3. Both of these two models can simulate the anisotropic two-phase turbulence, but underpredict the two-phase fluctuation velocities, and the difference between the two model predictions is not obvious.

4. The nonlinear k-ε-k_p model has no problem of convergence encountered in the implicit algebraic stress model.

5. In two-dimensional flows with small geometrical sizes the NKP model can save about 50% computation time than the USM model. However, in three-dimensional flows with large geometrical sizes the NKP model can save much more computation time.

Acknowledgment

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A Finite Element Model and Electronic Analogue of Pipeline Pressure Transients With Frequency-Dependent Friction

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A finite element model and its equivalent electronic analogue circuit has been developed for fluid transients in hydraulic transmission lines with laminar frequency-dependent friction. Basic equations are approximated to be a set of ordinary differential equations that can be represented in state-space form. The accuracy of the model is demonstrated by comparison with the method of characteristics. [DOI: 10.1115/1.1522415]

1 Introduction

If external perturbations are superimposed onto a steady system, the equilibrium no longer exists. These perturbations will propagate in the form of waves. Some waves chase each other to form weakly discontinuous rarefaction waves and strongly discontinuous shock waves. In hydraulic systems, transient flow usually occurs when there is either a retardation of the flow due to closure of a pump valve or an acceleration due to the opening of the valve. This may cause damage to hydraulic components, reduce volumetric efficiency, and hence disturb normal operations by forming the rarefaction wave, commonly known as waterhammer, and the shock wave generated by cavitation.

Dynamic analysis in a time domain is one of the most important parts of computer simulation of hydraulic systems. Most transient distributed models can be described by hyperbolic partial differential equations. The method of characteristics, [1–4], has been widely and successfully used in tackling the problems of fluid transients such as waterhammer, [5,6], and cavitation, [7,8]. In the calculation procedures of the method of characteristics, due to the determination between time-step and space-step, there is some difficulty in determining the compatible time-step and boundary conditions to connect with other hydraulic lumped or distributed models, especially in the design of expert systems, large-scale power system packages and parallel processing. For example, at least 2000 computing elements are needed for a time-step with 10^{-2} millisecond along a tube 20 m in length. When variable time-steps are required, the calculation for intermediate interpolations becomes very expensive. Another disadvantage of using the method of characteristics is that some mathematical transformations are required before a numerical scheme is implemented.

In an attempt to overcome these limitations, finite element formulations were devised by Rachford and Ramsey [9], Watt et al. [10], Paygude et al. [11], and others listed in [12] without considering frequency-dependent friction. These formulations produce many numerical oscillations due to using a conventional uniformly spaced grid system with two degrees-of-freedom, pressure and flow rate. In the work that follows, the Galerkin finite element method, [13], is re-examined by including frequency-dependent friction and using one degree-of-freedom, pressure, or flow rate. The objective of this paper is to illustrate a simple and well-defined problem as an example to demonstrate how the Galerkin finite element method is applied in the space variables only, and gives rise to an initial value problem for a system of ordinary differential equations in order to decouple time-step from space-step. In addition, an equivalent electronic analogue circuit is established. There is no difficulty in applying the presented procedures to solve other waterhammer problems and even cavitation problems.

2 Basic Governing Equations

Before introducing the procedure itself, the physical problem to which it will be applied is described. Because of sudden valve closure at the start of a fluid transmission line, the flow within the transmission line is to be calculated under the assumptions of one-dimensional, unsteady and compressible flow. The independent variables of space and time are denoted x and t. The dependent variables are P, the pressure and Q, the flow rate. In the analysis of fluid transients, two basic principles of mechanics namely (a) the conservation of mass law and (b) Newton's momentum law give rise to two partial differential equations.

