Do We Need Ramsey Taxation? Our Existing Taxes Are Largely Corrective*

Eric Yan         Qu Feng        Yew-Kwang Ng

April 1, 2020

Abstract

Due to the importance of environmental disruption (both in production and consumption), relative competition between individuals (including conspicuous consumption and keeping up with the Joneses), the diamond effect, excessive consumerism or the materialist bias, most taxes in most countries, though mainly designed for revenue collection, are largely corrective than distortive. There is thus no need for Ramsey taxation. In this paper, a theoretical model is built to make a comparison between the social optimality attained by an income tax and the individual optimality attained without an income tax. Relative competition and environmental disruption reinforce each other in causing excessive work and excessive pollution. An income tax is shown to reduce these double departures (from social optimality) of both leisure and environmental quality. The empirical test conducted on the data from the World Bank and the International Labor Organization conforms to this theoretical finding. More concretely, when the labor tax increases by 1 standard deviation from the average level, the average working time may be reduced by 1.125%. And when there is a higher profit tax than the average level by 1 standard deviation, about 6% of the cross-country average level of carbon damage in the sample may be reduced.

JEL Classification: O3, I2, E24

Keywords: Corrective taxation; environmental disruption; optimal taxation; Ramsey taxation; relative consumption.

---

*Feng: Economics Division, School of Social Sciences, Nanyang Technological University, Singapore (Email: qfeng@ntu.edu.sg). Ng: Special Chair Professor, School of Economics, Fudan University; Emeritus Professor, Monash University.
1 Introduction

Government spending requires government revenue. Though some governments rely on the sale of their assets including land, usually the most important sources of government revenues are various forms of taxation, including income and consumption taxes, import tariffs, and taxes on property transaction and ownership. Raising taxes is usually needed but it should be done efficiently. A very deep ingrained idea of economists is that taxes are distortive, e.g.

‘One of the most significant insights — obvious from current perspectives — was the recognition that all taxes induce distortions, but that the total dead weight loss of the tax system was not minimized by minimizing the number of distortions’ (Stiglitz (2002, p.341).

Most discussion of optimal commodity taxation follows the Ramsey’s (1927) tradition of trying to minimize the excess burden or distortionary costs of taxation. Though a proper analysis should take the whole economy (all goods, taken to subsume services) into account including the consumption and the production sides, the principal idea can be seen by the textbook supply-demand analysis of the excess burden of the taxation of a single good. For any given amount of tax revenue raised, the more elastic the demand and supply, the larger is the amount of distortion and the amount of excess burden. The idea of optimal taxation is to take account of this and, for any given amount of total revenue needed, design a tax-rate structure over all goods to minimize the total distortionary costs.

Though this principle is well-taken in the given framework, actually no distortionary costs of taxation need to be incurred at all for most cases in the real world. The production and consumption of most goods impose both significant environmental disruption and relative-competition (or positional-preferences) costs on others. Estimates of just the relative-competition effects alone justify a taxation rate of 33% or higher (Blanchflower and Oswald (2004); on the importance of relative competition; see also Frank (1999), Knight and Gunatilaka (2010), Dohmen et al. (2011), Garrard (2012), and others). Adding the significant effects of congestion, pollution, and environmental disruption in general probably requires corrective tax rates higher than needed for revenue raising for most countries. Ideally, we may

\footnote{By analyzing the welfare influences of relative consumption, Alvarez-Cuadrado (2007), Aronsson and Johansson-Stenman (2008), Aronsson and Johansson-Stenman (2010), Alexeev and Chih (2015), Bruce and Peng (2018) and others also suggest that the distortion arising with envy and comparison can be reduced by taxation.}
have a general income and/or consumption tax rate of 20-30% to take account of
general relative competition and moderate environmental disruption, plus higher
taxes imposed on activities/goods with more serious pollution and/or with more
serious conspicuous effects (Blanchflower and Oswald, 2004).

Moreover, due to excessive consumerism or the materialist bias (Ng, 2003), tax-
ing some goods can cause a negative excess burden (i.e. the excess burden may be
negative and hence not really a burden but an efficiency improvement). Also, goods
valued for their values rather than their intrinsic consumption effects are called di-
amond goods. A tax on a pure diamond good of $100 millions imposed not only no
excess burden of $30 millions; it imposes no burden at all, as shown in Ng (1987).
The percentage of negative burden is 100% (See Mandler (2018) for an extended
analysis). Though not many goods are pure diamond goods, there are many mixed
diamond goods (valued both for their values and their intrinsic consumption effects)
that still provides sources for revenue with negative excess burdens. As the amount
of revenue raised from corrective taxation and taxes on diamond goods is likely to
meet government spending in most cases, no additional taxes are needed purely for
raising revenue. For corrective taxes, the amounts of efficiency gains are maximized
by imposing taxes in accordance to the marginal damages (or the marginal costs
of abatement; see below) involved irrespective of the relevant demand and supply
elastics. This is the case, since corrective taxes generate efficiency gains rather
than impose distortionary costs. Ideally, a Pigovian tax on an external cost should
be in accordance to the marginal damages imposed.

However, the sizes of various environmental damages, especially for the global
warming problem that affects the whole world and for centuries to come, are very
difficult to estimate. Ng (2004) argues that, for most cases where some abatement
spending is desirable, taxes imposed should be at least equal to the marginal costs
of abatement investment. This is much easier to estimate than the marginal damages.
Moreover, the amount of revenue collected will be larger than the optimal amount of
abatement investment, thus solving also the financing problem for this investment;
see also Barrios et al. (2013) on the efficiency of green taxes and Kaplow (2008)
on more general issues of taxation and public economics. Despite the conceptual
superiority and viability of estimating the appropriate rates of corrective taxes
as discussed above, these corrective taxes have not been commonly used due to
political, administrative, and institutional factors. And institutional and political
factors, along with market conditions, lifestyle evolution, etc, may, in turn, bring
an adverse effect on the environmental quality (Bimonte and Stabile, 2017).

\footnote{For example, in Candau and Dienesch (2017), it is shown that many corrupted officers in}
In the absence (true in almost all cases) of the adequate Pigovian taxation of disruption and relative competition, a rough (even just a proportional one, ignoring questions if inequality) tax on income is still helpful. Arguing that a higher ratio of tax revenue to GDP can be desirable, Neild (2018, p.20) takes Austria, Germany, Italy, and Sweden as examples as opposed to the UK where public services and health and social care degrade simultaneously.

As discussed above, both the factor of environmental disruption and that of relative competition seems to call for a tax on income, but a formal analysis of the two effects together will be helpful to confirm this intuition. In our study, a theoretical model is built to allow for both these effects and to make a comparison between the social optimality attained by an income tax and the individual optimality attained without an income tax. It is shown that the representative individual has longer leisure time under social optimality than under individual optimality, while he consumes less under social optimality than under individual optimality. Therefore, when relative competition and environmental disruption are present, an income tax helps to reduce the double-double departures (from both relative competition and pollution) of leisure time and environmental quality from social optimality. (The first double refers to the reinforcing, rather than offsetting, effects of both relative competition and pollution; the second refers to the reduction in both leisure and environmental quality from optimality. We use ‘pollution’ and environmental disruption largely interchangeably.)

We use a numerical example to show that, due to the tax on income, environmental quality can improve with longer leisure time and less consumption, and that, in overall terms, the level of welfare increases. We also estimate the influences of income tax on carbon damage and working time empirically using World Bank’s (2018) and International Labor Organization’s (2018) data, which cover 111 countries over the period of 2005-2017 as an unbalanced panel data set. It is found that the average working time may be reduced by 1.125% when the labor tax increases by 1 standard deviation from the average level (i.e., 11.25 percentage points). And about 6% of the cross-country average level of carbon damage in the sample may be reduced when there is a higher profit tax than the average level by 1 standard deviation (i.e., 10.28 percentage points).

