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TRANSIENT MARANGONI WAVES DUE TO IMPULSIVE MOTION OF A SUBMERGED BODY

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The Oseen problem in a viscous fluid is formulated for studying the transient free-surface and Marangoni waves generated by the impulsive motion of a submerged body beneath a surface with surfactants. Wave asymptotics and wavefronts for large Reynolds numbers are obtained by employing Lighthill's two-stage scheme. The results obtained show explicitly the effects of viscosity and surfactants on Kelvin wakes.

Keywords: transient Marangoni waves, Kelvin wakes, wavefronts, surfactants

1. Introduction. This study is concerned with the transient surface waves generated by the impulsive motion of a submerged body through an incompressible viscous fluid. The surface waves are commonly formed in two different types: free-surface waves [1] and Marangoni waves [2]. Free-surface waves imply that the effect of surface tension is insignificant, while Marangoni waves appear when the effect of surface tension is significant due to the contamination of a surface-active material called surfactant.

Lord Kelvin [1] ignored the surface shear stress and developed a theory to determine the kinematics and dynamics of the steady free-surface waves generated by a moving body with constant velocity in an inviscid fluid of infinite depth. He found that the steady free-surface wave pattern consists of a series of so-called diverging waves and transverse waves. The diverging waves spread on each side of the moving body at an acute angle in relation to the body's moving direction, whereas the transverse waves move in the same direction as the moving body. The two wave systems intersect along the so-called "cusp locus" on both sides of the moving body. The angle between this line and the body's moving direction can be calculated as 19°28′. The lines constitute the outer edge of the so-called Kelvin wake. He also deduced that the diverging waves have a propagation direction of 35°16′ compared with the moving direction at a certain distance from the body's navigation route. Wehausen and Laitone [3], Chan and Chwang [4], and Shu [5] enriched the theory with the consideration of the viscous effect.

In the real ocean environment, the surface waves are complicated by the contaminant of surfactant due to the Marangoni effect [2], which is a phenomenon of liquid flowing along the surface from places with low surface tension to places with higher surface tension. The surfactant concentration varies with the motion of the surface, causing a surface-tension gradient that must be balanced by a nonzero surface shear stress. In the present paper, we adopt an analytical approach to study the transient interaction of the impulsive motion of a submerged body with a contaminated surface, and the consequence in terms of transient Marangoni waves. We are interested in the effects of viscosity and surfactant on a point force moving beneath a surface covered with a viscoelastic film of negligible thickness. The point force solution can be used to construct solutions for a submerged body of any shape.

2. Governing Equations. We consider a point force submerged in a viscous incompressible fluid that occupies initially the lower half space $z < 0$ in a Cartesian coordinate system. The point force is located at a distance z_0 below the surface of the fluid, being suddenly started from rest and made to move with uniform velocity $U^* \vec{e}_x$, where $\vec{e}_x$ denotes the unit vector along the x direction. Let us nondimensionalize the time by U^* / g, the distance by U^{*2} / g, the velocity by U^*, and the pressure by ρU^{*2},

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where g is the gravitational constant and ρ is the density of the fluid. Because only the wave profile at large distances downstream for high Reynolds numbers is investigated and the surfactant is assumed to act as a linear, viscoelastic film, the dimensionless Navier–Stokes equations describing the fluid flow, induced by an external body force with dimensionless strength $\vec{F}\mathrm{H}(t)\,\delta(\vec{x}-\vec{x}_0)$, may be linearized as

$$\nabla\cdot\vec{u}^* = 0, \qquad \frac{\partial\vec{u}^*}{\partial t}+\frac{\partial\vec{u}^*}{\partial x}=-\nabla p^* +\varepsilon\nabla^2\vec{u}^* +\vec{F}\mathrm{H}(t)\,\delta(\vec{x}-\vec{x}_0). \tag{1}$$

