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Abstract This paper gives similarity transformations for laminar film condensation on a vertical flat plate with variable temperature distribution and finds analytical solutions for arbitrary Prandtl numbers and condensation rates. The work contrasts with Sparrow and Gregg's assertion that wall temperature variation does not permit similarity solutions. To resolve the long debatable issue regarding heat transfer of non-isothermal case, some useful formulas are obtained, including significant correlations for varying Prandtl numbers. Results are compared with the available experimental data.

Keywords Similarity solutions · Laminar film condensation · Non-isothermal plate

الخلاصة

تعطي هذه الورقة التحويلات المتشابهة لطبقة تكثيف على لوحة مسطحة رأسية مع توزيع متغير لدرجة الحرارة، وإيجاد حلول تحليلية لأرقام براندل التعسفية ومعدلات التكثيف. يتعارض هذا العمل مع ما ذهب إليه سبارو وجريج بأن درجة اختلاف درجة حرارة الجدار لا تسمح بحلول متشابهة. واستناداً إلى الحلول المتشابهة، تم إيجاد معادلات مفيدة بما في ذلك الارتباط الهام لإعداد براندل المتغيرة. وتمت مقارنة النتائج مع البيانات التجريبية المتوفرة.

1 Introduction

A theory of laminar film condensation was first formulated by Nusselt [1,2] who considered condensation onto an isothermal flat plate maintained at a constant temperature below the saturation temperature of the surrounding quiescent vapour. Expressions for condensate film thickness and heat transfer characteristics were obtained using simple force and heat balance arguments. Effects due to inertia forces, thermal convection and interfacial shear were neglected, as was surface tension and the possible presence of waves on the condensate film surface. Later, authors have subsequently refined Nusselt's theory to include some of these omissions. The effects of thermal convection were first examined by Bromley [3] and then by Rohsenow [4] each of whom proposed modifications to the latent heat of condensation to be used in assessing heat transfer at the plate. Sparrow and Gregg [5] were the first to recognize the close parallels between natural convection boundary layers and laminar film condensation. Accordingly, they presented a boundary layer treatment for condensation in the presence of an isothermal cold wall which enabled them to incorporate both thermal convection and inertia effects. Similarity solutions of the governing parabolic equations were derived and detailed numerical solutions were obtained for a wide range of Prandtl numbers and condensation rates. Excellent agreement

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between the exact numerical results and Rohsenow's correlation was demonstrated at large Prandtl numbers. The greatest departures from Nusselt's theory were observed at low Prandtl numbers.

In view of the parallels between natural convection and laminar film condensation, it was a natural progression for Sparrow and Gregg [5] to consider the possible existence of similarity solutions for non-isothermal conditions at the plate. They concluded that the families of wall temperature distributions, power-law $T_w(x) = T^* - T_0 x^a$ or exponential-law $T_w(x) = T^* - T_0 e^{bx}$, where $T^*, T_0 > 0$ and a, b are constants, did not in fact permit similarity solutions. In particular, this appeared to preclude the existence of such a solution for the standard boundary condition of constant heat flux at the plate, which they had solved successfully in the natural convection setting in [6]. The influence of variations in wall temperature on the laminar film condensation was studied by Nagendra and Tirunarayanan [7], but unfortunately their presented results were questioned by Subrahmaniyam [8]. The work that follows this assertion is re-examined. Similarity solutions for variable wall temperatures $T_w(x) = T^* - D(x)$ are found and the associated flow and heat transfer characteristics are presented.

2 Physical Model and Governing Equations

The physical model to be examined is illustrated in Fig. 1. A semi-infinite flat plate is aligned vertically with its leading edge uppermost. The surrounding ambient is pure quiescent, saturated vapor maintained at a temperature T^* . The plate temperature $T_w(x) = T^* - D(x)$. As a result, the lower temperature at the plate condensation occurs and a continuous laminar film flows downwards along the plate. If the flow is assumed to be in a steady state the governing equations expressing conservation of mass, momentum and energy are, respectively,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = g \left(\frac{\rho - \rho^*}{\rho} \right) + \frac{\mu}{\rho} \frac{\partial^2 u}{\partial y^2}, \quad u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2}, \quad (1)$$

where (u, v) are velocity components associated with increasing coordinates (x, y) measured along and normal to the plate from the leading edge of the plate and T is temperature within the condensate film.

The physical properties, ρ^* , the density of the vapor, ρ, μ, C_p, k , the density, the dynamic viscosity, specific heat at constant pressure and the thermal conductivity of the condensate, respectively, are assumed to be constant and g is the acceleration due to gravity. Viscous dissipation and the effects of interfacial shear on the free surface of the condensate are assumed to be negligibly small.