Lowering an individual’s disposable income, an income tax is equivalent to levying a uniform tax on all commodities (Atkinson and Stiglitz 1972) where the rev-developing economies like China execute environmental regulations in a very slack way despite stringent environmental laws. The reason why they risk to do so is that they want to extract economic rent while introducing foreign investment that may originally be banned from entering the economy due to accompanying environmental problems.
The optimal income taxation literature pioneered by Mirrlees (1971) and the Ramsey optimal commodity taxation do not allow for the important external costs of environmental disruption and relative competition, and thus focus on distortion-minimization taking account of supply-and-demand elasticities (Ramsey) and the equality-efficiency tradeoff (Mirrlees) respectively. Differently, by allowing for the important external costs of environmental disruption and relative competition, we show that taxes sufficient to meet revenue requirement may be corrective and involve no distortion and no efficiency loss at all. Because of this reason, we then do not have to use Ramsey taxation in practice. The Mirrlees optimal income taxation approach is still practically relevant as the question of inequality has not been addressed in our approach that uses the representative individual method. However, in our era of severe relative competition on the one hand, and environmental disruption in general and global warming in particular on the other hand, our result that these two effects are cumulative and may be addressed to a large extend by income taxation provides both conceptual insights and practical relevance.

Ng and Ng (2001) discuss the importance of environmental disruption and relative-income effect during economic growth. If the two effects are not offset, economic growth may reduce welfare. The welfare reduction effect may still apply despite the provision of more public goods and disruption abatement with economic growth. This implies that, to ensure improvement in social welfare with economic growth, it is important to reduce the departures due to environmental disruption and relative competition.

Heijdra et al. (2015) propose the package of Pigovian tax and environmental abatement to be the solution to the double departures. In their model, they assume the path of economic development to be non-linear. Using the approach of Shallow-Lake Dynamics, they derive multiple equilibria. Implementing the policy package may move an economy from the equilibrium with high pollution and intensive work to the equilibrium with low pollution and more leisure. We agree with the finding of Heijdra et al. (2015). However, given the previously mentioned difficulties in implementing Pigovian tax, income taxes may remain the simplest and practicable approach.

The remainder of this paper is composed of a theoretical model in Section 2, a numerical analysis in Section 3, an empirical analysis in Section 4, and a concluding section in the end of this paper. All mathematical proofs and a supplementary section are relegated to the Appendix.
2 Model

In the model, we first analyze individuals’ optimal choice of consumption and leisure. Then, we study social optimality attained under the government’s selection of income tax and public goods. Ignoring factors like interpersonal differences, technological and population changes as is normal for a simple welfare-theoretical analysis, we may adopt a representative individual approach.

2.1 Individual optimality

Assume that the utility function of this individual is a quasi-concave function and takes the following general functional form:

\[ U = U(C, X, R, E) \] (1)

where \( C \) denotes consumption, \( X \) denotes leisure, \( R \) denotes relative income, \( E \) denotes environmental quality, and \( p \) denotes wage rate. \( X \) ranges between 0 and 1 by normalizing time endowment to be 1. And we take \( R = \frac{Y}{A} \) where \( Y \) is the income of the individual and \( A \) is the income of an average person with whom the representative individual is compared.

The individual’s budget constraint is

\[ C = Y = (1 - X)p. \] (2)

The individual’s budget constraint is depicted by (2). Hereafter, unless specially mentioning its meaning, we use subscript of a variable to represent that variable’s partial derivative, i.e. \( U_C = \frac{\partial U}{\partial C} \). In (1), the marginal utilities of all variables are positive. However, environmental quality \( E \) and an average person’s income \( A \) as aggregate variables are taken as given and beyond his control by the individual under individual optimization. The first-order condition for \( U \) with respect to \( X \) (which determines \( Y \) and \( C \) through Eq. (2)) subject to (2) is

\[ U_X = pU_C + p \frac{U_R}{A}. \] (3)

If we ignore \( pU_A \), then (3) is just the textbook equation of the marginal rate of substitution (MRS) between leisure and income, i.e. \( \frac{U_X}{U_C} \) with the wage-rate \( p \), or the tangency of the indifference curve for consumption-leisure choice with the budget line whose absolute slope is the wage rate. Our more general (3) allows for relative competition. It shows that, if individuals work to earn income not just
to finance for own consumption but also to engage in relative-income competition, income contributes to his utility both through consumption and relative income, making him willing to work longer than in the absence of the relative-income effect.

Let \( \eta \) with double superscript denote elasticity, e.g. \( \eta_{UX} \equiv \frac{XU}{U} \), \( \eta_{UC} \equiv \frac{CU}{U} \), and \( \eta_{UR} \equiv \frac{RU}{U} \). Let \( i \) indicate individual optimality. From (3), we may express individual optimal \( X \) to be a function of elasticities as follows:

\[
X^* = \frac{\eta_{UX}}{\eta_{UX} + \eta_{UC} + \eta_{UR}}.
\] (4)

Similarly, we may obtain individually optimal \( C \) as follows by plugging (4) into (2):

\[
C^* = \left( \frac{\eta_{UC} + \eta_{UR}}{\eta_{UX} + \eta_{UC} + \eta_{UR}} \right)p.
\] (5)

From the utility function (1), since the individual takes \( E \) as given and beyond his control, there are three variables that affect his utility positively: consumption \( C \), leisure \( X \), and relative income \( R \). With his total time endowment normalized to unity (Eq. (2) above), he thus spends his full income (total time endowment, or material income plus leisure) on leisure by a proportion equal to the elasticity of utility with respect to leisure, or the degree leisure contributes to utility in proportionate terms, divided by the sum of all three elasticities (of leisure \( X \), consumption \( C \), and relative income \( R \)) as Eq. (4) depicts. And, since earning income \( Y \) simultaneously increases consumption and relative income (at the individual level, this is true), the amount of consumption \( C \) is decided by the sum of the two elasticities (of consumption and relative income) over the sum of all the three elasticities.

In (4), a negative relationship exists between \( \eta_{UC} \) and \( X^i \) and also between \( \eta_{UR} \) and \( X^i \). This manifests the fact that an individual will have a lower level of optimal leisure time if his utility is more sensitive to an increase either in consumption (reflected in a larger \( \eta_{UC} \)) or in the relative competition effect (reflected in a larger \( \eta_{UR} \)). The negative relationship between \( \eta_{UR} \) and \( X^i \) means that the individual will have a lower level of optimal leisure time if his utility is more sensitive to a proportionate increase in relative income (reflected in a positive \( \eta_{UR} \)). And the positive relationship between \( \eta_{UX} \) and \( X^i \) suggests that he will have a higher level of optimal leisure time if his utility is more sensitive to an increase in leisure time.
2.2 Social optimality

On top of the model discussed above, the role of a government is introduced in two steps. First, let the government raise revenue by imposing an uniform income tax rate \( t \), with all the revenue used to spend on public good \( G \). As the question of redistribution or poverty reduction is not our focus here, ignoring the possible progressivity in the income tax simplifies the maths. Next, the government is assumed to use a proportion of its tax revenue to provide abatement investment to mitigate against environmental disruption.

2.2.1 The case without abatement investment

In this part, we want to analyze the case where the government imposes a proportionate income tax to finance general public spending, but does not invest in abatement to alleviate environmental disruption.

Consider the society/government problem of the choice of the income-tax rate \( t \) (and hence the public good supply \( G \)) to maximize the utility of the representative individual, taking into account of the effects of \( t \) on all relevant variables, including the work disincentive effect through \( \frac{dX}{dt} \). Income tax here is essentially the same as the tax on consumption as in our simple a-temporal model of static optimization as is usual for welfare analysis where income, consumption and wealth equal each other due to the absence of saving. For simplicity, we assume as usual that the productivity of the economy \( p \) is independent of \( t \) and \( G \). The possible disincentive effect is only through the income-leisure choice \( \frac{dX}{dt} \). Moreover, unlike the case of individual optimality, the government takes the effects of environmental quality \( E \) on utility \( U \) into account. \( E \) is taken as a negative function of \( Y \), i.e. \( E = E(Y) \), where \( \frac{dE}{dY} \leq 0 \).