(a) Continuity Equation

$$\frac{1}{c_0^2}\frac{\partial P}{\partial t} + \frac{\rho}{\pi r_0^2}\frac{\partial Q}{\partial x} = 0. \tag{1}$$

(b) Momentum Equation

$$\frac{\rho}{\pi r_0^2}\frac{\partial Q}{\partial t} + \frac{\partial P}{\partial x} + F(Q) + \rho g \sin\theta_0 = 0, \tag{2}$$

Table 1 n_i and m_i

i	1	2	3	4	5	6	7	8	9	10
n_i	2.63744 $\times 10^1$	7.28033 $\times 10^1$	1.87424 $\times 10^2$	5.36626 $\times 10^2$	1.57060 $\times 10^3$	4.61813 $\times 10^3$	1.36011 $\times 10^4$	4.00825 $\times 10^4$	1.18153 $\times 10^5$	3.48316 $\times 10^5$
m_i	1.0	1.16725	2.20064	3.92861	6.78788	1.16761 $\times 10^1$	2.00612 $\times 10^1$	3.44541 $\times 10^1$	5.91642 $\times 10^1$	1.01590 $\times 10^2$

where c_0 is the acoustic velocity, ρ is the density, g is the acceleration due to gravity, r_0 is the internal radius of the transmission line tube, and θ_0 is the angle of the transmission line tube inclined with the horizontal. The friction term $F(Q)$ can be expressed as a steady friction term F_0 plus a frequency-dependent friction term, for which a model applicable to laminar flow has been developed by Zielke [14] and Kagawa et al. [15].

$$F = F_0 + \frac{1}{2}\sum_{i=1}^{k} Y_i \qquad (3)$$

$$\begin{cases} \dfrac{\partial Y_i}{\partial t} = -\dfrac{n_i \mu}{\rho r_0^2} Y_i + m_i \dfrac{\partial F_0}{\partial t} \\ Y_i(0) = 0 \end{cases} \qquad (4)$$

μ is the viscosity. The constants n_i and m_i are tabulated in Table 1 and the number of terms k to be selected should be determined according to the relationship between the break frequency of the approximated weighting function and the frequency range of the system, [15], which is assumed in steady state when time $t = 0$.

3 Galerkin Finite Element Method

Finite element formulations using the Galerkin method in time domain analysis have been presented by Rachford and Ramsey [9], Watt et al. [10], and Paygude et al. [11] without considering frequency-dependent friction by using a conventional uniformly spaced grid system with two degrees-of-freedom, pressure, and flow rate. These formulations produce many numerical oscillations. In the work that follows, the Galerkin finite element method is re-examined by including frequency-dependent friction and using one degree-of-freedom, pressure, or flow rate.

Let $U = \rho/\pi r_0^2 Q$, $F_0(Q) = 8\mu/\pi r_0^4 Q$, $R = 8\mu/\rho r_0^2$ and $H_0 = \rho g \sin\theta_0$ for the case of linear steady friction, the Eq. (1), (2), and (4) can be rearranged in terms of the operator equations
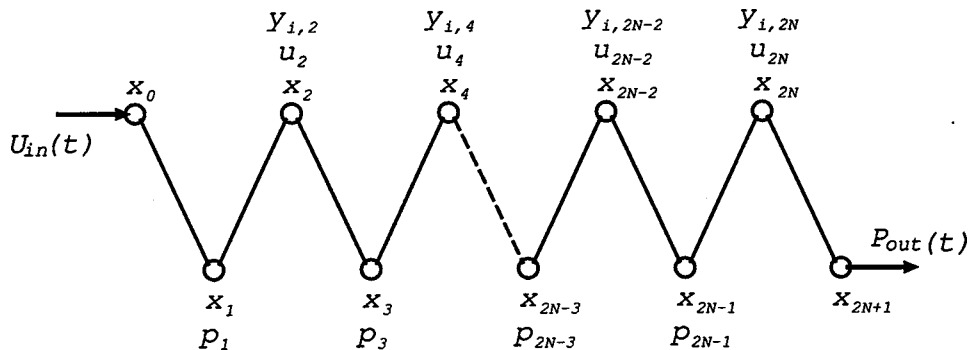
$$L_1(U,P,Y_i) \equiv \frac{1}{c_0^2}\frac{\partial P}{\partial t} + \frac{\partial U}{\partial x} = 0 \qquad (5)$$

$$L_2(U,P,Y_i) \equiv \frac{\partial U}{\partial t} + \frac{\partial P}{\partial x} + RU + \frac{1}{2}\sum_{i=1}^{k} Y_i + H_0 = 0 \qquad (6)$$

$$L_3(U,P,Y_i) \equiv \frac{\partial Y_i}{\partial t} + \frac{n_i R}{8} Y_i - m_i R \frac{\partial U}{\partial t} = 0. \qquad (7)$$