Equations (1) are boundary layer equations derived under the assumption that $\frac{\partial}{\partial x} \ll \frac{\partial}{\partial y}$ and that the condensation Grashof number,

$$Gr_{rx} = \frac{g\rho(\rho - \rho^*)x^3}{4\mu^2} \gg 1.$$

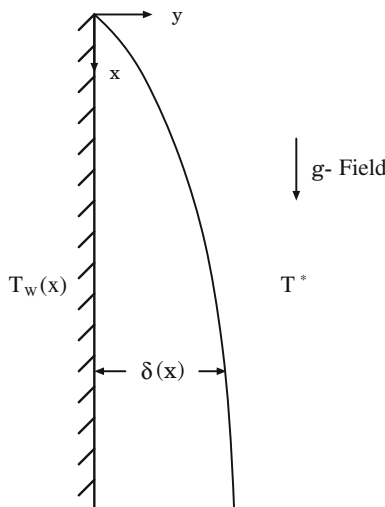


Fig. 1 Physical model and co-ordinate system

To the level of approximation involved, the v -momentum equation is satisfied identically.

Flow characteristics are to be established under the hydrodynamic boundary conditions of no slip at the plate and zero shear on the condensate free surface, namely

$$u = v = 0 \quad \text{on } y = 0; \quad \frac{\partial u}{\partial y} = 0 \quad \text{on } y = \delta(x), \tag{2}$$

where $\delta(x)$ is the thickness of the condensate film. The prescribed temperature conditions for variable temperatures at the plate and saturation temperature on the free surface require

$$T = T_w(x) = T^* - D(x) \quad \text{on } y = 0; \quad T = T^* \quad \text{on } y = \delta(x). \tag{3}$$

3 Similarity Equations

In the actual situation, the function $D(x)$ is a holomorphic function, which can be expanded into a convergent power series, so suppose

$$D(x) = \sum_{n=0}^{+\infty} T_n x^{p_n}, \tag{4}$$

where $\{T_n\}$ is a sequence of numbers and $\{p_n\}$ is a non-negative increasing sequence. The continuity equation in (1) is identically satisfied if a stream function formulation

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x},$$

is introduced. It then remains to invoke, if possible, similarity transformations which reduce the parabolic, partial differential equations (1) and their boundary conditions to ordinary differential equations. Because the third equation in (1) is linear for temperature T , this possibility is readily examined by the preliminary transformations

$$\psi \sim x^r f(\eta), \quad \eta \sim \frac{y}{x^s} \quad \text{and} \quad T \sim x^{p_n} \theta_n(\eta).$$

The second equation in (1) reduces to an ordinary differential equation in η only if $2r - 2s - 1 = r - 3s = 0$, i.e. $s = \frac{1}{4}, r = \frac{3}{4}$. Such results are entirely in keeping with the findings of Sparrow and Gregg [5]. If the third equation in (1) is also to become an ordinary differential equation in η then $r + p_n - s - 1 = p_n - 2s$. Thus, $s = \frac{1}{4}, r = \frac{3}{4}$ and p_n provide similarity scalings which ensure that the governing equations do indeed reduce to a pair of ordinary differential equations.

To examine solutions at extremes of low and high Prandtl numbers, $Pr = \frac{\mu C_p}{k}$, two forms of transformations are instructive.

$$Pr \leq 1 : \quad \psi = \frac{4\mu}{\rho} G_{rx}^{\frac{1}{4}} f(\eta) \quad Pr > 1 : \quad \psi = \frac{4\mu}{\rho} \frac{G_{rx}^{\frac{1}{4}}}{Pr^{\frac{3}{4}}} \tilde{f}(\tilde{\eta}) \tag{5a}$$

$$\eta = \frac{y}{x} G_{rx}^{\frac{1}{4}} \quad \tilde{\eta} = \frac{y}{x} G_{rx}^{\frac{1}{4}} Pr^{\frac{1}{4}} \tag{5b}$$

$$T - T^* = - \sum_{n=0}^{+\infty} T_n x^{p_n} \theta_n(\eta) \quad T - T^* = - \sum_{n=0}^{+\infty} T_n x^{p_n} \tilde{\theta}_n(\tilde{\eta}). \tag{5c}$$