Let \( \eta^{UE} \equiv \frac{EU}{U} \) and \( \eta^{EY} \equiv \frac{YE}{Y} \), respectively, denote, in proportionate terms, an individual’s utility change in response to a change in environmental quality and the effect on environmental quality of an increase in output. Possible tax/subsidy and spending to affect productivity are ignored as separate problems. Consistent with the focus of this paper, possible pollution/environmental taxes/subsidies are also assumed absent for this part of the analysis. The linear taxation of income to finance for \( G \) is the only government instrument considered in this sub-section. Possible government environmental spending and environmental regulations to directly improve environmental quality are also not explicitly considered (on which
see the next sub-section). We thus have the utility function (6):

\[ U = U(C, X, R, E, G) \]  

where we assume that environmental quality \( E \) and the amount of public goods \( G \) sourced from income tax, respectively, as

\[ E = E(Y) \]  

\[ G = tY = t(1 - X)p. \]  

We let the government maximize \( U \) with respect to the choice of \( t \) (which also determines \( G \) through Eq.(8)). In implicit form, the first-order condition for maximizing \( U \) with respect to \( t \) is

\[ U_C \frac{dC}{dt} + U_X \frac{dX}{dt} + U_R \frac{dR}{dt} + U_E \frac{dE}{dt} + U_G \frac{dG}{dt} = 0, \]  

where \( E_Y \) is negative due to the environmental disruption effect of production/consumption. The influence of \( X \) can be channelled through \( C, R, E \) and \( G \). We may take the third term in (9) as being equal to zero, since the relative competition effect cancels out at the social level; the relative level of the representative person cannot change. Thus, we obtain the following equation:

\[ U_X = pU_C + p \left( E_Y U_E + (t - \frac{1 - X}{dX/dt})(U_G - U_C) \right) \]  

Now compare (3) with (10). Starting with the case without any income taxation \((t = 0)\). Then the first terms of the right-hand side (R.H.S.) of both equations are the same. In (3), we have an additional second term on the relative competition effect that individuals take account of. In (10), this term does not appear, as the improvement in relative standing from earning higher incomes than other individuals does not materialize at the social level, being mutually offsetting between

---

\(^3\)It may be argued that the provision of public goods also generates pollution, making it not different from private goods. In fact, this effect is already allowed for in our model, as we let environmental quality \( E \) be a negative function of aggregate output \( Y \), not just of private consumption \( C \). One important difference between the environmental disruption effects of private and public spending is that, being an external effect, pollution is not (appreciably) taken into account by each private consumer or producer, while the society presumably should take account of the pollution of public spending in its decision including the size of public spending. By contrast, leisure itself does not pollute, leisure activities like tourism may involve pollution, but these activities are included under \( Y \).
individuals; any person’s increase in relative standing is offset by a decrease in that of others.

Instead, in (10), we have a negative environmental disruption effect $E_Y U_E$ coming from production/consumption. The part of environmental disruption is largely not taken into account by individuals as the effect of any individual on the social/global level of pollution is negligible. Individuals want to have less leisure and more consumption/income than the social optimal levels. Alternatively, they want to work longer than is socially optimal, being concerned with relative competition and not concerned with (in the decision on how much to work) environmental disruption. Therefore, we see a double deviation. The second disruption term on the R.H.S. of (10) says that individuals should consider the marginal utility of consumption net of the disruption effect; the second positive relative-competition term on the R.H.S. of (3) says that individuals in fact consider the marginal utility of consumption plus (or gross of) relative-competition effect. Thus, by letting $s$ indicate social optimality, this comparison of (3) and (10) shows:

**Lemma 1. (Reinforcement of relative competition and pollution):** If neither income/consumption nor pollution is taxed, individual optimization results in double departures from social optimality; the relative-competition effect and the environmental disruption effect add together (rather than offsetting), making the excessive work/production/consumption much worse than the case where only one of the two factors is present. That is, $X^i < X^s$, $C^i > C^s$ and $E^s > E^i$.

The proof of Lemma 1 is provided in the Appendix. Compared with the case of social optimality, the case of individual optimality has more serious problems of environmental quality and relative competition. If pollution and relative competition may be taxed directly at low costs, this may be a better way to tackle this problem. However, if this is not possible (e.g. due to information, administrative costs, political constraints, etc.), a positive income tax $t$ may serve to address the issue. This is because a sufficiently high $t$ lowering excessive work/production/consumption/pollution, according to (10), may help to reduce the relative-competition effect and the environmental disruption effect.

**Proposition 1. (Income tax as a remedy to the departure from social optimality):** In the presence of untaxed environmental disruption and relative-competition effects, a tax on income, if not excessive, is corrective, instead of being distortive. The optimal amount of the tax rate should account for both these two effects which are reinforcing rather than offsetting.
There may be some offsetting/reinforcing effects on top of the double departure effects that we want to focus on. When we do not have optimal lump-sum transfer to ensure that $U_C$ approximately equals $U_G$, there may exist the complication of the possible distortive effects of taxation, making the optimality conditions and the value of optimal social $C$ and $X$ more complicated. The implication behind this is that, along with this environmental disruption in the second term of (10), there may exist a sort of ‘second-best’ adjustment factor, when the marginal utility of $G$ is not equal to that of $C$. However, if we take the case that the government is reasonably efficient in choosing the appropriate level of $G$, then the marginal utility of $C$ and that of $G$ should be about the same. This will in fact be the case if we introduce a lump-sum tax/transfer on/to individuals. As the ‘second-best’ adjustment factor may go either way and is not our main focus here, we will simplify the analysis below by ignoring this factor, or take the simple case where $U_G = U_C$.

Given $U_G = U_C$, we can use (10) to determine $X$ of the case without the government’s abatement investment in proportionate/elasticity terms as follows:

$$
X^{**} = \frac{\eta^{UX}}{\eta^{UC}(1-t) + (\eta^{EY} \eta^{UE} + \eta^{UX})}.
$$

(11)

And the optimal consumption level of the individual under social optimality is written as follows by plugging (11) into $C = (1-t)(1-X)p$:

$$
C^{**} = \frac{(\frac{\eta^{UC}}{1-t} + \eta^{EY} \eta^{UE})p}{\frac{\eta^{UC}}{1-t} + (\eta^{EY} \eta^{UE} + \eta^{UX})}.
$$

(12)

Compare (11) with (4). The relative income effect $R$ affects the level of leisure (and also consumption) under individual optimality. However, it has no influence under social optimality as $R$ does not change at the social level, or individual competition cancels out at the social level. Instead, environmental quality (which is ignored at the individual level) is taken into account in social optimization.

Income tax is chosen by the government to maximize social welfare, so, in Eqs. (11) and (12), we interpret $t$ as the dependent variable and regard $X^*$ and elasticities as independent variables.

Other things being equal, Eq. (11) depicts a positive relationship between environmental disruption effect $|\eta^{EY} \eta^{UE}|$ and optimal leisure time $X^*$, given that $\eta^{EY} \eta^{UE} < 0$ due to $E_Y < 0$ and $U_E > 0$. This implies the necessity of longer leisure time when a stronger environmental disruption effect is present. Meanwhile, given $X^*$ and other things, a larger $|\eta^{EY} \eta^{UE}|$ holds along with a lower $(1-t)$, suggesting
that, in the presence of a stronger environmental disruption effect, a higher income tax $t$ should be set for social optimality.

According to Eq. (12), a stronger environmental disruption effect implies that the individual should consume less, given the negative relationship between $C^*$ and $|\eta EY \eta UE|$ depicted by (12). Meanwhile, given $C^*$, a higher income tax $t$ should be set for social optimality when a stronger untaxed environmental disruption effect is present.

### 2.2.2 The case with abatement investment

In this part, in addition to public goods, we also take account of the government’s abatement investment (denoted by $B$) conducive to environmental quality. Abatement investment, regulations, and public goods provided by the government are all related to income level but influence individual’s utility in different ways. Public goods $G$ increasing with income $Y$ directly affect individual’s utility. By contrast, income $Y$ typically affects utility through its negative influence on environmental quality $E$, given that people demand higher $E$ when they have higher $Y$. Thus, the government responds by investing more in abatement to improve $E$. We assume the direct influence of $Y$ on $E$ to be negative and the indirect influence of $Y$ on $E$ to be positive. Along with income $Y$, abatement investment $B$ affects environmental quality $E$ through $E = E(Y, B)$ that satisfies $\frac{dE}{dB} \geq 0$ and $\frac{dE}{dY} \leq 0$. For simplicity, we assume that $B$ is an $\alpha$ ratio of total tax revenue and public goods $G$ is a $(1 - \alpha)$ ratio of total tax revenue, respectively, as follows:

$$B = \alpha tY \quad (13)$$

$$G = (1 - \alpha)tY. \quad (14)$$

Express the representative individual’s utility function as

$$U = U(C, X, R, E(Y, B), G),$$

where $C = (1 - t)Y$ and $R = \frac{Y}{A}$.