At negative time $t<0$, everything is at rest, $\vec{u}^* =\vec{O}$, $p^* =0$, $\xi=0$, $\zeta=0$, and $\eta=0$. On the surface, the tangential stress balances the surface-tension gradient induced by the surfactant, and the normal stress has a jump proportional to the surface tension and mean curvature. On $z=0$, we linearize these surface conditions [6] together with the kinematic boundary condition to yield

$$\varepsilon\left(\frac{\partial u^*}{\partial z}+\frac{\partial w^*}{\partial x}\right)=\left(\lambda+\kappa\frac{\partial}{\partial t}\right)\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\xi, \quad \varepsilon\left(\frac{\partial v^*}{\partial z}+\frac{\partial w^*}{\partial y}\right)=\left(\lambda+\kappa\frac{\partial}{\partial t}\right)\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\zeta,$$

$$\eta-p^* +2\varepsilon\frac{\partial w^*}{\partial z}=\sigma\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\eta, \quad \left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}\right)\xi=u^*, \quad \left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}\right)\zeta=v^*, \quad \left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}\right)\eta=w^*. \tag{2}$$

The fluid velocity and pressure vanish at infinity, $\vec{u}^* \to\vec{O}$, $p^* \to 0$, as $z\to-\infty$. Here the variables $\vec{u}^* =(u^*, v^*, w^*)^T$ and p^* represent the nondimensional perturbed velocity and perturbed pressure in the fluid; ξ,ζ, and η are three displacements of the surface along the x, y, and z directions; $\vec{x}=(x,y,z)^T$ and $\vec{x}_0=(0,0,-z_0)^T$ are the field point and source point; the dimensionless parameters ε and σ can be regarded as the reciprocal of the Reynolds number and the Weber number; λ and κ are the elasticity and viscosity of the viscoelastic film; and $\mathrm{H}(\cdot)$ and $\delta(\cdot)$ are Heaviside's step function and Dirac delta function, respectively. The solution to Eqs. (1) for an unbounded fluid is given by Shu and Chwang [7] as

$$\vec{u}_0=-\frac{\mathrm{H}(t)}{4\pi}\vec{F}\cdot\left(\mathbf{I}\nabla^2-\nabla\nabla\right)\int_0^t\frac{\mathrm{erf}\left(r^*/2\sqrt{\varepsilon\tau}\right)}{r^*}\,d\tau, \qquad p_0=\frac{\mathrm{H}(t)\,\vec{F}\cdot\vec{x}}{4\pi r^3}, \tag{3}$$

where $\vec{x}^* =\vec{x}-\tau\vec{e}_x$, $r^* =||\vec{x}^* -\vec{x}_0||$, and $r=||\vec{x}-\vec{x}_0||$.

Now let the entire solution be written as $\vec{u}^* =\vec{u}_0+\vec{u}$, $p^* =p_0+p$. To reduce the number of variables involved, we represent the motion as a potential flow plus a rotational flow. Thus, we define two new functions φ and $\vec{\omega}=(\omega_x, 0, \omega_z)^T$ by

$$\vec{u}=\nabla\phi+\nabla\times\vec{\omega} \tag{4}$$

such that

$$\nabla^2\phi=0, \qquad \frac{\partial\vec{\omega}}{\partial t}+\frac{\partial\vec{\omega}}{\partial x}=\varepsilon\nabla^2\vec{\omega}, \qquad p=-\frac{\partial\phi}{\partial t}-\frac{\partial\phi}{\partial x} \tag{5}$$

subject to the conditions $\xi=0$, $\zeta=0$, and $\eta=0$ for $t<0$. By means of the Laplace transform in t and the Fourier transforms in x and y

$$[\bar{\phi},\bar{\vec{\omega}}](s,\alpha,\beta,z)=\int_0^\infty\int_{-\infty}^\infty\int_{-\infty}^\infty[\phi,\vec{\omega}](t,x,y,z)\exp\left\{-st-i\alpha x-i\beta y-[A,B](z+z_0)\right\}dtdxdy, \tag{6}$$

$$[\bar{\eta},\bar{\xi},\bar{\zeta}](s,\alpha,\beta)=\int_0^\infty\int_{-\infty}^\infty\int_{-\infty}^\infty[\eta,\xi,\zeta](t,x,y)\exp\left(-st-i\alpha x-i\beta y\right)dtdxdy, \tag{7}$$

it then follows from Eqs. (5) that A and B must satisfy

$$A=\sqrt{\alpha^2+\beta^2}, \quad B=\sqrt{\alpha^2+\beta^2+\frac{s+i\alpha}{\varepsilon}}. \tag{8}$$