Equations (1) reduce to

$$f''' + 3ff'' - 2f'^2 + 1 = 0 \quad \tilde{f}''' + \frac{1}{Pr} (3\tilde{f}\tilde{f}'' - 2\tilde{f}'^2) + 1 = 0 \tag{6a}$$

$$\theta_n'' + Pr (3f\theta_n' - 4p_n f'\theta_n) = 0 \quad \tilde{\theta}_n'' + 3\tilde{f}\tilde{\theta}_n' - 4p_n \tilde{f}'\tilde{\theta}_n = 0, \tag{6b}$$

with boundary conditions

$$f(0) = f'(0) = 0; \quad \theta_n(0) = 1; \quad \tilde{f}(0) = \tilde{f}'(0) = 0; \quad \tilde{\theta}_n(0) = 1; \tag{7a}$$

$$f''(\eta_\delta) = \theta_n(\eta_\delta) = 0; \quad \tilde{f}''(\tilde{\eta}_\delta) = \tilde{\theta}_n(\tilde{\eta}_\delta) = 0, \tag{7b}$$

where

$$\tilde{f} = P_r^{\frac{3}{4}} f, \quad \tilde{\eta} = P_r^{\frac{1}{4}} \eta, \quad \tilde{\theta}_n = \theta_n. \tag{8}$$

η_δ denotes the value of η on the condensate free surface $y = \delta(x)$ and $\tilde{\eta}_\delta$ here denotes $\frac{d}{d\tilde{\eta}}$.

$$u = \frac{4\mu G_{rx}^{\frac{1}{2}}}{\rho x} \frac{\partial f}{\partial \eta} = \frac{4\mu G_{rx}^{\frac{1}{2}}}{\rho P_r^{\frac{1}{2}} x} \frac{\partial \tilde{f}}{\partial \tilde{\eta}}, \quad v = \frac{\mu G_{rx}^{\frac{1}{4}}}{\rho x} \left[\eta \frac{\partial f}{\partial \eta} - 3f \right] = \frac{\mu G_{rx}^{\frac{1}{4}}}{\rho P_r^{\frac{3}{4}} x} \left[\tilde{\eta} \frac{\partial \tilde{f}}{\partial \tilde{\eta}} - 3\tilde{f} \right]. \tag{9}$$

Let the condensate film flow rate per unit width of the wall be

$$Q_x = \int_0^{\delta(x)} \rho u \, dy. \tag{10}$$

Then, from (5) and (9),

$$Q_x = 4\mu G_{rx}^{\frac{1}{4}} f(\eta_\delta) = 4\mu P_r^{-\frac{3}{4}} G_{rx}^{\frac{1}{4}} \tilde{f}(\tilde{\eta}_\delta). \tag{11}$$

Although Eqs. (6) and (7) may be solved quite satisfactorily for prescribed P_r and η_δ or $\tilde{\eta}_\delta$, it remains to relate the non-dimensional condensate film thicknesses to the prevailing physical conditions under which condensation occurs. Such conditions are reflected by the condensation parameter $\frac{C_p \Delta T}{h_{fg}}$ where C_p is the specific heat at constant pressure, h_{fg} is the latent heat of condensation and ΔT is the local temperature differential between T^* and the temperature at the plate. The required correlation is obtained from the overall energy balance

$$-\int_0^x \kappa \left(\frac{\partial T}{\partial y} \right)_{y=0} dx + \int_0^{\delta(x)} \rho u h_{fg} dy + \int_0^{\delta(x)} \rho u C_p (T^* - T) dy = 0, \tag{12}$$

where the first term is the heat transferred from the condensate to the plate, the second represents the latent heat of condensation and the third is due to subcooling of the condensate below saturation temperature. Negligible heat conduction across the liquid-vapour interface has been assumed.

In terms of the transformation variables, this leads to the result

$$\sum_{n=0}^{+\infty} \frac{C_p \Delta T_n}{h_{fg}} \frac{\theta'_n(\eta_\delta)}{4 p_n + 3} = -P_r f(\eta_\delta) \quad \text{or} \quad \sum_{n=0}^{+\infty} \frac{C_p \Delta T_n}{h_{fg}} \frac{\tilde{\theta}'_n(\eta_\delta)}{4 p_n + 3} = -\tilde{f}(\eta_\delta), \tag{13}$$

where $\Delta T_n = T_n x^{p_n}$, i.e., $T^* - T_w(x) = \sum_{n=0}^{+\infty} \Delta T_n$.

The quantities that appear in Eq. (13) are output data from the solutions of the appropriate equations. As pointed out by Sparrow and Gregg [5], a unique correlation, therefore, exists between the condensation parameter and η_δ or $\tilde{\eta}_\delta$.