Government chooses $t$ and $\alpha$ to maximize $U$. According to (13) and (14), the first-order condition for $U$ with respect to $\alpha$ is

$$U_E E_B = U_G. \quad (15)$$
The first-order condition for $U$ with respect to $t$ is written as

$$U_C\left(\frac{dC}{dt}\right) + U_X\left(\frac{dX}{dt}\right) + U_R\left(\frac{dR}{dt}\right) + U_E\left(\frac{dY}{dt} + E_B\frac{dB}{dt}\right) + U_G\left(\frac{dG}{dt}\right) = 0. \quad (16)$$

Assume that, in (16), $\frac{dR}{dt} = 0$, given that the effect of $R$ does not exist under social optimality. Then, (16) can be reduced to

$$U_X = p\left[\frac{1 - X}{dX/dt} + (1 - t)\right]U_C + pU_EE_Y + p\left[t - \frac{(1 - X)}{dX/dt}\right][(1 - \alpha)U_G + \alpha U_E E_B]. \quad (17)$$

Plugging (15) into (16) gives

$$U_X = pU_C + p\left(E_Y E + (t - \frac{(1 - X)}{dX/dt})(U_G - U_C)\right). \quad (18)$$

Equation (18) is the same as equation (10) where $G$ does not enter $E$. That is, the choice of $X$ and $C$ only relates to the relative importance of $C$, $X$, $G$, and $E$ in the utility function, no more on $B$. The reason behind this is that the choice of $\alpha$ already optimizes between $B$ and $G$. For simplicity, we consider the case where $U_G = U_C$ as mentioned previously. Then, we derive the same results as (11) and (12).

3 Numerical Analysis

Proposition 1 suggests that an income tax can serve as a remedy to excessive working time and environmental disruption. In this part, we want to show this proposition using the following utility function:

$$U = \left(\nu_C C^\sigma + \nu_X X^\sigma + \nu_R R^\sigma + \nu_E E^\sigma + \nu_G G^\sigma\right)^{\frac{1}{\sigma}} \quad (19)$$

Two cases of governmental influence are considered on the basis of (19) to compare with individual optimality. In the first case, we consider the provision of public goods but preclude environmental abatement:

**Case 1. (No abatement spending $B$)** In (19), we have that $G = tY$ and $E = (E_0 - \delta Y)$ (i.e. environmental endowment $E_0$ minus environmental pollution caused by output $\delta Y$), and have relative income $R = 1$ at the social level.

In the second case, the provision of public goods and environmental abatement are simultaneously considered:
Case 2. (With abatement spending $B$) In (19), we have that $G = (1 - \alpha)tY$, $B = \alpha tY$, and $E = (E_0 - \delta Y + B)$, and have relative income $R = 1$ at the social level.

Table 1 lists consumption, welfare and environmental quality with respect to given leisure time simulated with Cases 1 and 2. The finding suggests that, whether the government conducts abatement investment or not, higher consumption level $C$ under individual optimality is accompanied by lower leisure time $X$, lower utility $U$, and lower environmental quality $E$.

Then, focus on the case where $B$ is absent. As can be seen from the difference between panels (1) and (3) and that between panels (5) and (7), consumption is lower in the case $t = 0.1$ than the case $t = 0$, whether $\sigma$ is chosen to be 0.3 or 1.1. However, the government can use the income tax revenue it collects to provide the individual with public goods such as education, parks, etc. Thus, given $X$, utility is higher in the case $t = 0.1$ than the case $t = 0$.

Next, look at the case where $B$ and $G$ coexist. The case is observed from panels (2), (4), (6), and (8). Like the case where $B$ is absent but $G$ exists, in this case, environmental quality and utility are both higher in the case $t = 0.1$ than in the case $t = 0$. Nevertheless, while the contrast between panels (1) and (2) suggests that $U$ is, in overall terms, higher in this case than in the above case, the contrast between panels (5) and (6) suggests that $U$ may be lower in this case than in the above case. Even so, the numerical analysis allows us to conclude that, despite lowering an individual's consumption level, a higher income tax can enhance utility by improving environmental quality and increasing leisure time.

4 Empirical Analysis

In this section, we estimate the effects of tax on labor supply and environmental emission and thus empirically test the hypotheses implied by Lemma 1 and Proposition 1 using cross-country data. The empirical test is composed of two parts. In the first part, we separate the roles of the producer and the worker. In developing economies, carbon emissions are mainly accounted for by production. Analyzing in this way approaches the real situation of the world composed mostly of developing economies. In the second part, a regression analysis is conduct on a representative individual playing the roles of the producer and the worker simultaneously. Analyzing in this way fits our theoretical model, and allows us to test if income taxes can reduce the double departures.
In the case where the producer and the worker are separate, we estimate the following equations:

\[
\text{workingtime} = \alpha_0 + \alpha_1 \text{labortax} + \text{control variables} + \epsilon, \tag{20}
\]

\[
\text{damage} = \beta_0 + \beta_1 \text{profttax} + \text{control variables} + \epsilon', \tag{21}
\]

where \(\epsilon\) and \(\epsilon'\) are error terms. The key parameters of interest here are \(\alpha_1\) and \(\beta_1\), denoting the effects of tax on labor supply and environmental emission, respectively.

Table 2 reports the data sources, definitions and summary statistics of variables from World Bank (2018) or International Labor Organization (2018), with CIPS tests results in the last column of the table. There are eight variables in this table. As an indicator of the departure of leisure, the variable \(\text{working time}\) is measured by the average weekly hours worked per employed person. Carbon damage (denoted by \(\text{damage}\)), as a ratio of GNI, is considered as an indicator for environmental departure for two reasons. First, unlike many other pollutants, carbon emissions are more intractable because they are unavoidable to all economic activities. Although rain forests absorb carbon dioxide, their function has become limited due to the fast and large emissions of carbon dioxide accompanying economic growth. Second, global warming is now an imminent environmental problem that may bring human beings a catastrophe within decades (Weitzman, 2009), while, if not being largely removed using some specific technologies like storage or carbon capture, carbon dioxide will exist in the atmosphere for a very long time.

Income tax considered in our empirical analysis has two categories. One is a tax on labor income (put simply as \(\text{labortax}\)) proxied by World Bank’s (2018) ‘labour tax and contributions as a percentage of commercial profits’. The other is \(\text{proftax}\) as a percentage of total commercial profits, also from World Bank (2018). The former is related to an individual’s leisure time, as a higher tax on working time is hypothesized to frustrate his work incentive. The latter related to producer’s profit can lower output level, and is, thus, hypothesized to reduce environmental departure. Since the data used are in macro level, we also include \(\text{income}\) (and its square, de-
noted by \textit{incomesq}) as control variables in the analysis. To consider heterogeneous effects of income and tax levels, we also divide economies into groups. For economies with a higher (lower) income level than the median of surveyed economies, they are indexed as \( HIINC = 1(0) \) and for those imposing a higher (lower) labor income tax than the median of all surveyed economies, they are indexed as \( HITAX = 1(0) \).

Table 3 reports the effect of \textit{labor tax} on \textit{working time}. In this table, the data on 111 economies for period 2005-2017 contains at most 797 observations. Columns (1) -(10) represent regressions with different specifications and control variables. Time dummies are controlled in the regressions except in columns (3), (8) and (10), where a time trend is used instead. In addition, in column (4) a lagged value of labor tax is used as the independent variable, and income proxied by GDP per capita is included as an additional control variable in the regression reported in column (5). Random-effects (RE) or GLS estimates are reported in column (1) and fixed-effects (FE) estimates are given in other columns. Columns (1)-(3) show that the estimated coefficient of \textit{labor tax} is \(-0.001\) and significant, implying that if the labor tax increases by 1 standard deviation from the average level, i.e., 11.25 percentage points, the average working time may be reduced by 1.125%. This magnitude suggests that people tend to work less by approximately 2 hours per month or 23 hours per year. The estimated coefficients of \textit{labor tax} are similar in column (4) using the lagged value, in column (5) controlling for income and in column (7) using subsample of low income economies (with dummy \( HIINC = 0 \)). The subsample estimate in column (6) using low-labor-tax group (with dummy \( HITAX = 0 \)) remains negative and significant, but with a bigger magnitude of \(-0.002\).