When the transmission line is divided into $2N+1$ equal elements, each Δx in length, as shown in Fig. 1 and a suitable finite dimensional space is spanned by shape functions, [13],

$$w_j^+(x) = \frac{x_{j+2}-x}{x_{j+2}-x_j} \quad w_j^-(x) = \frac{x-x_j}{x_{j-2}-x_j}. \qquad (8)$$

The Galerkin method consists in finding approximations to U, P, and Y_i of the form

$$U(x,t) = u_{2j}(t)w_{2j}^+(x) + u_{2j+2}(t)w_{2j+2}^-(x) \qquad (9)$$

$$P(x,t) = p_{2j+1}(t)w_{2j+1}^+(x) + p_{2j+3}(t)w_{2j+3}^-(x) \qquad (10)$$

$$Y_i(x,t) = y_{i,2j}(t)w_{2j}^+(x) + y_{i,2j+2}(t)w_{2j+2}^-(x) \qquad (11)$$

for $j = 0,1,\ldots,N-1$, where the unknown coefficients u, p, and y_i, which are nodal values of U, P, and Y_i, respectively, are determined by an inner product $(\bullet,\bullet)$ so that

$$(L_i, w_j^+) \equiv \int_{x_j}^{x_{j+2}} w_j^+ L_i dx = 0 \quad \text{and} \quad (L_i, w_j^-) \equiv \int_{x_j}^{x_{j-2}} w_j^- L_i dx = 0 \qquad (12)$$
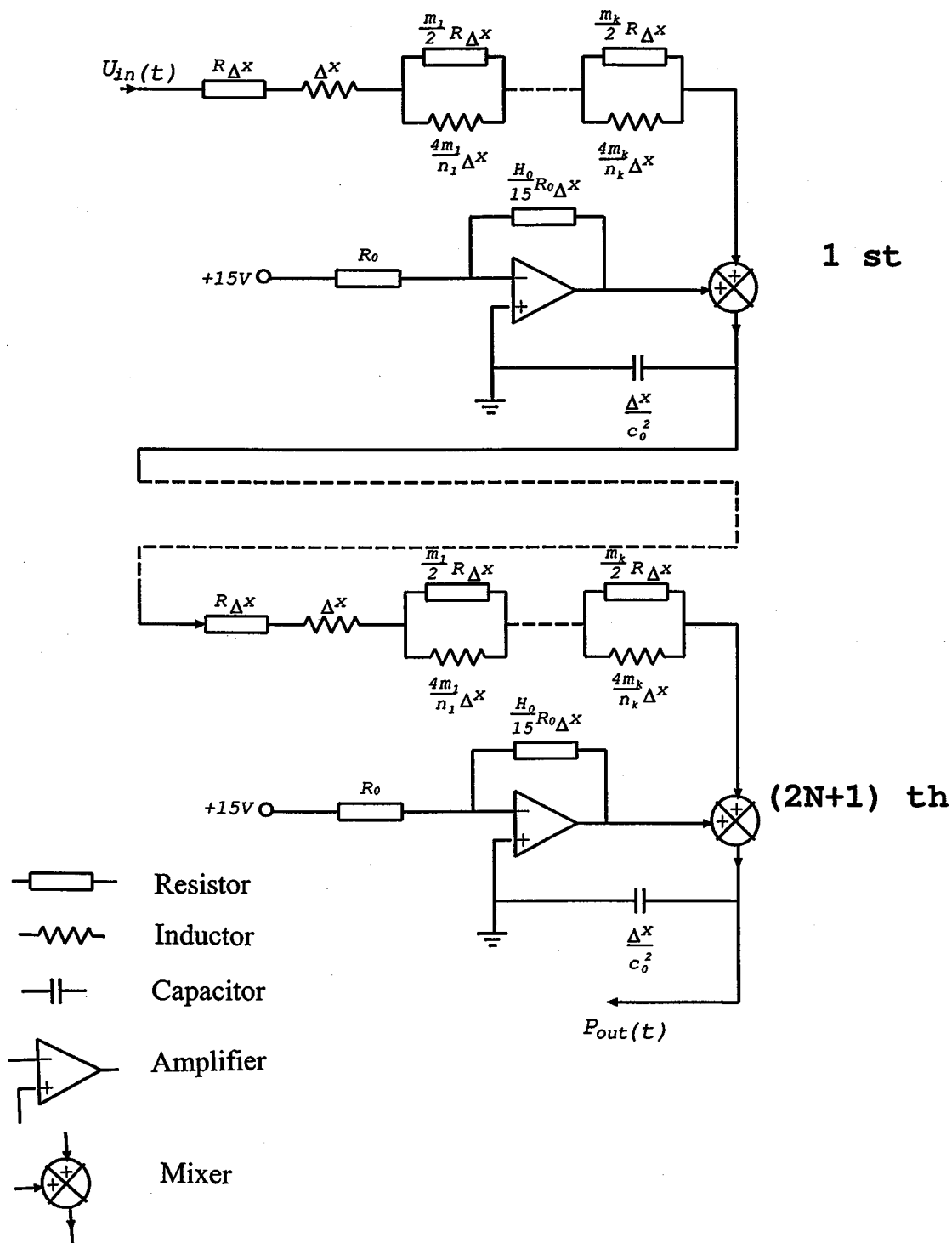