4 Analytical Solutions

Analytical solutions, instead of numerical solutions, of the Eqs. (6) and (7) can be obtained by means of series solutions so that some useful formulas and properties of condensation heat transfer can be readily found. The asymptotic solutions of the Eqs. (6) and (7) are

$$f = \eta^2 \left[\frac{1}{2} \eta_\delta - \frac{1}{12} \eta_\delta^5 - \frac{1}{6} \eta + \frac{1}{120} \eta_\delta^2 \eta^3 + O(\eta_\delta^9) \right] \tag{14}$$

$$\theta_n = 1 - \left[\frac{1}{\eta_\delta} + \frac{P_r}{30} (3 + 8p_n) \eta_\delta^3 \right] \eta + \frac{2P_r}{3} p_n \eta_\delta \eta^3 + \frac{P_r}{8} (1 - 4p_n) \eta^4 - \frac{P_r}{40} (1 - 4p_n) \frac{\eta^5}{\eta_\delta} + O(\eta_\delta^8) \tag{15}$$

$$\tilde{f} = \tilde{\eta}^2 \left[\frac{1}{2} \tilde{\eta}_\delta - \frac{1}{12P_r} \tilde{\eta}_\delta^5 - \frac{1}{6} \tilde{\eta} + \frac{1}{120P_r} \tilde{\eta}_\delta^2 \tilde{\eta}^3 + O(\tilde{\eta}_\delta^9) \right] \tag{16}$$

$$\tilde{\theta}_n = 1 - \left[\frac{1}{\tilde{\eta}_\delta} + \frac{1}{30} (3 + 8p_n) \tilde{\eta}_\delta^3 \right] \tilde{\eta} + \frac{2}{3} p_n \tilde{\eta}_\delta \tilde{\eta}^3 + \frac{1}{8} (1 - 4p_n) \tilde{\eta}^4 - \frac{1}{40} (1 - 4p_n) \frac{\tilde{\eta}^5}{\tilde{\eta}_\delta} + O(\tilde{\eta}_\delta^8). \tag{17}$$

5 Approximate Formulas

The most important result to be given in problems of the present type is an expression for heat transfer. This can be presented most conveniently by introduction of the local Nusselt number

$$Nu_x = \frac{x}{T^* - T_w(x)} \left. \frac{\partial T}{\partial y} \right|_{y=0}, \tag{18}$$

where $T_w(x)$ is the wall temperature. The Nusselt number and the Grashof number are traditionally combined in one expression, which in our case leads to

$$\frac{Nu_x}{G_{rx}^{\frac{1}{4}}} = - \frac{\sum_{n=0}^{+\infty} \frac{C_p \Delta T_n}{h_{fg}} \theta'_n(0)}{\sum_{n=0}^{+\infty} \frac{C_p \Delta T_n}{h_{fg}}} \quad \text{or} \quad \frac{Nu_x}{G_{rx}^{\frac{1}{4}}} = - \frac{P_r^{\frac{1}{4}} \sum_{n=0}^{+\infty} \frac{C_p \Delta T_n}{h_{fg}} \tilde{\theta}'_n(0)}{\sum_{n=0}^{+\infty} \frac{C_p \Delta T_n}{h_{fg}}}. \tag{19}$$

The asymptotic approximation is

$$\frac{Nu_x}{G_{rx}^{\frac{1}{4}}} \approx \left(\sum_{n=1}^{+\infty} \frac{3}{3 + 4p_n} \frac{C_p \Delta T_n}{P_r h_{fg}} \right)^{-\frac{1}{4}}, \tag{20}$$

for arbitrary wall temperature variations when $P_r \rightarrow 0$ or $P_r \rightarrow +\infty$.

6 Power-Law Temperature Variation

For the case of the power-law temperature variation, we have $D(x) = T_0 x^a$, where $T_0 > 0$ and a are two constants, so that (13) and (19) give

$$\frac{C_p \Delta T}{h_{fg}} = -(4a + 3) P_r \frac{f(\eta_\delta)}{\theta'(\eta_\delta)} = -(4a + 3) \frac{\tilde{f}(\eta_\delta)}{\tilde{\theta}'(\eta_\delta)}, \tag{21}$$

where $\Delta T = T^* - T_w(x) = T_0 x^a$ and

$$\frac{Nu_x}{G_{rx}^{\frac{1}{4}}} = -\theta'(0) = -P_r^{\frac{1}{4}} \tilde{\theta}'(0). \tag{22}$$

Hence it can be shown that

$$N_{ux} \left[\frac{\frac{C_p \Delta T}{h_{fg}}}{\frac{g C_p \rho (\rho - \rho^*) x^3}{4 \mu \kappa}} \right]^{\frac{1}{4}} \approx \left(\frac{3 + 4a}{3} \right)^{\frac{1}{4}} \left\{ 1 - \frac{3 [9 - (27 + 52a) P_r] C_p \Delta T}{160 (3 + 4a) P_r h_{fg}} \right\}, \quad (23)$$

for low or high Prandtl numbers. It is worth to mention to this end that the corrected power-law Nusselt's formula concerning the questionable results presented in [7,8] should be

$$\frac{N_{ux}(a)}{N_{ux}(a = 0)} \approx \left(\frac{3 + 4a}{3} \right)^{\frac{1}{4}}. \quad (24)$$