In columns (1)-(7) of Table 3, the regressor \textit{labor tax} is assumed exogenous. Nevertheless, \textit{labor tax} could be endogenous when it is affected by the declining trend of \textit{working time} (as a tax base). In this case, we run FE IV regressions using \textit{income} as an instrument for the endogenous labor tax in columns (8) and (9). After considering potential endogeneity issue, the IV estimates of the \textit{labor tax} coefficient become \(-0.008\) and significant, with a much bigger magnitude than the FE or RE estimates. To deal with potential heteroskedasticity and serial correlation in errors, GMM estimates after controlled for a time trend are reported in column (10). Similar to FE IV estimates in column (8), the effect of labor tax is still negative though not significant. Summarizing the results reported in Table 3, the robust evidence on the negative effect of \textit{labor tax} coefficients suggests that a labor income tax can help reduce the departure of leisure time by lowering an individual’s
working time.

Table 4 presents the estimation results on the effect of profit tax on carbon damage. In this table, the data on 186 economies for period 2006-2017 contains at most 2,062 observations. As in Table 3, all columns of Table 4 report results of regressions with different specifications and control variables. In particular, column (1) reports RE (or GLS) estimates, and other columns consider the FE model. Time effects are controlled by dummies in the regressions in columns (1), (2), (4)-(7), (9) and by a time trend in (3), (8) and (10). In addition, for robustness checks, in column (4) a lagged value of profit tax is considered, and in column (5) income is added in the regression. The FE estimate of profit tax coefficient in column (2) is $-0.007$ and significant at the 5% level, implying that raising profit tax by 1 standard deviation, i.e., 10.28 percentage points, from the average level, the carbon damage could be reduced by 0.072 percentage point, which is about 6% of the cross-country average level of carbon damage in the sample. In column (3) time effects are modelled by a time trend instead of dummies, and in column (4) a lagged value of profit tax is used. In these two columns, the estimated coefficients of profit tax are $-0.003$ and $-0.004$, respectively. They are not as significant as in column (2).

In columns (5) and (6), variables income and its square are included in the regression, the coefficients of profit tax are $-0.006$ and $-0.003$, respectively. The positive sign of income and negative sign of its square empirically verify the EKC hypothesis that carbon damage increases with economic growth and then decreases after certain income level. We also report the subsample estimates using high income economies in column (7), and the estimated coefficient of profit tax is -0.011 and significant at the 1% level. Similar to Table 3, FE IV estimates are reported in columns (8) and (9) allowing that profit tax is endogenous and instrumented by income. The estimated effects of profit tax on carbon damage remain negative, but with a much bigger magnitude than those in other columns assuming exogeneity. Similarly, GMM estimates are added as column (10) to deal with nonspherical errors, and a negative effect of profit tax is found, albeit insignificant. Combining all these regressions in Table 4, the strong evidence on the negative impact of profit tax on carbon damage indicates that a profit tax can help reduce environmental departure.

\footnote{The p-values of CD tests are reported in columns (2) and (3) in Table 3, showing that the null hypothesis of cross-sectional independence is rejected. However, these results should not be considered as an alarming message for estimation and inference. The reason is that the largest number of time periods in this data set is 13, not large enough to obtain accurate estimates of idiosyncratic errors in the panel data regression. In addition, the derivation of asymptotical distribution of CD test under the null also requires a large number of time periods. This is also the case in columns (2) and (3) in Tables 4 and 5 below.}
As our paper follows most simplified welfare-theoretical analysis in using an a-temporal and a representative-individual framework, we make no distinction between consumption and income, profits earners or wages earners, etc. Thus, we may lump taxes together and call them income taxes. We want to see if such a generalized income taxes may help to address the double departures discussed above. Empirically, partly based on Argenziano and Gilboa (2017), we depict the tax as \( \text{incometax} = (\text{labortax})^\gamma (\text{profittax})^{1-\gamma} \), a Cobb-Douglas function.

Taking the log of \( \text{incometax} \), we obtain the linear combination of \( \text{labortax} \) and \( \text{profittax} \) that can be found a similarity in Fullerton and West’s (2010) study on pollution tax where the taxes on engine size, gasoline, and newness of a car are the linear combinations of the optimal taxes with respect to different engine size, gasoline, and newness.

We set \( \gamma \) as 0.7 by reference to World Bank’s (2018) average employment population ratio (of male) of the globe in 2017, since it is related to the population size of workers in an economy. Thus, we run the following regression models by replacing \( \text{labortax} \) in (20) and \( \text{profittax} \) in (21) with \( \text{incometax} \):

\[
\text{workingtime} = \alpha_0 + \alpha_1 \text{incometax} + \text{control variables} + \epsilon, \quad (22)
\]

\[
\text{damage} = \beta_0 + \beta_1 \text{incometax} + \text{control variables} + \epsilon’. \quad (23)
\]

The regression results of (22) are listed in Panel A of Table 5. The specifications of models are the same as those in Table 3. Accordingly, columns (1)-(5) and (7) of Panel A share almost the same coefficients of income tax as those of labor tax in columns (1)-(5) and (7) of Table 3. By contrast, the result in column (6) of Panel A is not significant. Column (6) specifies the group with lower income tax than the average of all surveyed economies.

The regression results of (23) are listed in Panel B of Table 5. The table shares

---

6 Based on Weber’s law which says that ‘the change in a stimulus that will be just noticeable is a constant ratio of the original stimulus’, and assuming independence, Argenziano and Gilboa (2017) show that an economic agent’s utility function is a Cobb-Douglas function of the relevant variables. Likewise, assuming that the two taxes on the producer and the worker are independent, we can depict the tax lumping the taxes on the producer’s and the worker’s incomes as a Cobb-Douglas function under the following two prerequisites that can be compared to Eqs. (2)-(5) and the assumption of Separability in Argenziano and Gilboa (2017): (i) The tax function incorporating the two income taxes keeps the original relationship between the two taxes. That is, if the rate of profit income tax is larger than the rate of wage income tax, then the functional value of the income tax with respect to the rate of profit income tax alone will be larger than that with respect to the rate of wage income tax alone. This feature is compared to Eqs. (2)-(5) of Argenziano and Gilboa (2017); (ii) Choose a value between the two income tax rates and combine the value with each of the two taxes as two different pairs. While the values of the two taxes can be compared, the order of the two pairs is consistent with the order of the two taxes. This is compared to the assumption of Separability in Argenziano and Gilboa (2017).
the same model specifications as those in Table 4. By comparing the coefficients in columns (1)-(7) with those in Table 4, we know that income tax has a bigger impact on carbon damage. For example, in column (1) of Panel B, the coefficient of income tax is -0.011, while the coefficient of profit tax in column (1) of Table 4 is -0.008. In column (2) of Panel B, after controlling for income and its square term, the impact of tax is -0.011 and significant, contrasted with -0.003 and insignificant in column (6) of Table 4.

5 Conclusion

An individual’s welfare increases with environmental quality and leisure, which, due to his competition of consumption with other individuals, are often sacrificed in exchange of higher income, accompanied by excessive working time and the degradation of environmental quality. In this study, we show that a corrective tax in the form of income tax can redress the double departures of environmental quality and leisure. Using World Bank’s (2018) and International Labor Organization’s (2018) data, we confirm income tax as the remedy to the two problems.

Despite the robust findings, some works, however, might be left to continue at a micro level due to the following shortcomings of macro-level data. First, income tax rates we collect from World Bank (2018) are weighted results at an economy-wide level that cannot fully reflect the genuine tax burdens on heterogeneous households and producers—Take Benczúr et al. (2018) as an example. When facing the same change in income tax, high-income and low-income workers respond differently in their labor supply. This, as a consequence, may turn out to affect an economy’s income inequality. Second, the variations of working time and taxes are rather small relative to their means. This also limits our scope on the differences among individuals/producers. The current findings explain the world of homogeneous individuals/producers that we model. We are curious about further policy implications of income taxes in the world of heterogeneous families and monopolistic competitive manufacturing sector.

Different from in Tables 3 and 5, FEIV estimates are not reported in Table 5 as income is no longer a valid instrument for income tax.
References


Fullerton, D and S. E. West, “Tax and Subsidy Combinations for the Control of Car Pollution,” *The B.E. Journal of Economic Analysis & Policy*, 2010, 10(1).