Fig. 1 Interlacing grid system

Fig. 2 Equivalent electronic analogue circuit

Table 2 Parameter list

Upstream Pressure (MPa)	Downstream Pressure (MPa)	Radius r_0 (mm)	Length L (km)	Density ρ (kg/m²)	Acoustic Velocity c_0 (m/s)	Viscosity μ (cP)
3	2	4	0.02	871	1392	50.518

when $i=1$, j is chosen as an odd number and when $i=2$ or 3, j is chosen as an even number.

Taking account of the boundary conditions that flow rate $u_0 = U_{in}(t)$ is known at this end and pressure $p_{N+1} = P_{out}(t)$ is known at another end, the resultant ordinary differential equation is of the following form:

$$
\frac{d}{dt}\begin{pmatrix} \mathbf{u} \\ \mathbf{p} \\ \mathbf{y} \end{pmatrix}
$$

$$
= \begin{pmatrix}
-R\mathbf{I} & \dfrac{1}{16\Delta x}\mathbf{A} & -\dfrac{1}{2}(\boldsymbol{\Xi}\otimes\mathbf{I})^T \\[2ex]
\dfrac{c_0^2}{16\Delta x}\mathbf{B} & \mathbf{O} & \mathbf{O} \\[2ex]
-R^2\mathbf{M}\otimes\mathbf{I} & \dfrac{R}{16\Delta x}\mathbf{M}\otimes\mathbf{A} & -\dfrac{R}{8}[\mathbf{F}+4\mathbf{M}\otimes(\boldsymbol{\Xi}\otimes\mathbf{I})^T]
\end{pmatrix}
$$

$$
\times\begin{pmatrix} \mathbf{u} \\ \mathbf{p} \\ \mathbf{y} \end{pmatrix}
+ \frac{c_0^2}{16\Delta x}U_{in}\begin{pmatrix} \mathbf{O} \\ \mathbf{C} \\ \mathbf{O} \end{pmatrix}
+ \frac{1}{16\Delta x}P_{out}\begin{pmatrix} \mathbf{D} \\ \mathbf{O} \\ R\mathbf{M}\otimes\mathbf{D} \end{pmatrix}
$$

$$
- H_0\begin{pmatrix} \mathbf{E} \\ \mathbf{O} \\ R\mathbf{M}\otimes\mathbf{E} \end{pmatrix} \tag{13}
$$

Here the abbreviations for matrix and vector elements appear in expanded form in the Appendix for clarity.

4 Electronic Analogue

The new ordinary differential equation allows us to simulate the fluid transients by constructing an equivalent electronic analogue circuit as shown in Fig. 2. The value and type for each electronic component are indicated.

