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This document gives a presentation of mathematical tools used for financial risk modeling and related analytics, and is divided into three parts. The first part focuses on stochastic modeling using diffusion processes (Chapter 1), time series (Chapter 2), jump processes and insurance risk (Chapter 3), and random dependence structures (Chapter 4). The second part covers classical risk measures, starting with the superhedging risk measure, which is constructed in Chapter 5 from basic financial derivatives. Value at risk (VaR) is considered in Chapter 6, followed by tail value at risk (TVaR), conditional tail expectation (CTE) and expected shortfall (ES) in Chapter 7. The third part deals with credit risk, starting with Chapter 8 which builds on the geometric Brownian motion model of Chapter 1 for the pricing of default bonds. Chapters 9 and 10 consider credit default via defaultable bonds, credit default swaps (CDS) and collateralized debt obligations (CDOs), and involve more advanced knowledge of stochastic processes. Finally, Chapter 11 considers credit scoring. The concepts presented are illustrated by 119 Python codes and 119  scripts, and accompanied with 168 figures and 82 exercises with complete solutions.