We can express the surface condition (2) in terms of $\bar{\phi}, \bar{\omega}_x, \bar{\omega}_z, \bar{\eta}, \bar{\xi}$, and $\bar{\zeta}$ as

$$\mathbf{C}\vec{V}=\vec{C}^{\{A\}}\exp\left[-A\,(z+z_0)\right]+\vec{C}^{\{B\}}\exp\left[-B\,(z+z_0)\right], \tag{9}$$

where the superscripts "$\{A\}$" and "$\{B\}$" denote contributions by the potential flow and the rotational flow, respectively, $\vec{V}=(\bar{\phi}, \bar{\omega}_x, \bar{\omega}_z, \bar{\eta}, \bar{\xi}, \bar{\zeta})^T$,

$$\mathbf{C}=\begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{21} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{31} & C_{32} & 0 & 0 & 0 & 0 \\ A & -i\beta & 0 & -(s+i\alpha) & 0 & 0 \\ i\alpha & 0 & i\beta & 0 & -(s+i\alpha) & 0 \\ i\beta\sqrt{\varepsilon} & B\sqrt{\varepsilon} & -i\alpha\sqrt{\varepsilon} & 0 & 0 & -(s+i\alpha)\sqrt{\varepsilon} \end{bmatrix} \tag{10}$$

and

$$\vec{C}^{\{A\}}=\frac{\alpha F_x+iF_z}{2s(s+i\alpha)A}\begin{bmatrix} \alpha A\left[A\,(\lambda+s\kappa)-2(s+i\alpha)\,\varepsilon\right]\sqrt{\varepsilon} \\ \beta A\left[A\,(\lambda+s\kappa)-2(s+i\alpha)\,\varepsilon\right]\sqrt{\varepsilon} \\ i\left[A-(s+i\alpha)^2+A^3\sigma-2A^2\,(s+i\alpha)\,\varepsilon\right] \\ 0 \\ 0 \\ 0 \end{bmatrix}, \tag{11}$$

$$\vec{C}^{\{B\}}=\frac{F_x}{2s\,(s+i\alpha)\,B\sqrt{\varepsilon}}\begin{bmatrix} -A^2(s+i\alpha)(\lambda+s\kappa) \\ 0 \\ -i\alpha\left(1+A^2\sigma\right)B\sqrt{\varepsilon} \\ 0 \\ 0 \\ 0 \end{bmatrix}+O(\sqrt{\varepsilon}), \tag{12}$$

where

$$C_{11}=i\alpha A\left[A\,(\lambda+s\kappa)+2\,(s+i\alpha)\,\varepsilon\right]\sqrt{\varepsilon}, \quad C_{12}=\alpha\beta\,(s+i\alpha)\,\varepsilon^{3/2}, \tag{13}$$

$$C_{13}=i\beta\left[A^2\,(\lambda+s\kappa)+B\,(s+i\alpha)\,\varepsilon\right]\sqrt{\varepsilon}, \quad C_{21}=i\beta A\,(s+i\alpha)\left[A\,(\lambda+s\kappa)+2\,(s+i\alpha)\,\varepsilon\right]\sqrt{\varepsilon}, \tag{14}$$

$$C_{22}=\left[A^2B\,(\lambda+s\kappa)+\left(\beta^2+B^2\right)(s+i\alpha)\,\varepsilon\right]\sqrt{\varepsilon}, \quad C_{23}=i\alpha\left[A^2(\lambda+s\kappa)+B\,(s+i\alpha)\,\varepsilon\right]\sqrt{\varepsilon}, \tag{15}$$

$$C_{31}=(s+i\alpha)^2+A\left[1+A^2\sigma+2A\,(s+i\alpha)\,\varepsilon\right], \quad C_{32}=-i\beta\left[1+A^2\sigma+2B\,(s+i\alpha)\,\varepsilon\right]. \tag{16}$$

The solution for the Laplace–Fourier transform of the vertical surface displacement η may be expressed as

$$\bar{\eta}=-\frac{i(\alpha F_x+iAF_z)\left\{(s+i\alpha)^2\left[(s+i\alpha)^2-A\left(1+A^2\sigma\right)\right]+A^4\left(1+A^2\sigma\right)(\lambda+s\kappa)\,\varepsilon\right\}\exp\left(-Az_0\right)}{2sA\Delta}+O\left(\varepsilon^{3/2}\right), \tag{17}$$

where

$$\Delta\left(s,\alpha,\beta\right)=(s+i\alpha)^2\left[(s+i\alpha)^2+A\left(1+A^2\sigma\right)\right]+A^2\left[4\,(s+i\alpha)^3-A^2\left(1+A^2\sigma\right)(\lambda+s\kappa)\right]\varepsilon. \tag{18}$$