Appendix

A Mathematical proofs

Proof of (4) and (5)

Given $\eta^{UX} = \frac{XU}{U}$, $\eta^{UR} = \frac{RU}{U}$, $\eta^{UC} = \frac{CU}{U}$, $C = Y$, and $Y = (1-X)p$, we can rewrite (3) as

$$\frac{XU_X}{U} = p\left(\frac{X}{C}\right)\frac{CU_C}{U} + p\left(\frac{X}{R}\right)\frac{RU_R}{AU}$$

$$\Rightarrow \eta^{UX} = \frac{\eta^{UC}X}{(1-X)} + \frac{\eta^{UR}X}{(1-X)}$$

Thus,

$$X^i = \frac{\eta^{UX}}{\eta^{UX} + \eta^{UC} + \eta^{UR}}.$$ 

Plugging it into $C = Y$, where $Y = (1-X)p$, we have

$$C^i = \left(\frac{\eta^{UC} + \eta^{UR}}{\eta^{UX} + \eta^{UC} + \eta^{UR}}\right)p.$$ 

Proof of (10).

An individual’s utility function is $U = U(C,X,R,E,G)$, where $E = E(Y)$, $Y = (1-X)p$, $G = t(1-X)p$, and $C = (1-t)Y$. Maximizing utility with respect to income tax $t$ gives

$$U_C \frac{dC}{dt} + U_X \frac{dX}{dt} + U_E \frac{dE}{dt} + U_G \frac{dG}{dt} = 0 \quad (A1)$$

where $\frac{dC}{dt} = -(1-X)p - (1-t)p\frac{dX}{dt}$, $\frac{dE}{dt} = -E_Y p\frac{dX}{dt}$, and $\frac{dG}{dt} = p[(1-X) - t\frac{dX}{dt}]$. 

22
Given that $X(1 - t)$ under social optimality despite a higher income tax. This

Thus,

$$U_X = p[(1 - X) + (1 - t)]U_C + pE_Y U_E + p[t - \left(\frac{1 - X}{dt}\right)]U_G$$

which can be further reduced to the following:

$$U_X = p(t - \frac{1 - X}{dt})(U_G - U_C) + pU_C + pE_Y U_E$$

**Proof of Lemma (1).**

We want to show that, in equilibrium, $X^{ss} > X^{is}$ and $C^{ss} < C^{is}$—Here, we use asterisk to show a variable in equilibrium. To that end, we start from deriving the difference between $\frac{X^{ss}}{C^{ss}}$ and $\frac{X^{is}}{C^{is}}$. Given that $C^{is} = (1 - X^{is})p$ and $C^{ss} = (1 - t)(1 - X^{ss})p$, we can express $\frac{X^{ss}}{C^{ss}} - \frac{X^{is}}{C^{is}}$ as

$$\frac{X^{ss}}{(1 - t)(1 - X^{ss})p} - \frac{X^{is}}{(1 - X^{is})p} = \frac{X^{ss}(1 - X^{is}) - (1 - t)(1 - X^{ss})X^{is}}{(1 - t)(1 - X^{ss})(1 - X^{is})p}$$

$$= \frac{X^{ss} - X^{is}[1 - t + tX^{ss}]}{(1 - t)(1 - X^{ss})(1 - X^{is})p}$$

Given that $X^{ss} \in (0, 1)$, the value of $tX^{ss}$ is smaller than $t$, so we have

$$\frac{X^{ss}}{C^{ss}} - \frac{X^{is}}{C^{is}} > \frac{X^{ss} - X^{is}[1 - t + t]}{(1 - t)(1 - X^{ss})(1 - X^{is})p}$$

$$= \frac{X^{ss} - X^{is}}{(1 - t)(1 - X^{ss})(1 - X^{is})p}.$$ 

Thus, $\frac{X^{ss}}{C^{ss}} > \frac{X^{is}}{C^{is}}$ if $X^{ss} - X^{is} > 0$. Because $X^{ss} - X^{is} > 0$ is equivalent to $(1 - t)(1 - X^{ss}) < (1 - X^{ss}) < (1 - X^{is})$, i.e. $C^{ss} < C^{is}$ that satisfies $\frac{X^{ss}}{C^{ss}} > \frac{X^{is}}{C^{is}}$, we know that $C^{ss} < C^{is}$ if $X^{ss} > X^{is}$.

Next consider the opposite case $C^{ss} > C^{is}$. Under this circumstance, an individual consumes more under social optimality despite a higher income tax. This

---

8 The above can be reduced to

$$U_X \frac{dX}{dt} = -(1 - X)p - (1 - t)p \frac{dX}{dt} U_C + U_E E_Y p \frac{dX}{dt} + pU_G[(1 - X) - t \frac{dX}{dt}]$$

That is,

$$U_X = p[\left(\frac{1 - X}{dt}\right) + (1 - t)]U_C + pE_Y U_E + p[t - \left(\frac{1 - X}{dt}\right)]U_G$$
implies that a higher income tax might have no negative, but even have a positive influence on consumption level, so \((1 - t)Y > Y\) may hold for a given \(Y\), a contradiction. Therefore, it must be true that \(C^{ss} < C^{is}\) if \(X^{ss} > X^{is}\).

Moreover, given that \(E = E(Y)\), where \(E_Y < 0\), we have \(E^{ss} = E((1 - t)(1 - X^{ss})p)\) and \(E^{is} = E((1 - X^{is})p)\) satisfying \(E^{ss} > E^{is}\) due to a positive \(t\) and a larger \(X^{ss}\) in the case of social optimality and a zero \(t\) and a smaller \(X^{is}\) in the case of individual optimality.

Proof of (11) and (12).

As assumed in the text, \(U_{GC} = U_C\). Then, from (10), we have

\[
\frac{XU_X}{U} = \eta^{UX} = p\left(\frac{X}{U}\right)(U + E_YY)
= p\left(\frac{X}{U}\right)
\left(\frac{C + IU}{U} + \left(\frac{U}{E}\right)\left(\frac{Y}{E} - \left(\frac{E_Y}{Y}\right)\left(\frac{EU}{U}\right)\right)\right)
= p\left(\frac{X}{(1 - t)(1 - X)p}\right)\eta^{UC} + \left(\eta^{EY}E^{UE}\right)\eta^{UX}
\]

Let \(s\) denote social optimality. Then the above finding can be reorganized as follows

\[
X^{ss} = \frac{\eta^{UX}}{\eta^{UC}(1 - t)} + \left(\eta^{EY}E^{UE} + \eta^{UX}\right)
\]

and

\[
C^{ss} = (1 - t)\left(\frac{\eta^{UC}}{1 - t} + \left(\eta^{EY}E^{UE} + \eta^{UX}\right)\right)\eta^{UX}
\]

Proof of (17).

We want to reduce (16). Given \(\frac{dR}{dt} = 0\),

\[
UC\left(\frac{dC}{dt}\right) + UX\left(\frac{dX}{dt}\right) + UE\left(E_Y\frac{dY}{dt} + E_B\frac{dB}{dt}\right) + U_G\left(\frac{dG}{dt}\right) = 0
\]

\[B = \alpha Y\text{ (according to (13)) and } G = (1 - \alpha)\text{ (according to (14))}\]

\[
\Rightarrow UC\left(\frac{d[(1 - t)(1 - X)p]}{dt}\right) + UX\left(\frac{dX}{dt}\right) + UE\left(E_Y\frac{d[(1 - X)p]}{dt} + E_B\frac{d[\alpha t(1 - X)p]}{dt}\right)
+ U_G\left(\frac{d[(1 - \alpha)t(1 - X)p]}{dt}\right) = 0
\]

\[
\Rightarrow UC[-(1 - X) - (1 - t)\frac{dX}{dt}]p + UX\left(\frac{dX}{dt}\right) + UE\left(E_Y\frac{-pdX}{dt} + \alpha E_B[(1 - X)p - tp\frac{dX}{dt}]\right)
+ (1 - \alpha)U_G[(1 - X)p - tp\frac{dX}{dt}] = 0
\]
Then, we arrive at

$$U_X = p \left( \frac{(1 - X)}{\frac{\text{d}X}{\text{d}t}} + (1 - t) \right) U_C + p U_E E_Y + p \left[ t - \frac{(1 - X)}{\frac{\text{d}X}{\text{d}t}} \right] [(1 - \alpha) U_G + \alpha U_E E_B]$$
B Supplementing ‘No need for Ramsey Taxes’

In this section/appendix, we provide a graphical illustration for the points that

1. Optimal Ramsey commodity taxation depends on the elasticities of demand and supply;

2. Optimal Pigovian taxation of external effects does not depend on these elasticities but on the marginal damages of pollution.