For a section of transmission line of length Δx, this can be considered to offer a resistor due to the steady fluid friction, an inductor due to the fluid inertia and a capacitor due to the fluid compressibility. The frequency-dependent friction can simply be formed by a set of subcircuits, each of which is made of a resistor and a inductor hooked in parallel. The inverting operational amplifier indicates a constant pressure drop due to gravity for an inclined tube. On reading the analogue circuit, we may get the impression that, without the frequency-dependent friction, the circuit is almost identical to a π-type LC butterworth passive low-pass filter, [16,17]. With time, the wave generated by changes of boundary conditions becomes smoother and smoother due to the rejection of the high frequencies superimposed upon the wave. The parallel RL subcircuit, which is representative of frequency-dependent friction, enhances the function of the low-pass filter by means of raising passing impedance to high-frequency signals.

The great advantage in using the analogue circuit to analyses the fluid transients has been found in the following three major points:

a. A quantitative analysis for the analogue circuit is able to help us fully to understand the fluid transients by means of existing well-established electronic theory.
b. An equivalent electronic analogue circuit is able to provide a monitoring base, such as differentiating cavitation noise from system noise.
c. The low-pass filter design theory is able to modify the circuit to get more precise mathematical models, such as the frequency-dependent friction model.

5 Numerical Results

Transient responses of the finite element formulation have been compared with those of the method of characteristics to demonstrate the accuracy of the Galerkin method. Comparison with previously published data, [18], listed in Table 2, for studying pressure changes at the vicinity of an upstream valve after the sudden closure of the valve in a flowing liquid line, show good agreement between experimental results and numerical computation by means of the method of characteristics for transient flow under negative pressure, in which liquids can withstand tensile strength against vaporization.

It is to be noted at this stage that ordinary differential equations, [13], are based on the Galerkin finite element method in the space
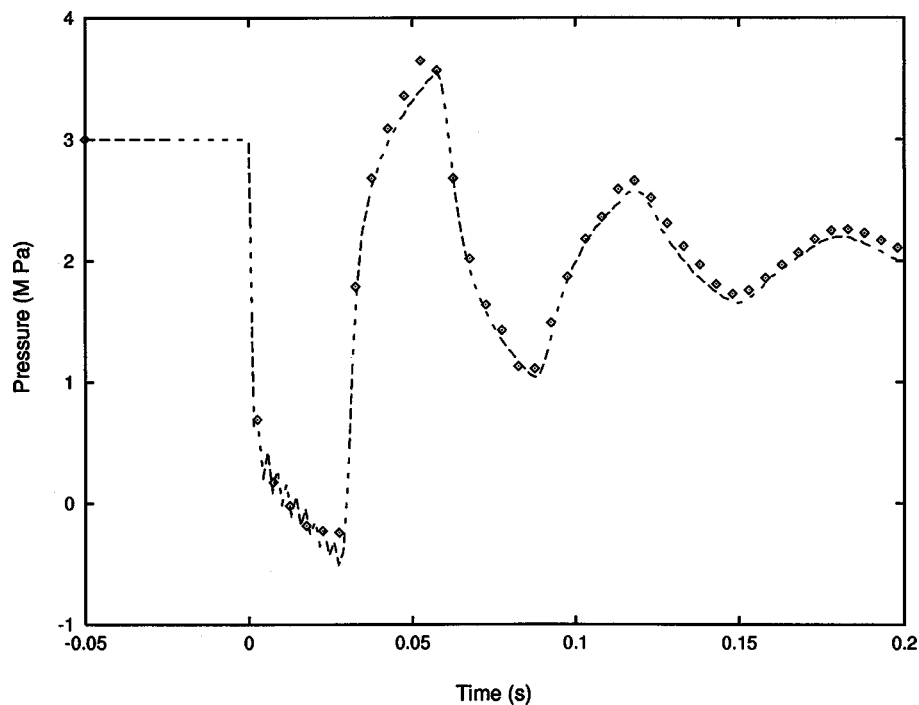

Fig. 3   Finite element method (point) versus method of characteristics (line)

domain only. If the finite element is to be adopted for the time domain also, the problem becomes unmanageable even on large computers. A compromise is to have another numerical scheme in the time domain. Although many different methods are available for solving the ordinary differential equations, [19], the Dormand and Prince fifth-order Runge-Kutta method, [20], has been adopted here.