3. Wave Asymptotics. The problem posed here is that of finding the far-field asymptotic behavior of surface waves, induced by impulsive motion of a submerged point force $\vec{F}=(F_x,0,F_z)$, where F_x and F_z are the dimensionless drag and lift forces, respectively. We begin with introducing the cylindrical coordinates (R,θ) on the surface through

$$x=R\cos\theta, \quad y=R\sin\theta. \tag{19}$$

To obtain the leading terms in the far-field asymptotic representation for small ε (large Reynolds number) and small s (large time), we will employ Lighthill's two-stage scheme [8], which in essence involves calculating the α-integration by residues [9] and the R-integration by the method of steepest descent [10].

For the first stage of Lighthill's scheme, we consider the roots of the pole equation

$$\Delta(s,\alpha,\beta)=(s+i\alpha)^2\left[(s+i\alpha)^2+A\left(1+A^2\sigma\right)\right]+A^2\left[4(s+i\alpha)^3-A^2\left(1+A^2\sigma\right)(\lambda+s\kappa)\right]\varepsilon. \tag{20}$$

For small ε and s, the roots $(\alpha^{(j)}, j=1,2)$ take the form

$$\alpha^{(j)}=(-1)^{j-1}A_2-\frac{2iA_1^2 s}{3A_2^2-2A_1(A_1+1)}-\frac{A_1^4\left[4iA_2+(-1)^{j-1}A_1\lambda\right]\varepsilon}{A_2\left[3A_2^2-2A_1(A_1+1)\right]}+O\left(\varepsilon^{3/2}+\varepsilon s+s^2\right), \tag{21}$$

where $A_2=\sqrt{A_1(1+A_1^2\sigma)}$ and A_1 satisfies the cubic equation $\sigma A_1^3-A_1^2+A_1+\beta^2=0$; i.e.,

$$A_1=\begin{cases}\dfrac{1}{3}\left[1+2\sqrt{1-3\sigma}\cos\left(\dfrac{\pi+\phi}{3}\right)\right], \cos\phi=\dfrac{27\beta^2\sigma^2+9\sigma-2}{2(1-3\sigma)^{3/2}},\ 0\le\phi\le\pi, & \text{if } \sigma\le\dfrac{1}{4},\\ \text{no real positive root} & \text{if } \sigma>\dfrac{1}{4}.\end{cases} \tag{22}$$

Using the residue theorem for a meromorphic function with respect to α, we can write the leading terms contributing significantly to the asymptotic expressions about $\varepsilon=0$ and $s=0$ of the surface elevation as

$$\int_0^\infty \eta\exp(-st)\,dt=-\frac{1}{2\pi s}\sum_{j=1}^{2}\int_{-\infty}^{\infty}\frac{\left[A_2F_x+(-1)^{j-1}iA_1F_z\right]A_1A_2}{3A_2^2-2A_1(A_1+1)}\exp\left(-A_1z_0+iRh_j\right)\left[1+O(\varepsilon+s)\right]d\beta, \tag{23}$$

where $h_j(\beta|\alpha^{(j)})=\alpha^{(j)}\cos\theta+\beta\sin\theta$.

For the second stage of Lighthill's scheme, we consider the saddle points that satisfy the derivative of the exponent of the Fourier kernel,

$$\frac{\partial}{\partial\beta}h_j(\beta|\alpha^{(j)})=\frac{\partial\alpha^{(j)}}{\partial\beta}\cos\theta+\sin\theta=0. \tag{24}$$