Most of the existing experimental data deal with the average heat transfer coefficient. It is, therefore, necessary for the sake of comparison between theory and experiment to consider the definition of the average heat transfer coefficient for the case where the surface temperature varies as in the present case. Since $(\frac{\mu^2}{g \rho^2})^{\frac{1}{3}}$ has a unit of length, an average Nusselt number may be defined by

$$N_{um}^* = \frac{q_m}{\kappa \Delta T_m} \left(\frac{\mu^2}{g \rho^2} \right)^{\frac{1}{3}}, \quad (25)$$

and the Reynolds number may be defined by

$$R_e^* = \frac{4 q_m L}{\mu h_{fg}}, \quad (26)$$

where the mean temperature difference is

$$\Delta T_m = \frac{1}{L} \int_0^L \Delta T \, dx, \quad (27)$$

and the average heat transfer rate is

$$q_m = \frac{1}{L} \int_0^L \left(\kappa \frac{\partial T}{\partial y} \Big|_{y=0} \right) dx. \quad (28)$$

For $R_e^* < \frac{16}{3} P_r^{-\frac{3}{4}} G_{rL}^{\frac{1}{4}}$, the asymptotic approximation becomes

$$N_{um}^* \approx \frac{4}{3} \left(\frac{\rho - \rho^*}{\rho} \right)^{\frac{1}{3}} \left(\frac{3}{4} R_e^* \right)^{-\frac{1}{3}} \left[1 + \frac{81 + 264a + 208a^2 - \frac{9}{P_r} (3 + 4a)}{120 (3 + 7a)} \left(\frac{3 R_e^* P_r^{\frac{3}{4}}}{16 G_{rL}^{\frac{1}{4}}} \right)^{\frac{4}{3}} \right]. \quad (29)$$

Letting $\frac{3 R_e^* P_r^{\frac{3}{4}}}{16 G_{rL}^{\frac{1}{4}}} \ll 1$ and $\rho \gg \rho^*$, (29) becomes

$$N_{um}^* \approx \frac{4}{3} \left(\frac{3}{4} R_e^* \right)^{-\frac{1}{3}}, \quad (30)$$

which agrees with Fujii et al.'s approximate solution [9] for the case of uniform surface heat flux.

7 Isothermal Case and Comparisons of Previous Correlations and Experimental Values

Previous theoretical analyses of laminar film condensation have been mostly centered on isothermal surfaces. In the isothermal case, $D(x) = \text{constant}$ such that $p_n = 0$ for any n .

The asymptotic approximate becomes

$$\begin{aligned}
 N_{ux} \left[\frac{\frac{C_p \Delta T}{h_{fg}}}{\frac{g C_p \rho (\rho - \rho^*) x^3}{4 \mu \kappa}} \right]^{\frac{1}{4}} &= 1 + \frac{9 \left(3 - \frac{1}{Pr} \right) C_p \Delta T}{160 h_{fg}} - \frac{39355 - \frac{9650}{Pr} - \frac{7069}{Pr^2}}{1075200} \left(\frac{C_p \Delta T}{h_{fg}} \right)^2 \\
 &+ O \left[\left(\frac{C_p \Delta T}{h_{fg}} \right)^3 \right].
 \end{aligned}
 \tag{31}$$

Comparison of the approximate formula (31) with exact numerical result is presented in Table 1. These Nusselt number results for large and low Prandtl numbers are presented in Fig. 2. The approximate formula (31) deviates from the exact numerical result for large values of $\frac{C_p \Delta T}{h_{fg}}$, because the truncation error in (31) is $O\left[\left(\frac{C_p \Delta T}{h_{fg}}\right)^3\right]$.