While a diagrammatic demand/supply analysis may be regarded as too partial-equilibrium an analysis, actually for a two sector model, it is also a fully general-equilibrium analysis, as from Walras’s law, equilibrium in \((n - 1)\) sectors implies equilibrium in all the \(n\) sectors, where \(n\) is the number of goods/sectors. Moreover, when combined with the Hicks composite theorem, many analytical cases like the ones here may be simplified into two sectors, making the diagrammatical demand/supply analysis much more generally applicable ([Ng 2011](#) Section 5.4). In Fig 1, the horizontal axis measures the quantities of the relevant commodities. For the case where the supply and demand of the commodity concerned is represented by \(D^1\) and \(S^1\), a substantial tax yielding a large amount of tax revenue represented by the large rectangle, while the amount of deadweight loss, distortionary cost, or excess burden of taxation is only the small triangle. For the case where the supply and demand of the commodity concerned is represented by \(D^2\) and \(S^2\), a smaller tax yielding a much smaller amount of tax revenue represented by the small rectangle, while the amount of excess burden of taxation is the slightly larger triangle. This illustrates the point of Ramsey taxation that the larger the absolute elasticities of demand and supply, the larger is the excess burden of taxation on that commodity. To minimize the total amount of excess burden for any given amount of revenue raised, higher taxes should be imposed on commodities of lower (absolute) elasticities of demand and supply. At the optimal structure of taxes, an equal proportionate deviation (from the untaxed position) in quantities transacted for all commodities is involved.

For the case of taxing a commodity to account for the pollution involved, as illustrated in Fig 2, a tax in accordance to the amount of the marginal damage shifts the supply curve from \(S\) to \(S'\). For simplicity, we allow here for variation in the elasticity of demand only (allowing for the variation in the elasticity of supply complicates the diagram without changing the result). Here, whether the demand is the less elastic \(D^1\) or the more elastic \(D^2\), the optimal amount of tax on the relevant commodity depends on the marginal damage of the external cost, not on the difference in elasticity.
If desired, we could also use our mathematical model to show this more rigorously. Or, just cite: If a tax/charge/fine on pollution is feasible, its optimal amount depends on the marginal damage, with some adjustment to account for any disincentive effect of income taxation (which, if positive, increases the amount of the optimal fine), not on the elasticities of demand and supply, as shown in Ng [2004, Appendix 1].
Figure 1: Price elasticities important for Ramsey’s taxes

Figure 2: Price elasticities not important for pollution taxes
<table>
<thead>
<tr>
<th></th>
<th>Welfare, consumption, and environmental quality</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B is absent but G exists</td>
<td>B and G coexist</td>
</tr>
<tr>
<td></td>
<td>(1) $\sigma=0.3$ and $t=0.1$</td>
<td>(2) $\sigma=0.3$ and $t=0.1$</td>
</tr>
<tr>
<td>X</td>
<td>C</td>
<td>U</td>
</tr>
<tr>
<td>0.1</td>
<td>0.81</td>
<td>0.121</td>
</tr>
<tr>
<td>0.16</td>
<td>0.759</td>
<td>0.139</td>
</tr>
<tr>
<td>0.21</td>
<td>0.707</td>
<td>0.152</td>
</tr>
<tr>
<td>0.27</td>
<td>0.656</td>
<td>0.162</td>
</tr>
<tr>
<td>0.39</td>
<td>0.553</td>
<td>0.177</td>
</tr>
<tr>
<td>0.44</td>
<td>0.501</td>
<td>0.182</td>
</tr>
<tr>
<td>0.5</td>
<td>0.45</td>
<td>0.186</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3) $\sigma=0.3$ and $t=0$</td>
<td>(4) $\sigma=0.3$ and $t=0$</td>
</tr>
<tr>
<td>X</td>
<td>C</td>
<td>U</td>
</tr>
<tr>
<td>0.1</td>
<td>0.9</td>
<td>0.042</td>
</tr>
<tr>
<td>0.16</td>
<td>0.843</td>
<td>0.051</td>
</tr>
<tr>
<td>0.21</td>
<td>0.786</td>
<td>0.059</td>
</tr>
<tr>
<td>0.27</td>
<td>0.729</td>
<td>0.066</td>
</tr>
<tr>
<td>0.39</td>
<td>0.614</td>
<td>0.078</td>
</tr>
<tr>
<td>0.44</td>
<td>0.557</td>
<td>0.083</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>0.087</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(5) $\sigma=1.1$ and $t=0$</td>
<td>(6) $\sigma=1.1$ and $t=0$</td>
</tr>
<tr>
<td>X</td>
<td>C</td>
<td>U</td>
</tr>
<tr>
<td>0.1</td>
<td>0.81</td>
<td>0.308</td>
</tr>
<tr>
<td>0.16</td>
<td>0.759</td>
<td>0.313</td>
</tr>
<tr>
<td>0.21</td>
<td>0.707</td>
<td>0.318</td>
</tr>
<tr>
<td>0.27</td>
<td>0.656</td>
<td>0.323</td>
</tr>
<tr>
<td>0.39</td>
<td>0.553</td>
<td>0.335</td>
</tr>
<tr>
<td>0.44</td>
<td>0.501</td>
<td>0.341</td>
</tr>
<tr>
<td>0.5</td>
<td>0.45</td>
<td>0.347</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(7) $\sigma=1.1$ and $t=0$</td>
<td>(8) $\sigma=1.1$ and $t=0$</td>
</tr>
<tr>
<td>X</td>
<td>C</td>
<td>U</td>
</tr>
<tr>
<td>0.1</td>
<td>0.9</td>
<td>0.286</td>
</tr>
<tr>
<td>0.16</td>
<td>0.843</td>
<td>0.293</td>
</tr>
<tr>
<td>0.21</td>
<td>0.786</td>
<td>0.299</td>
</tr>
<tr>
<td>0.27</td>
<td>0.729</td>
<td>0.306</td>
</tr>
<tr>
<td>0.39</td>
<td>0.614</td>
<td>0.321</td>
</tr>
<tr>
<td>0.44</td>
<td>0.557</td>
<td>0.328</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>0.336</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
<td>obs</td>
</tr>
<tr>
<td>------------</td>
<td>-----------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>working time</td>
<td>Average weekly hours actually worked per employed person</td>
<td>1242</td>
</tr>
<tr>
<td>damage</td>
<td>Adjusted savings: carbon dioxide damage (% of GNI)</td>
<td>7509</td>
</tr>
<tr>
<td>profit tax</td>
<td>Profit tax (% of commercial profits)</td>
<td>2299</td>
</tr>
<tr>
<td>labor tax</td>
<td>labor tax and contributions (% of commercial profits)</td>
<td>2299</td>
</tr>
<tr>
<td>income</td>
<td>GDP per capita</td>
<td>8863</td>
</tr>
<tr>
<td>incomesq</td>
<td>the square of GDP per capita</td>
<td></td>
</tr>
<tr>
<td>HIINC</td>
<td>economic group with per capita GDP no lower than the 50% quantile of all surveyed economies</td>
<td></td>
</tr>
<tr>
<td>HITAX</td>
<td>economic group with labor income tax no lower than the 50% quantile of all surveyed economies</td>
<td></td>
</tr>
</tbody>
</table>

**Notes:**
WDI is the abbreviation of World Development Indicators and ILO is the abbreviation of International Labor Organization. WDI's data are available for the period 2005-2017; ILO's data are available for the period 2005-2017.
Table 3 The influence of labor tax on working time