For small ε and s, the roots $\beta_\pm^{(j)}$ take the form

$$\beta_\pm^{(j)}=(-1)^{j-1}\frac{3A_4^2-2\left(A_3+1\right)A_3}{3A_4^2-2A_3}\left\{1+\frac{(-1)^{j-1}4i\left(3A_4^2-4A_3\right)A_4^3 s}{3A_4^6-6A_3^2A_4^4+14A_3A_4^4-28A_3^2A_4^2+8A_3^4+8A_3^3}\right.$$
$$\left.+\frac{\left[(-1)^j 8i\left(3A_4^2-4A_3^2\right)A_4^3-\left(3A_4^4-6A_3^2A_4^2+8A_3A_4^2-4A_3^3-4A_3^2\right)A_3\lambda\right]A_3^2\varepsilon}{3A_4^6-6A_3^2A_4^4+14A_3A_4^4-28A_3^2A_4^2+8A_3^4+8A_3^3}\right\}A_4\tan\theta+O\left(\varepsilon^{3/2}+\varepsilon s+s^2\right), \tag{25}$$

where $A_4=\sqrt{A_3(1+A_3^2\sigma)}$ and A_3 satisfies the sextic equation

$$(1+A_3^2\sigma)\left(3A_3^2\sigma-2A_3+1\right)^2\tan^2\theta+\left(A_3^2\sigma-A_3+1\right)\left(3A_3^2\sigma+1\right)^2=0. \tag{26}$$

Equation (26) cannot be solved by rational operations and root extraction on coefficients. For small σ, the root A_3 takes the form

$$A_3 = M_\pm \left[1 + \frac{(5G_\pm - 6)\, M_\pm^2 \sigma}{G_\pm - 2} + O(\sigma^2) \right] \tag{27}$$

with the $G_\pm$ and $M_\pm$ written as

$$M_\pm = \frac{G_\pm + 1}{2}, \qquad G_\pm = \frac{1 \pm \sqrt{1 - 8\tan^2\theta}}{4\tan^2\theta}. \tag{28}$$

Hence

$$A_1 = M_\pm + (-1)^j \frac{4i\left(G_\pm - 1\right) M_\pm^{1/2} s}{G_\pm \left(G_\pm - 2\right)} + \frac{\left(G_\pm - 1\right) M_\pm^{1/2} \left[(-1)^{j-1} 8i\left(2G_\pm - 1\right) M_\pm^{3/2} + \left(5G_\pm - 2\right) M_\pm^2 \lambda\right] \varepsilon}{G_\pm \left(G_\pm - 2\right)}$$

$$+ \frac{\left(5G_\pm - 6\right) M_\pm^3 \sigma}{G_\pm - 2} + O\left(\varepsilon^{3/2} + \varepsilon\sigma + \sigma^2 + \varepsilon s + \sigma s + s^2\right), \tag{29}$$

$$A_2 = M_\pm^{1/2} + (-1)^j \frac{2i\left(G_\pm - 1\right) s}{G_\pm \left(G_\pm - 2\right)} + \frac{\left(G_\pm - 1\right) M_\pm^{1/2} \left[(-1)^{j-1} 8i\left(2G_\pm - 1\right) M_\pm^{3/2} + \left(5G_\pm - 2\right) M_\pm^2 \lambda\right] \varepsilon}{2G_\pm \left(G_\pm - 2\right)}$$

$$+ \frac{\left(3G_\pm - 4\right) M_\pm^{5/2} \sigma}{G_\pm - 2} + O\left(\varepsilon^{3/2} + \varepsilon\sigma + \sigma^2 + \varepsilon s + \sigma s + s^2\right), \tag{30}$$

$$A_4 = M_\pm^{1/2} \left[1 + \frac{\left(3G_\pm - 4\right) M_\pm^2 \sigma}{G_\pm - 2} + O(\sigma^2) \right], \tag{31}$$

$$\alpha_\pm^{(j)} = (-1)^{j-1} M_\pm^{1/2} \left\{ 1 + (-1)^{j-1} \frac{i\left(G_\pm - 3\right) s}{\left(G_\pm - 2\right) M_\pm^{1/2}} + \frac{\left[(-1)^{j-1} 2i\left(5G_\pm - 7\right) M_\pm^{3/2} + \left(3G_\pm - 4\right) M_\pm^2 \lambda\right] \varepsilon}{G_\pm - 2} \right.$$

$$\left. + \frac{\left(3G_\pm - 4\right) M_\pm^2 \sigma}{G_\pm - 2} \right\} + O\left(\varepsilon^{3/2} + \varepsilon\sigma + \sigma^2 + \varepsilon s + \sigma s + s^2\right), \tag{32}$$