From the coefficients of the first degree and the second degree of $\frac{C_p \Delta T}{h_{fg}}$ in (31), we know that there are two special values $Pr_1 = \frac{1}{3}$ and $Pr_2 \approx 0.56379651$. When $Pr \geq Pr_2$, the curves are monotone increasing and convex. When $Pr_2 > Pr \geq Pr_1$, the curves are monotone increasing but concave. When $Pr < Pr_1$, the curves are monotone decreasing and concave. Comparing with Nusselt's simple theory in the type problem [1,2], which predicted that

$$N_{ux} \left[\frac{\frac{C_p \Delta T}{h_{fg}}}{\frac{g C_p \rho (\rho - \rho^*) x^3}{4 \mu \kappa}} \right]^{\frac{1}{4}} = 1,
 \tag{32}$$

the result shows formula (32) is approximately satisfied only at $Pr = \frac{1}{3}$. As the condensate film thickness increases, inertia forces tend to decrease the heat transfer, while sub-cooling tends to increase the heat transfer. When $Pr > \frac{1}{3}$, sub-cooling wins out over inertia forces and the curves rise monotonically. When $Pr < \frac{1}{3}$, inertia forces win out over sub-cooling and the curves fall monotonically. When $Pr = \frac{1}{3}$, sub-cooling first matches and then wins out over inertia forces.

Table 1 Comparison of approximate formula with exact numerical result for Prandtl number = 2.58

$\frac{C_p \Delta T}{h_{fg}}$	$N_{ux} \left[\frac{\frac{C_p \Delta T}{h_{fg}}}{\frac{g C_p \rho (\rho - \rho^*) x^3}{4 \mu \kappa}} \right]^{\frac{1}{4}}$	
	Exact result	Approximate formula (31)
0.0001	1.0000	1.0000
0.0016	1.0002	1.0002
0.0081	1.0012	1.0012
0.0257	1.0038	1.0038
0.0632	1.0092	1.0092
0.1328	1.0190	1.0189
0.2511	1.0350	1.0349
0.4419	1.0595	1.0587
0.7402	1.0947	1.0912
1.1997	1.1431	1.1300
1.9047	1.2076	1.1633
2.9923	1.2908	1.1520

