

Erratum for the book

# Spectral Methods

## Algorithms, Analysis and Applications

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[Updated on December 28, 2015 with corrections in shaded text]

### 1 Chapter 1

1. Page 16, Line 12: Change  $h = \frac{2\pi}{N}$  to  $h = \frac{2\pi}{N+1}$ .

### 2 Chapter 2

1. Page 28 in (2.25), replace  $D_{N/2-1}(x - x_j)$  by  $\mathcal{D}_{N/2-1}(x - x_j)$ .
2. Page 29 in Table 2.1, replace  $2\tilde{u}_{N/2-1}$  by  $\tilde{u}_{N/2-1}$ , and replace  $\tilde{u}_{N/2}$  by  $2\tilde{u}_{N/2}$ .
3. Page 36, four lines below (2.53), change “ $e^{i(p-k)x_j} = 1$ ” to “ $\frac{1}{2N} \sum_{j=0}^{2N-1} e^{i(p-k)x_j} = 1$ ”

### 3 Chapter 3

1. Page 69, remove “3.1.8.1” before “A Short Summary of this Section”.
2. Page 83, Line 7: change “ $G_{N-1}^{\alpha, \beta+1}$ ” and “ $\tilde{G}_{N-1}^{\alpha, \beta+1}$ ” to “ $G_{N-2}^{\alpha+1, \beta+1}$ ” and “ $\tilde{G}_{N-2}^{\alpha+1, \beta+1}$ ”, respectively.
3. Page 86, “ $\pi$ ” is missing in (3.147), which should be

$$\frac{2j+1}{2N+3}\pi \leq \theta_j \leq \frac{2j+2}{2N+3}\pi, \quad 0 \leq j \leq N.$$

4. Page 94, remove the factor  $2^n$  in the denominator of the terms in the sum of (3.169).
5. Page 104, a comma is missing in the last line of (3.203).
6. Page 106,  $T(x)$  should be  $T_n(x)$  in (3.212a).
7. Page 118, Theorem 3.35 above (3.257), add the condition on  $l$ : “ $0 \leq l < N + 1$ ”
8. Page 120, the line above (3.267), change “ $0 \leq l \leq m$ ” to “ $1 \leq l \leq m$ ”, and two lines below (3.267), remove “,  $\beta$ ”

9. Page 130, the proof in lines 2-6 should be changed to

$$\begin{aligned}
\|I_N^c u\|_\omega^2 &= \frac{\pi}{N+1} \sum_{j=0}^N \hat{u}^2(\theta_j) \leq \frac{\pi}{N+1} \sum_{j=0}^N \left\{ \max_{\theta \in K_j} |\hat{u}(\theta)| \right\}^2 \\
&\leq \sum_{j=0}^N \left( \|\hat{u}\|_{L^2(K_j)} + \frac{\pi}{N+1} \|\partial_\theta \hat{u}\|_{L^2(K_j)} \right)^2 \\
&= \sum_{j=0}^N \|\hat{u}\|_{L^2(K_j)}^2 + \frac{2\pi}{N+1} \sum_{j=0}^N \|\hat{u}\|_{L^2(K_j)} \|\partial_\theta \hat{u}\|_{L^2(K_j)} + \frac{\pi^2}{(N+1)^2} \sum_{j=0}^N \|\partial_\theta \hat{u}\|_{L^2(K_j)}^2 \\
&\leq \|\hat{u}\|_{L^2(0,\pi)}^2 + \frac{2\pi}{N+1} \left( \sum_{j=0}^N \|\hat{u}\|_{L^2(K_j)}^2 \right)^{\frac{1}{2}} \left( \sum_{j=0}^N \|\partial_\theta \hat{u}\|_{L^2(K_j)}^2 \right)^{\frac{1}{2}} + \frac{\pi^2}{(N+1)^2} \|\partial_\theta \hat{u}\|_{L^2(0,\pi)}^2 \\
&\leq \|\hat{u}\|_{L^2(0,\pi)}^2 + \frac{2\pi}{N+1} \|\hat{u}\|_{L^2(0,\pi)} \|\partial_\theta \hat{u}\|_{L^2(0,\pi)} + \frac{\pi^2}{(N+1)^2} \|\partial_\theta \hat{u}\|_{L^2(0,\pi)}^2 \\
&= \left( \|\hat{u}\|_{L^2(0,\pi)} + \frac{\pi}{N+1} \|\partial_\theta \hat{u}\|_{L^2(0,\pi)} \right)^2.
\end{aligned}$$

This implies

$$\|I_N^c u\|_\omega \leq \|\hat{u}\|_{L^2(0,\pi)} + \frac{\pi}{N+1} \|\partial_\theta \hat{u}\|_{L^2(0,\pi)}.$$

Finally, the inverse change of variable  $\theta \rightarrow x$  leads to (3.297).

