-valued x . Note: $\{Y(i, j) = L_{i-1}(x_j)\}_{1 \leq i, j \leq n}$.
- (3) **[x, w]=legs(n);**
Compute the Legendre-Gauss quadrature nodes and weights $\{x_j, w_j\}_{j=1}^n$, where the Newton's method is used for computing the nodes.
- (4) **x=legsndm(n);**
Compute the Legendre-Gauss points $\{x_j\}_{j=1}^n$ by the eigen-method.
- (5) **[x, w]=legslb(n);**
Compute the Legendre-Gauss-Lobatto quadrature nodes and weights $\{x_j, w_j\}_{j=1}^n$, where the Newton's method is used for computing the nodes.
- (6) **x=legslbndm(n);**
Compute the Legendre-Gauss-Lobatto points $\{x_j\}_{j=1}^n$ by the eigen-method.
- (7) **w=legslbwt(n);**
Compute the Legendre-Gauss-Lobatto weights $\{w_j\}_{j=1}^n$ by the weight expression in Yakimiw [3].
- (8) **[x, w]=legsrld(n);**
Compute the Legendre-Gauss-Radau quadrature nodes and weights $\{x_j, w_j\}_{j=1}^n$, where the eigen-method is used for computing the nodes.
- (9) **D=legsdiff(n,x); D=legslbdiff(n,x); D=legsrddiff(n,x);**
Compute the first-order differentiation matrix D (of size $n \times n$) associated with the Legendre-Gauss, Legendre-Gauss-Lobatto and Legendre-Gauss-Radau points $\{x_j\}_{j=1}^n$, respectively.
- (10) **s=ledisctran(n,x,w,p,iflag);**
Perform discrete Legendre transforms on the Legendre-Gauss-Lobatto points and weights $\{x_j, w_j\}_{j=1}^n$. If iflag=0, it conducts the forward transform from the physical values (stored in p) to the Legendre expansion coefficients (output in s), while if iflag $\neq 0$, it conducts the backward transform from the

Legendre expansion coefficients (stored in p) to the physical values (output in s). See Page 101 of [1].

(11) **duh=lefreqdiff(n,uh);**

Perform spectral differentiation in the frequency space. Given the Legendre expansion coefficients (stored in uh), it returns the Legendre expansion coefficients of the first-order derivative (output in duh). See Page 105 of [1].

2. Chebyshev Spectral Methods

(1) **D=chebdiff(n);**

Compute the first-order differentiation matrix (of size $(n + 1) \times (n + 1)$) associated with the Chebyshev-Gauss-Lobatto points. See Page 80 of [2].

(2) **w=chebfft(v);**

Perform Chebyshev spectral differentiation via FFT. Given the physical values (stored in v) on the Chebyshev-Gauss-Lobatto points, it returns the first-order derivative values (output in w). See Page 81 of [2].

————— Many More Routines Are Coming —————

Bibliography

- [1] J. Shen, T. Tang and L.L. Wang. *Spectral Methods: Algorithms, Analysis and Applications*. Springer Series in Computational Mathematics, Volumn 41, Springer, 2011.
- [2] L. Trefethen. *Spectral Methods in Matlab*, Volumn 10, SIAM, 2000.
- [3] E. Yakimiw. Accurate computation of weights in classical Gauss-Christoffel quadrature rules, *Journal of Computational Physics*, 129(2), 406-430, 1996.