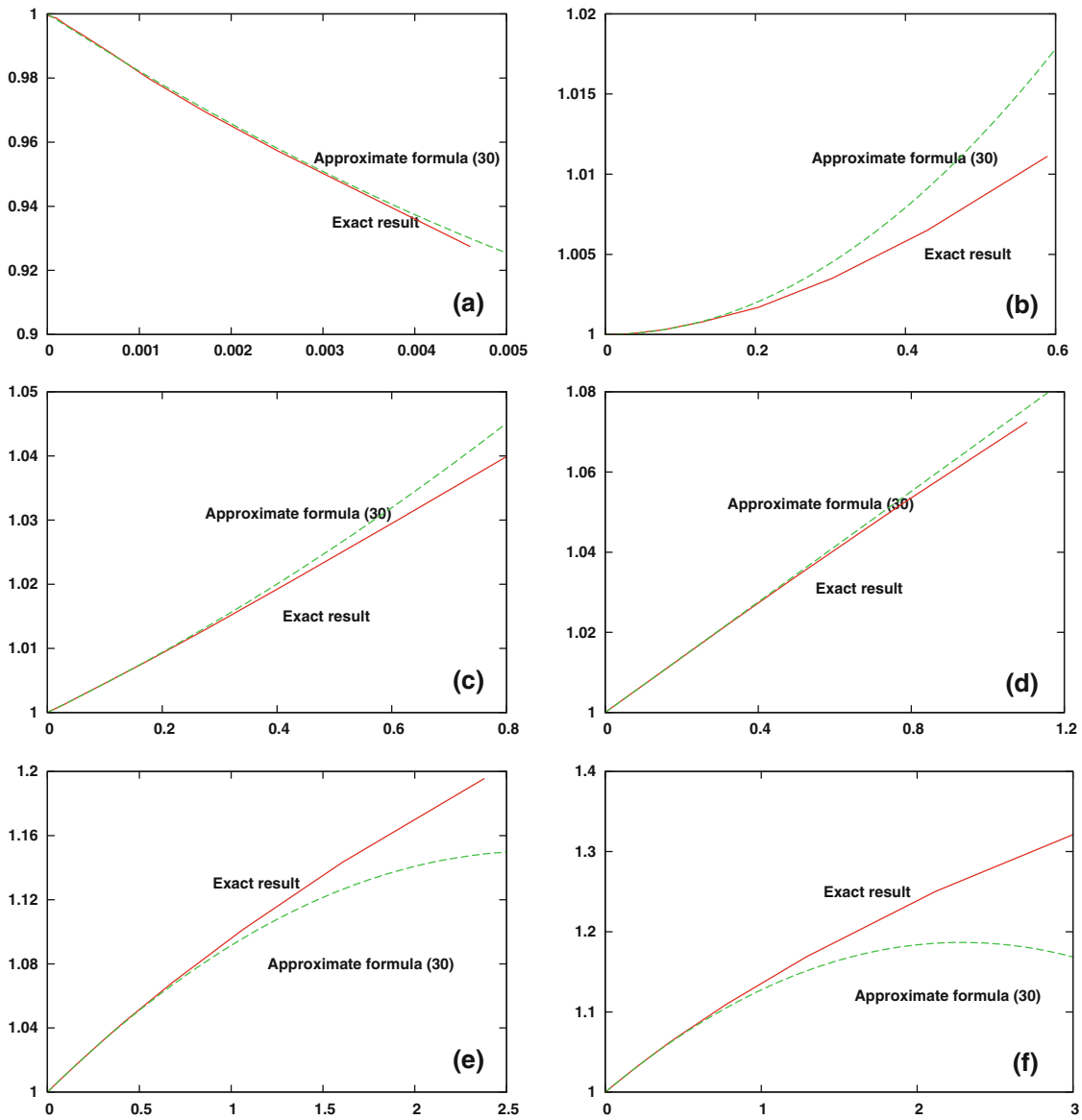


Fig. 2 Comparison of approximate formula (31) with exact numerical result for various Prandtl numbers $Pr = \mathbf{a} 0.003$ $\mathbf{b} \frac{1}{3}$, $\mathbf{c} 0.45$, $\mathbf{d} 0.56379651$, $\mathbf{e} 1$, $\mathbf{f} 10$ in terms of $N_{ux} \left[\frac{C_p \Delta T}{g C_p \rho (\rho - \rho^*) x^3} \right]^{\frac{1}{4}}$ versus $\frac{C_p \Delta T}{h_{fg}}$

Letting $Pr \rightarrow +\infty$, (31) becomes

$$\begin{aligned}
 N_{ux} \left[\frac{C_p \Delta T}{g C_p \rho (\rho - \rho^*) x^3} \right]^{\frac{1}{4}} &= \left\{ 1 + \frac{27 C_p \Delta T}{40 h_{fg}} + \frac{1643}{67200} \left(\frac{C_p \Delta T}{h_{fg}} \right)^2 + o \left[\left(\frac{C_p \Delta T}{h_{fg}} \right)^3 \right] \right\}^{\frac{1}{4}} \\
 &\approx \left[1 + 0.675 \frac{C_p \Delta T}{h_{fg}} + 0.0244449404 \left(\frac{C_p \Delta T}{h_{fg}} \right)^2 \right]^{\frac{1}{4}}. \tag{33}
 \end{aligned}$$

Table 2 Comparisons of Nusselt numbers of various observers condensing pure saturated steam on vertical flat plates or tubes

Observers	L	P_r	R_e^*	G_{rL}	$\frac{E}{E_N}$
Baker et al. [12]	20 ft	1.4823	510.65 ~ 5734.08	8.6565×10^{15}	$\frac{18.10}{18.38}$
Meisenburg et al. [13]	12 ft	1.5072	216.80 ~ 4730.85	1.8108×10^{15}	$\frac{26.21}{26.60}$
Hebbard and Badger [14]	11.969 ft	1.5274	632.61 ~ 3557.93	1.7518×10^{15}	$\frac{31.93}{32.26}$
Garrett and Wighton [15]	12.5 in	2.1170	247.74 ~ 327.50	6.4192×10^{11}	$\frac{12.40}{13.20}$
Mills and Seban [16]	5 in	6.2625	7.18 ~ 8.02	6.0926×10^9	$\frac{-17.95}{-17.81}$

If Rohsenow’s correlation [4] is now referred, it is apparent that

$$N_{ux} \left[\frac{\frac{C_p \Delta T}{h_{fg}}}{\frac{g C_p \rho (\rho - \rho^*) x^3}{4 \mu \kappa}} \right]^{\frac{1}{4}} = \left(1 + 0.68 \frac{C_p \Delta T}{h_{fg}} \right)^{\frac{1}{4}}, \tag{34}$$

which is nearly identical to this paper’s result, when the inertia forces are neglected, that is, the Prandtl number approaches infinity and only the first approximation is considered. It is quite evident that the correlation (31) is more accurate than previous correlations, especially at finite and low Prandtl numbers.