10. The above modification also applies to line 7-10 on Page 132.

11. Page 137, above Theorem 3.44, change ‘‘Chap. 5’’ to ‘‘Chap. 6’’

12. Page 138, the last line, change ‘‘ $\sum_{n=k}^N$ ’’ to ‘‘ $\sum_{n=0}^{N-k}$ ’’

## 4 Chapter 4

1. Page 141, the line below (4.3), change ‘‘necessary’’ to ‘‘sufficient’’

2. Page 146, replace  $a_-b_+$  by  $a_-b_+$ .

3. Page 151, Line 4, replace  $O(N^2)$  by  $O(N^2)$ .

4. Page 169, in the proof of Theorem 4.3, add ‘‘and with integration by parts’’ after ‘‘Under the assumption (4.3)’’; and remove  $H^2(I) \cap$  in the definition of  $X$  in the line below (4.85).

## 5 Chapter 6

1. Page 202: Last line: replace (6.3) by (6.4).

2. Page 212, above (6.37), ‘‘ $\phi_k(x) = \gamma_k J_{n+3}^{-2,-1}(x)$ ’’ should be ‘‘ $\phi_k(x) = \gamma_k J_{k+3}^{-2,-1}(x)$ ’’.

- Page 223, the part of the proof below (6.68) should be changed to: ....., while for  $m = 1$ , we verify from the Cauchy-Schwarz inequality that

$$\|\partial_x \tilde{u}\| \leq c(\|\partial_x u\| + |u(-1) - u(1)|) \leq c\left(\|\partial_x u\| + \left|\int_{-1}^1 \partial_x u \, dx\right|\right) \leq c\|\partial_x u\|.$$

This ends the proof of Theorem 3.4.

- Page 236, Problem 6.4, change “(6.18)” to “(6.32)”

## 6 Chapter 7

- Page 241, Formula (7.15): change “if  $\alpha > 0$ ” in the second line, to “if  $-1 < \alpha < 0$ ”.
- Page 246, Eqn. (7.36): change “ $\forall p \cdot q \in \widehat{P}_{2N+\delta}$ ” to “ $\forall p \in \widehat{P}_K, \forall q \in \widehat{P}_L, K + L \leq 2N + \delta$ ”.
- Page 258, Eqn. (7.82): change “ $\forall p \cdot q \in \widehat{P}_{2N+1}$ ” to “ $\forall p \in \widehat{P}_K, \forall q \in \widehat{P}_L, K + L \leq 2N + 1$ ”.
- Page 268, right below Eqn. (7.119): change

$$-\frac{1}{4}((ue^{x/2}) - \Pi_N^{1,0}(ue^{x/2}), \mathbf{v}_N)_\omega \quad \text{to} \quad -\frac{1}{4}((ue^{x/2}) - \Pi_N^{1,0}(ue^{x/2}), \mathbf{w}_N)_\omega$$

and add the missing parentheses in the term

$$-\frac{1}{2} \int_0^{+\infty} [((ue^{x/2}) - \Pi_N^{1,0}(ue^{x/2})) \mathbf{w}_N]' e^{-x} \, dx$$

## 7 Chapter 8

- Page 302, above and in (8.12), change “ $(EA)^{-1}$  (resp.  $(EA)^{-T}$ )” to “ $(AE)^{-1}$  (resp.  $(AE)^{-T}$ )”
- Page 303, lines 7-8, change “ $(EA)^{-1}$ ” to “ $(AE)^{-1}$ ”, and change “ $(EA)^{-T}$ ” to “ $(AE)^{-T}$ ”
- Page 304, change  $E^{TBE}$  and  $E^{TAE}$  to  $E^T BE$  and  $E^T AE$ , respectively, in (8.20) and (8.21).

## 8 Chapter 9

- Page 371, change  $T(u) = -iu$  in (9.21) to  $T(u) = -iku$ .
- Page 394, replace the superscript  $n$  by  $k$  in the last line.
- Page 395, replace  $\psi^{k+1}$  by  $\phi^{k+1}$  in (9.118)-(9.119).
- Page 396,  $2p_N^k - p_N^{k-1}$  should be  $p_N^k$  in (9.121) corresponding to (9.102).
- Page 396, the first “Poisson equation” should be “Poisson-type equation” in the last line.