$$\beta_\pm^{(j)} = (-1)^j G_\pm M_\pm^{1/2} \left\{ 1 + (-1)^j \frac{4is}{\left(G_\pm - 2\right) M_\pm^{1/2}} + \frac{\left[(-1)^{j-1} 8i\left(2G_\pm - 1\right) M_\pm^{3/2} + \left(5G_\pm - 2\right) M_\pm^2 \lambda\right] \varepsilon}{G_\pm - 2} \right.$$

$$\left. + \frac{\left(5G_\pm - 2\right) M_\pm^2 \sigma}{G_\pm - 2} \right\} \tan\theta + O\left(\varepsilon^{3/2} + \varepsilon\sigma + \sigma^2 + \varepsilon s + \sigma s + s^2\right). \tag{33}$$

Using the method of steepest descent, we obtain the leading term

$$\int_0^\infty \eta \exp(-st)\, dt = \frac{1}{s} \sqrt{\frac{1}{2\pi R \cos\theta}} \left(1 - 8\tan^2\theta\right)^{-1/4} \sum_{j=1}^{2} \sum_{\pm} M_\pm^{-1/4} \left[M_\pm F_x + (-1)^{j-1} i M_\pm^{3/2} F_z \right]$$

$$\times \left[\exp\left(-M_\pm z_0 + (-1)^{j-1} iR \frac{M_\pm^{3/2}}{G_\pm} \left\{ 1 + (-1)^{j-1} \left[4i + (-1)^{j-1} M_\pm^{1/2} \lambda \right] M_\pm^{3/2} \varepsilon + M_\pm^2 \sigma \right\} \cos\theta \pm (-1)^j i \frac{\pi}{4} \right) + O\left(\varepsilon + \sigma + s + \frac{1}{R} \right) \right]$$

$$\times \exp\left(-\frac{2RM_{\pm}\cos\theta}{G_{\pm}}s\right). \tag{34}$$

Upon substitution and other mathematical manipulations, the surface elevation can formally be expressed as

$$\int_0^{\infty} \eta \exp(-st)dt = \frac{1}{s}\sqrt{\frac{1}{2\pi R\cos\theta}}\left(1-8\tan^2\theta\right)^{-1/4}$$

$$\times \sum_{\pm}\left[M_{\pm}^{3/4}P_{\pm}\left(F_x\cos\gamma_{\pm} - M_{\pm}^{1/2}F_z\sin\gamma_{\pm}\right) + O\left(\varepsilon+\sigma+s+\frac{1}{R}\right)\right]\exp\left(-\frac{2RM_{\pm}\cos\theta}{G_{\pm}}s\right), \tag{35}$$

where

$$P_{\pm} = \exp\left(-\frac{M_{\pm}\left(z_0 G_{\pm} + 4RM_{\pm}^2\varepsilon\cos\theta\right)}{G_{\pm}}\right), \quad \gamma_{\pm} = \frac{R\left[1+M_{\pm}^2(\sigma+\lambda\varepsilon)\right]M_{\pm}^{3/2}\cos\theta}{G_{\pm}} \mp \frac{\pi}{4}. \tag{36}$$

Using the inverse Laplace transform in s, we write the exact integral expression for the surface elevation as

$$\eta = \sqrt{\frac{2}{\pi R\cos\theta}}\left(1-8\tan^2\theta\right)^{-1/4}\sum_{\pm} M_{\pm}^{3/4}P_{\pm}\left(F_x\cos\gamma_{\pm} - M_{\pm}^{1/2}F_z\sin\gamma_{\pm}\right)\mathrm{H}\left(t-\frac{2RM_{\pm}\cos\theta}{G_{\pm}}\right) + O\left(\varepsilon+\sigma+\frac{1}{R}\right). \tag{37}$$

4. Conclusions. The asymptotic expansion has been performed for the transient free-surface and Marangoni waves due to the impulsive motion of a submerged point force. The new asymptotic expressions of surface elevations and wavefronts have been obtained, including the effects of viscosity and surfactants. It has been found that the presence of surfactants, such as viscoelastic surface films, changes the free-surface boundary conditions in the tangential direction and thus strongly modifies the flow pattern. As a consequence, wave energy is dissipated by the enhanced viscous damping in the short-gravity-wave region. Thus, viscosity reduces the surface wave amplitude, while the surfactants change the phase of the wave.

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