The comparison for p_0 is usually meaningful, but p_0 is not explicitly computed for the arrangement shown in Fig. 1. The interpolation using the Galerkin method

$$\int_{x_0}^{x_1} w_0^+ L_1(U,P,Y_i)dx = 0$$

gives
$$\frac{dp_0}{dt} = -\frac{1}{2}\frac{dp_1}{dt} + \frac{3c_0^2}{4\Delta x}(U_{in} - u_2). \tag{14}$$

The result for p_0 is plotted and compared as shown in Fig. 3 with the time-step$=5\times 10^{-3}$ seconds.

6 Conclusions

A finite element model with laminar frequency-dependent friction has been developed. The finite element model retains the accuracy of the method of characteristics. Based on the finite element model, an electronic analogue methodology has been presented for the problem of fluid transients. The use of the equivalent electronic analogue circuit provides an alternative way to understand the mechanism of transient fluid flow and improve the frequency-dependent friction model mathematically.

Acknowledgments

Appendix

Details of Abbreviations Used in the Ordinary Differential Eq. (13).

$$\mathbf{u} = \{u_2, u_4, \cdots, u_{2N}\}^T, \quad \mathbf{p} = \{p_1, p_3, \cdots, p_{2N-1}\}^T,$$

$$\mathbf{y} = \{y_{1,2}, y_{1,4}, \cdot, y_{k,2N}, y_{2,2}, \cdots, y_{k,2N}\}^T,$$

$$\mathbf{N} = \{n_1, n_2, \cdots, n_k\}^T, \quad \mathbf{M} = \{m_1, m_2, \cdots, m_k\}^T, \Xi = \{1,1,\cdots,1\}^T$$

$$\mathbf{A} = \begin{pmatrix}
10 & -12 & 2 \\
-1 & 11 & -11 & 1 \\
 & -1 & 11 & -11 & 1 \\
 & & \cdot & \cdot & \cdot & \cdot \\
 & & & \cdot & \cdot & \cdot & \cdot \\
 & & & & \cdot & \cdot & \cdot & \cdot \\
 & & & & & -1 & 11 & -11 & 1 \\
 & & & & & & -1 & 11 & -11 \\
 & & & & & & & -2 & 12
\end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix}
-12 & 2 \\
11 & -11 & 1 \\
-1 & 11 & -11 & 1 \\
 & \cdot & \cdot & \cdot & \cdot \\
 & & \cdot & \cdot & \cdot & \cdot \\
 & & & \cdot & \cdot & \cdot \\
 & & & -1 & 11 & -11 & 1 \\
 & & & & -1 & 11 & -11 & 1 \\
 & & & & & -2 & 12 & -10
\end{pmatrix}$$

$$\mathbf{C} = \{10, -1, 0, \cdot, \cdot, \cdot, 0, 0, 0\}^T \quad \mathbf{D} = \{0, 0, 0, \cdot, \cdot, \cdot, 0, 1, -10\}^T$$

$$\mathbf{E} = \{1, 1, 1, \cdot, \cdot, \cdot, 1, 1, 1\}^T$$

$$\mathbf{F} = \begin{pmatrix}
n_1\mathbf{I} \\
 & n_2\mathbf{I} \\
 & & \cdot \\
 & & & \cdot \\
 & & & & \cdot \\
 & & & & & n_k\mathbf{I}
\end{pmatrix}.$$

The symbol $\otimes$ defines the Kronecker product of a vector $\mathbf{Z} = \{z_1, z_2, \cdots, z_k\}^T$ and a matrix $\mathbf{G}$, that is, $\mathbf{Z}\otimes\mathbf{G} \equiv \{z_1\mathbf{G}, z_2\mathbf{G}, \cdots, z_k\mathbf{G}\}^T$.