For low or high Prandtl numbers and $R_e^* < \frac{16}{3} P_r^{-\frac{3}{4}} G_{rL}^{\frac{1}{4}}$, (29) is simplified as

$$N_{um}^* \approx \frac{4}{3} \left(\frac{\rho - \rho^*}{\rho} \right)^{\frac{1}{3}} \left(\frac{3}{4} R_e^* \right)^{-\frac{1}{3}} \left[1 + \frac{3 \left(3 - \frac{1}{P_r} \right)}{40} \left(\frac{3 R_e^* P_r^{\frac{3}{4}}}{16 G_{rL}^{\frac{1}{4}}} \right)^{\frac{4}{3}} \right]. \tag{35}$$

Letting $\frac{3 R_e^* P_r^{\frac{3}{4}}}{16 G_{rL}^{\frac{1}{4}}} \ll 1$ and $\rho \gg \rho^*$, (35) becomes

$$N_{um}^* = \frac{4}{3} \left(\frac{3}{4} R_e^* \right)^{-\frac{1}{3}}, \tag{36}$$

which is the same as Nusselt formula [1,2] for the case of uniform surface temperature. In the Table 2, L expresses an effective length of flat plate or tube. E and E_N express, respectively, Nusselt number relative errors of formula (35) and Nusselt formula (36) against experiment. The property values for water and steam are taken from Kaye and Laby [10] and Weast [11]. Table 2 shows that the present formula (35) appears to predict the experimental results better than Nusselt’s formula (36) except Mills and Seban’s case, where their tube was too short. Unfortunately, the above-measured experimental data are confined to $\frac{3 R_e^* P_r^{\frac{3}{4}}}{16 G_{rL}^{\frac{1}{4}}} < 0.2$. The improvement of the present formula (35) is not notable.

8 Exponential-Law Temperature Variation

For the case of exponential-law temperature variation, we have

$$D(x) = T_0 e^{bx} = T_0 \sum_{n=0}^{+\infty} \frac{b^n}{n!} x^n,$$

where $T_0 > 0$ and b are two constants. The following results can be obtained.

$$N_{ux} \left[\frac{\frac{C_p \Delta T}{h_{fg}}}{\frac{g C_p \rho (\rho - \rho^*) x^3}{4 \mu \kappa}} \right]^{\frac{1}{4}} \approx K^{-\frac{1}{4}} \left\{ 1 - \frac{1}{480} \{ 27K - [21 + 4K(15 + 32bx)] P_r \} \frac{C_p \Delta T}{P_r h_{fg}} \right\}, \tag{37}$$

where $K = M(1; \frac{7}{4}; -bx)$. $M(\alpha; \beta; x)$ denotes the Kummer function and is usually defined as [17]

$$M(\alpha; \beta; x) = \sum_{n=0}^{+\infty} \frac{(\alpha)_n}{n!(\beta)_n} x^n,$$

and $(x)_n$ denotes the Pochhammer polynomial and is defined as the n -fold product [17]

$$(x)_n = x(x+1)(x+2)\dots(x+n-1) \quad n = 1, 2, \dots,$$

with $(x)_0 = 1$. It is worth to mention to this end that the corrected exponential-law Nusselt's formula concerning the questionable results presented in [7, 8] should be

$$\frac{N_{ux}(b)}{N_{ux}(b=0)} \approx \left[\frac{1}{M(1; \frac{7}{4}; -bx)} \right]^{\frac{1}{4}}. \quad (38)$$

9 Concluding Remarks

A detailed examination of laminar film condensation against a non-isothermal vertical plate has been performed. Similarity transformations for laminar film condensation on a vertical flat plate and analytical solutions for arbitrary Prandtl numbers and condensation rates have been given here. Based on the similarity solutions, the respective formulations for low and high Prandtl numbers have been followed in parallel and asymptotic formulae for the associated heat transfer characteristics have been obtained. For specific physical conditions the results contract to the results of earlier workers thus providing a basic confirmation of the validity of the solutions obtained. Unfortunately, relevant experimental information is very scarce and often results have been surmised from inappropriate physical configurations and ambient conditions. There is currently no satisfactory agreement between even first order results and experimental results. Accordingly, it is difficult to gauge the significance and value of higher order terms. Nevertheless the range of non-isothermal conditions for which heat transfer estimates may be obtained has been extended and the associated asymptotic representations presented. Results are compared with the available experimental data. As a separate, but potentially related and extremely interesting application, the various problems [18–28] are involving the film formation due to condensation on non-isothermal conditions. To resolve the long debatable issue regarding the ratio of heat transfer results in the non-isothermal (NS) case with those in isothermal (IS) case, the corrected Nusselt's formula should be $\frac{N_{uNS}}{N_{uIS}} \approx (\frac{3+4a}{3})^{\frac{1}{4}}$ for power-law wall temperature distribution $T_w(x) = T^* - T_0x^a$ and $\frac{N_{uNS}}{N_{uIS}} \approx [\frac{1}{M(1; \frac{7}{4}; -bx)}]^{\frac{1}{4}}$ with the Kummer function $M(1; \frac{7}{4}; -bx)$ for exponential-law wall temperature distribution $T_w(x) = T^* - T_0e^{bx}$, respectively.

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