<table>
<thead>
<tr>
<th>Independent variables:</th>
<th>Panel regression</th>
<th>FEIV</th>
<th>GMM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>labor tax</td>
<td>-0.001***</td>
<td>-0.001***</td>
<td>-0.001***</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>lagged labor tax</td>
<td>-0.001**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>income</td>
<td>-0.003***</td>
<td>0.028**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.012)</td>
<td></td>
</tr>
<tr>
<td>time trend</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td></td>
<td>-0.003***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>year dummies</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>specification(s)</td>
<td>RE/GLS</td>
<td>FE</td>
<td>FE</td>
</tr>
<tr>
<td></td>
<td>FE</td>
<td>FE</td>
<td>FE</td>
</tr>
<tr>
<td></td>
<td>FE, HITAX=0</td>
<td>FE</td>
<td>FE</td>
</tr>
<tr>
<td></td>
<td>HIINC=0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>No. of obs</td>
<td>797</td>
<td>797</td>
<td>797</td>
</tr>
<tr>
<td></td>
<td>797</td>
<td>746</td>
<td>794</td>
</tr>
<tr>
<td></td>
<td>394</td>
<td>353</td>
<td></td>
</tr>
<tr>
<td></td>
<td>794</td>
<td>794</td>
<td></td>
</tr>
<tr>
<td></td>
<td>781</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.165</td>
<td>0.165</td>
<td>0.124</td>
</tr>
<tr>
<td></td>
<td>0.158</td>
<td>0.172</td>
<td>0.275</td>
</tr>
<tr>
<td></td>
<td>0.144</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CD test (p-value)</td>
<td>0.018</td>
<td>0.000</td>
<td></td>
</tr>
</tbody>
</table>

first-stage regression of labor tax

| income                 | -3.987***       | -3.987*** |
|                        | (1.356)         | (1.380)   |
| time trend             | -0.014          |           |
|                        | (0.038)         |           |
| year dummies           | no              | yes       |
| $R^2$                  | 0.089           | 0.087     |

Notes:
(i) working time is proxied by the average weekly hours per employee; labor tax is proxied by labor tax and contributions (% of commercial profits); income is proxied by GDP per capita; incomesq is the square of income.
(ii) FE refers to fixed-effects estimates; RE refers to random-effects GLS estimates.
(iii) Columns (6) and (7) are, respectively, for subsamples HITAX=0 and HIINC=0.
(iv) $R^2$ values reported are within R-square in columns (1)-(7), and are overall R-square in columns (8) and (9).
(v) Standard errors are reported in parentheses. The stars, *, ** and *** indicate the significance level at 10%, 5% and 1%, respectively.
### Table 4 The influence of profit tax on carbon damage

<table>
<thead>
<tr>
<th>Independent variables:</th>
<th>Panel regression</th>
<th>No. of obs</th>
<th>R²</th>
<th>CD test (p-value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>profit tax</td>
<td>-0.008***</td>
<td>2062</td>
<td>0.108</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td></td>
<td>(1.004)</td>
<td></td>
</tr>
<tr>
<td>lagged profit tax</td>
<td>-0.004*</td>
<td>2062</td>
<td>0.108</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td></td>
<td>(1.013)</td>
<td></td>
</tr>
<tr>
<td>income</td>
<td>0.813</td>
<td>1877</td>
<td>0.128</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>(0.569)</td>
<td></td>
<td>(0.030)</td>
<td></td>
</tr>
<tr>
<td>income square</td>
<td>-0.111***</td>
<td>2055</td>
<td>0.143</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>(0.037)</td>
<td></td>
<td>(0.017)</td>
<td></td>
</tr>
<tr>
<td>time trend</td>
<td>0.009***</td>
<td>1091</td>
<td>0.050</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td></td>
<td>(0.132)</td>
<td></td>
</tr>
<tr>
<td>year dummies</td>
<td>yes</td>
<td>2055</td>
<td>0.015</td>
<td>1.843*</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td></td>
<td>(1.004)</td>
<td></td>
</tr>
<tr>
<td>specification(s)</td>
<td>RE/GLS</td>
<td>2055</td>
<td>0.006</td>
<td>2.067**</td>
</tr>
<tr>
<td></td>
<td>FE</td>
<td></td>
<td>(1.013)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>FE</td>
<td>2055</td>
<td>0.005</td>
<td></td>
</tr>
<tr>
<td></td>
<td>FE, HIINC=0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>FE</td>
<td>2055</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td></td>
<td>FE</td>
<td>2055</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td></td>
<td>FE</td>
<td>2055</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td></td>
<td>FE</td>
<td>2055</td>
<td>0.000</td>
<td></td>
</tr>
</tbody>
</table>

Notes:
(i) *damage* is proxied by carbon dioxide damage (% of GNI); *profit tax* is proxied by profit tax (% of commercial profits); *income* is proxied by GDP per capita; *incomesq* is the square of *income*.
(ii) *FE* refers to fixed-effects model; *RE* refers to random-effects model; *IV* refers to IV regression.
(iii) Column (7) is for subsample HIINC=0.
(iv) *R²* values reported are within R-square in columns (1)-(7), and are overall R-square in columns (8) and (9).
(v) Standard errors are reported in parentheses. The stars, *, ** and *** indicate the significance level at 10%, 5% and 1%, respectively.

**First-stage regression of profit tax**

- income: 1.843* (1.004)
- time trend: -0.270*** (0.030)
- year dummies: yes

Overall *R²*: 0.006 (1.013)
Table 5: The influences of income tax on working time and carbon damage

**Panel A: Dependent variable: working time**

<table>
<thead>
<tr>
<th>Independent variables</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>income tax</td>
<td>-0.001***</td>
<td>-0.001***</td>
<td>-0.001***</td>
<td>-0.001***</td>
<td>-0.000</td>
<td>-0.001**</td>
<td>-0.001**</td>
</tr>
<tr>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.001)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>lagged income tax</td>
<td></td>
<td>-0.001***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.000)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>income</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>time trend</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>year dummies</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>specification(s)</td>
<td>RE/GLS</td>
<td>FE</td>
<td>FE</td>
<td>FE</td>
<td>FE</td>
<td>FE</td>
<td>HISYNTX = 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>FE, HIINC = 0</td>
</tr>
<tr>
<td>No. of obs</td>
<td>796</td>
<td>796</td>
<td>796</td>
<td>745</td>
<td>793</td>
<td>416</td>
<td>353</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.165</td>
<td>0.165</td>
<td>0.123</td>
<td>0.166</td>
<td>0.174</td>
<td>0.227</td>
<td>0.148</td>
</tr>
<tr>
<td>CD test (p-value)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.011</td>
</tr>
</tbody>
</table>

**Panel B: Dependent variable: damage**

<table>
<thead>
<tr>
<th>Independent variables</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>income tax</td>
<td>-0.011***</td>
<td>-0.013***</td>
<td>-0.010***</td>
<td>-0.015***</td>
<td>-0.011***</td>
<td>-0.026***</td>
<td></td>
</tr>
<tr>
<td>(0.003)</td>
<td>(0.003)</td>
<td>(0.004)</td>
<td>(0.003)</td>
<td>(0.004)</td>
<td>(0.005)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>lagged income tax</td>
<td></td>
<td></td>
<td>-0.012***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.003)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>income</td>
<td></td>
<td></td>
<td></td>
<td>1.142**</td>
<td>0.509</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.574)</td>
<td></td>
<td></td>
<td></td>
<td>(0.598)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>incomesq</td>
<td></td>
<td></td>
<td></td>
<td>-0.132***</td>
<td>-0.100***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.036)</td>
<td></td>
<td></td>
<td></td>
<td>(0.037)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>time trend</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.009***</td>
<td></td>
<td>0.027***</td>
</tr>
<tr>
<td>(0.003)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.004)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>year dummies</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>specification(s)</td>
<td>RE/GLS</td>
<td>FE</td>
<td>FE</td>
<td>FE</td>
<td>FE</td>
<td>FE</td>
<td>HIINC = 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No. of obs</td>
<td>2061</td>
<td>2061</td>
<td>2061</td>
<td>1876</td>
<td>2054</td>
<td>2054</td>
<td>1091</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.112</td>
<td>0.112</td>
<td>0.009</td>
<td>0.134</td>
<td>0.149</td>
<td>0.054</td>
<td>0.108</td>
</tr>
<tr>
<td>CD test (p-value)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.000</td>
</tr>
</tbody>
</table>

Notes:
(i) working time is proxied by the average weekly hours actually worked per employed person; damage is proxied by carbon dioxide damage (% of GNI); income tax is the combination of labor tax and profit tax shown in the text; income is proxied by GDP per capita; incomesq is the square of income.
(ii) FE refers to fixed-effects model; RE refers to random-effects model.
(iii) Columns (6) and (7) of Panel A are, respectively, for subsamples HISYNTX = 0 and HIINC = 0. Column (7) of Panel B is for subsample HIINC = 0.
(iv) $R^2$ values reported are within R-square.
(v) Standard errors are reported in parentheses. The stars, ** and *** indicate the significance level at 5% and 1%, respectively.