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Prediction of Centrifugal Slurry Pump Head Reduction: An Artificial Neural Networks Approach

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The feasibility of using artificial neural networks (ANN) in the prediction of head reduction of centrifugal pumps handling slurries is examined. An ANN model is proposed and compared with the empirical correlation given by the present authors earlier. The comparison showed that the ANN could successfully be used for the prediction of head reductions of centrifugal slurry pumps. The mean deviation between predicted and experimental values is 5.86% which is reasonable for slurry handling processes.
[DOI: 10.1115/1.1523062]

Introduction

It is well established that the performance of centrifugal pumps with slurries gets reduced in the presence of solids in the carrier liquid. The magnitude of the reduction is a function of concentration of solids in the mixture, physical properties of solids like their specific gravity, size and size distribution of particles, and pump size. For a given concentration of solids and volumetric flow rate at a constant shaft speed, the influence of slurry on a centrifugal pump performance is usually quantified by the head ratio (HR $=H_{\text{slurry}}/H_{\text{water}}$), and the efficiency ratio (ER$=E_{\text{slurry}}/E_{\text{water}}$). Another measure of the head reduction of a pump due to presence of solids in the carrying liquid is the head reduction factor (K $=1$-HR), which is widely used. Several correlations have been proposed in the literature to predict HR or K for a given operating condition, [1–7]. However, due to the complexity of the problem, they are usually restricted to narrow range of particular operating conditions. We seek an alternative model to estimate with acceptable accuracy the head reduction factor by using the artificial neural networks (ANN) method. This approach has received significant attention in recent years and it has been applied successfully

to a wide range of problems, [9–17], because of its flexibility and easy implementation with a high-speed computer.

The objective of this paper is to evaluate the head reduction factor (K) of a centrifugal slurry pump using ANN. A total of 206 experimental data on slurry pumps with various solids materials available in the literature, [1,2,4–6,8], were utilized. It was found that the ANN model gives satisfactory predictions for almost all cases and leads to lesser prediction errors compared to conventional prediction tools.
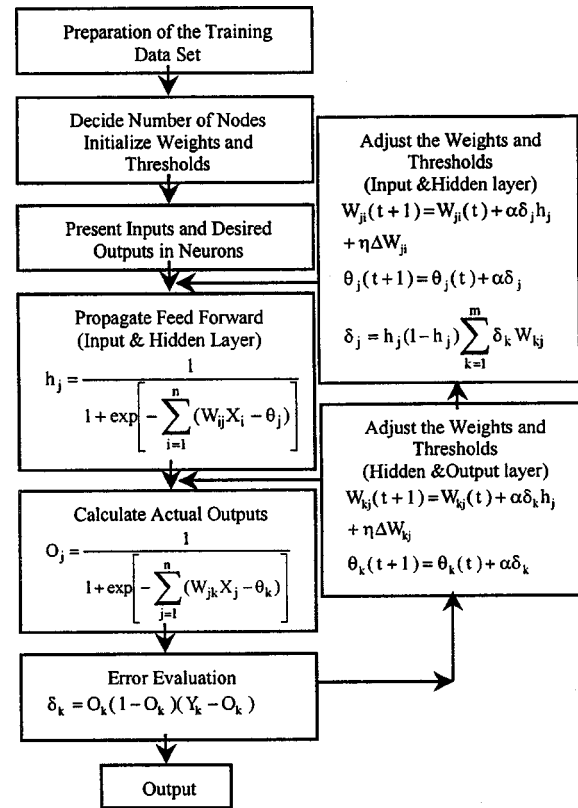
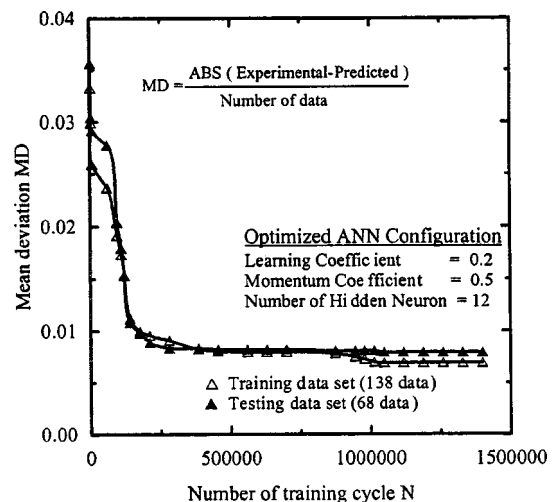Fig. 1   Flow chart for the back-propagation learning algorithm

Fig. 2   Performance of proposed neural network architecture