Determine L-observer

- Given Σ and $\Sigma' \subseteq \Sigma$, let $P: \Sigma^* \rightarrow \Sigma'^*$ be the natural projection.
- Let $L \subseteq \Sigma^*$. Suppose it is recognized by a nondeterministic automaton $G = (X, \Sigma, \xi, x_0, X_m)$, i.e. $L_m(G) = L$.
- For all $U \subseteq X$ and $\sigma \in \Sigma'$, let

$$f(U,\sigma,\Sigma') := \{x \in X \mid (\exists u,u' \in (\Sigma-\Sigma')^*)\ \xi(x,u\sigma u') \cap U \neq \emptyset\}$$

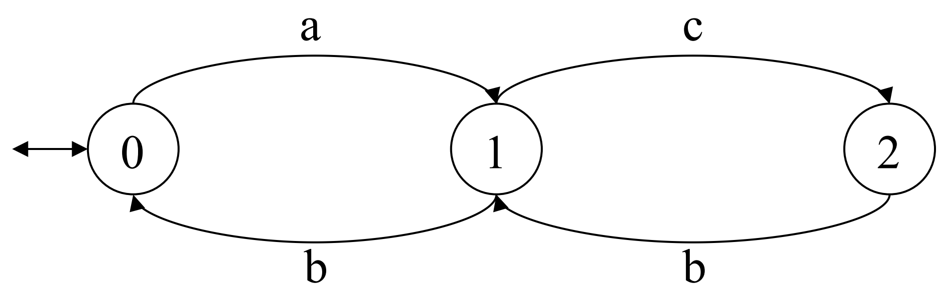
For all $\mathcal{U} \subseteq 2^X$, $g(\mathcal{U}) := \{f(U,\sigma,\Sigma') \mid U \in \mathcal{U} \wedge \sigma \in \Sigma'\} \cup \mathcal{U}$

- For all $\mathcal{U} \subseteq 2^X$, let $h(\mathcal{U}) := \{U \subseteq X \mid (\exists U', U'' \in \mathcal{U})\ U = U' - U''\} - \{\emptyset\}$
- For all $\mathcal{U} \subseteq 2^X$, let $q(\mathcal{U}) := \{U \in \mathcal{U} \mid (\forall U' \in \mathcal{U})\ U' \subseteq U \Rightarrow U = U'\}$
- Let $\mathcal{U}_0 = \{U = \{x \in X \mid x \in X_m \vee (\exists u \in (\Sigma-\Sigma')^*)\ \xi(x,u) \cap X_m \neq \emptyset\}, X - U\}$
- Let $\Omega(\Sigma', \mathcal{U}_0) := (q \circ h^{\infty} \circ g)^r(\mathcal{U}_0)$ for some $n_r, r \in \mathbb{N}$, such that

$$(q \circ h^{\infty} \circ g)^r(\mathcal{U}_0) = (q \circ h^{\infty} \circ g)^{r+1}(\mathcal{U}_0)$$

- Let $A := \{\sigma \in \Sigma \mid (\exists U_1, U_2 \in \Omega(\Sigma', \mathcal{U}_0) : U_1 \neq U_2)(x_1 \in U_1)(x_2 \in U_2)\ x_2 = \xi(x_1, \sigma)\}$.
- P is the L-observer if and only if $A \subseteq \Sigma'$.

- How to determine whether $P:\Sigma^* \to \Sigma'^*$ is an L-observer?



$\Sigma = \{a,b,c\}$
$\Sigma' = \{a,b\}$

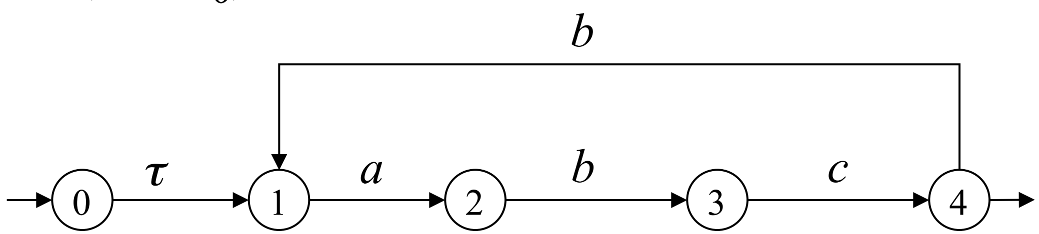
- Step 1: Compute S with $L_m(S) = P(L)$
- Step 2: Compute $\Omega(\Sigma', \mathcal{U}_0)$
  - $\mathcal{U}_0 = \{\{0\}, \{1,2\}\}$
  - $f(\{1,2\},a,\Sigma')=\{0\}$, $f(\{0\},a,\Sigma')=\varnothing$
  - $f(\{1,2\},b,\Sigma')=\{1,2\}$, $f(\{0\},b,\Sigma')=\{1\}$
  - $g(\mathcal{U}_0)=\{\varnothing,\{0\},\{1\},\{1,2\}\}$
  - $h(g(\mathcal{U}_0))=\{\{0\},\{1\},\{2\},\{1,2\}\}$
  - $q(h(g(\mathcal{U}_0)))=\{\{0\},\{1\},\{2\}\}$
  - $\Omega(\Sigma', \mathcal{U}_0) = \{\{0\},\{1\},\{2\}\}$

- $A = \{a,b,c\}$
- Since $A \not\subseteq \Sigma'$, P is not an L-observer.

Partition States based on Marking Weak Bisimulation

- Let $G = (X,\Sigma,\xi,x_0,X_m)$ be nondeterministic and $\Sigma' \subseteq \Sigma$.
- Let $\mathcal{U}_0 = \{X_m, X - X_m\}$
- Compute $\Omega(\Sigma', \mathcal{U}_0)$
- $X/\approx_{\Sigma'} = \Omega(\Sigma', \mathcal{U}_0)$

G



$\Sigma = \{\tau, a, b, c\}$
$\Sigma' = \{\tau, c\}$

- $X/\approx_{\Sigma'} = \Omega(\Sigma', \mathcal{U}_0) =$
  $\left\{<0>=\{0\},<1>=\{1,2,3\},<4>=\{4\}\right\}$

$G/\approx_{\Sigma'}$