Supplemental Materials

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This document includes the supplemental materials for the paper titled "Exploiting Reactive Mobility for Collaborative Target Detection in Wireless Sensor Networks."

Appendix A
Proof of Lyapunov Condition

Lemma 1. *Let $\{I_i : i = 1, \ldots, N\}$ be a sequence of mutually independent Bernoulli random variables, the Lyapunov condition holds for this sequence.*

Proof: Denote μ_i, σ_i^2, and r_i^3 as the mean, the variance, and the third central moment of I_i, respectively. The Lyapunov condition is formally stated as follows [1]: *For any i, r_i^3 is finite and $\lim_{N\to\infty} \rho(N) = 0$ where* $\rho(N) = \frac{\left(\sum_{i=1}^N r_i^3\right)^{\frac{1}{3}}}{\left(\sum_{i=1}^N \sigma_i^2\right)^{\frac{1}{2}}}$.

Suppose p_i is the success probability of I_i, the mean and variance of I_i are $\mu_i = p_i$, $\sigma_i^2 = p_i(1-p_i)$, respectively. And the third central moment of I_i is $r_i^3 = \mathbb{E}[|I_i - \mu_i|^3] = (1-p_i)^3 p_i + p_i^3(1-p_i)$. Obviously, for any i, r_i^3 is finite. Furthermore, as $0 \le p_i \le 1$, $r_i^3 \ge 0$. Accordingly, $\rho(N) \ge 0$. Moreover,

$$\sum_{i=1}^N r_i^3 = \sum_{i=1}^N (p_i - p_i^2) - 2\sum_{i=1}^N p_i^2(p_i - 1)^2 \le \sum_{i=1}^N (p_i - p_i^2) = \sum_{i=1}^N \sigma_i^2.$$

Therefore,

$$\lim_{N\to\infty} \rho(N) = \lim_{N\to\infty} \frac{\left(\sum_{i=1}^N r_i^3\right)^{\frac{1}{3}}}{\left(\sum_{i=1}^N \sigma_i^2\right)^{\frac{1}{2}}} \le \lim_{n\to\infty} \frac{\left(\sum_{i=1}^N \sigma_i^2\right)^{\frac{1}{3}}}{\left(\sum_{i=1}^N \sigma_i^2\right)^{\frac{1}{2}}} = 0.$$

As $\rho(N) \ge 0$, $\lim_{N\to\infty} \rho(N) = 0$. Hence, the Lyapunov condition is satisfied. □

Appendix B
Calculation of Joint Probabilities

This section discusses the Monte Carlo method to calculate the joint probabilities in Eq. (13), (15), (20) and (21). As $E_{i,j} = \sum_{k=1}^j e_{i,k}$ where $e_{i,j}$ are mutually independent Gaussian random variables given by Eq. (16) and (17), in order to simplify notation, we consider the following problem: *Suppose $X_1, X_2, \ldots, X_n$ are mutually independent Gaussian variables,* i.e., $X_i \sim \mathcal{N}(\mu_i, \sigma_i^2)$. *How to calculate the probability $p = \Pr(C)$, where C is the event $\bigcap_{i=1}^n \sum_{j=1}^i X_j \le \lambda$ and λ is a constant?* The Monte Carlo method for computing p is as follows. Denote vector $\mathbf{X} = [X_1, X_2, \ldots, X_n]$. We draw N samples of $\mathbf{X}$ and count the number of occurrences of event C, which is denoted as N'. The ratio N'/N is an unbiased estimate of p. According to the Chernoff's inequality [1], the inequality $|p - N'/N| \le \sqrt{-\frac{1}{N}\ln\frac{\delta}{2}}$ holds with a probability of at least $1-\delta$. For instance, in order to achieve an accuracy of $|p - N'/N| \le 0.01$ with a probability of at least 95%, we need to draw 36889 samples to estimate p. We note that the joint probabilities can be pre-computed on desktop computer and stored in a table on the cluster head for the movement scheduling algorithm.

TABLE 1
Evaluation of Monotonicity of $C(\lambda_2, \mathbf{X})$

N	n	$\lvert\mathbf{V}\rvert$	$\lvert\mathbf{V}^+\rvert$	$\lvert\mathbf{V}^+\rvert/\lvert\mathbf{V}\rvert$
2	200^2	588731550	582714192	0.9898
3	40^3	1225921021	1214844804	0.9910
4	16^4	853906020	850734897	0.9963
5	10^5	1125805664	1124525354	0.9989

Appendix C
Evaluation of Monotonicity of $C(\lambda_2, \mathbf{X})$

As the remainder term R_1 in the Taylor expansion of $C(\lambda_2, \mathbf{X})$ given by Eq. (23) depends on $\beta_{2,i}$, $C(\lambda_2, \mathbf{X})$ does not strictly decrease with $\sum_{i=1}^N \beta_{2,i}$. In this section, a numerical simulation is conducted to evaluate the monotonicity of $C(\lambda_2, \mathbf{X})$ with respect to $\sum_{i=1}^N \beta_{2,i}$. In the N-dimensional space $\{\beta_{2,i} | i \in [1, N], \beta_{2,i} \in (0,1)\}$, n points are uniformly chosen to calculate the value of $C(\lambda_2, \mathbf{X})$ and $\sum_{i=1}^N \beta_{2,i}$. The relationship between $C(\lambda_2, \mathbf{X})$ and $\sum_{i=1}^N \beta_{2,i}$ is evaluated for $\binom{n}{2}$ combinations of two points in the space. For each combination of two points P_1 and P_2, if $\sum_{i=1}^N \beta_{2,i}|_{P_1} > \sum_{i=1}^N \beta_{2,i}|_{P_2}$, this combination is included into set $\mathbf{V}$. Moreover, if $C(\lambda_2, \mathbf{X})|_{P_1} < C(\lambda_2, \mathbf{X})|_{P_2}$ for this combination, it is included into set $\mathbf{V}^+$. The evaluation results are listed in

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Table 1. Note that $|\mathbf{V}|$ and $|\mathbf{V}^+|$ represent the cardinalities of $\mathbf{V}$ and $\mathbf{V}^+$, respectively. Due to the exponential complexity with respect to the dimension number N, we reduce the resolution for choosing points for the cases with high dimension. If $\beta_{2,i}$ are independent from each other, the ratio $|\mathbf{V}^+|/|\mathbf{V}|$ in Table 1 indicates the probability that $C(\lambda_2, \mathbf{X})$ decreases with $\sum_{i=1}^{N} \beta_{2,i}$. The evaluation results show that $C(\lambda_2, \mathbf{X})$ decreases with $\sum_{i=1}^{N} \beta_{2,i}$ with a high probability ($> 98\%$). Therefore, such near-monotonic relationship ensures the value of $C(\lambda_2, \mathbf{X})$ found by maximizing $\sum_{i=1}^{N} \beta_{2,i}$ is near-minimum.

References

[1] R. B. Ash and C. A. Doléans-Dade, *Probability & Measure Theory*, 2nd ed. A Harcourt Science and Technology Company, 1999.