

# Analysis of the Noise Robustness Problem and a New Blind Channel Identification Algorithm

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**Abstract**—Blind channel identification has generated much interest in signal processing and communications. Although existing cross relation based blind channel identification algorithm can achieve promising results, one of the drawbacks is the performance degradation in a noisy environment. In this work, we show that the degradation in convergence performance of MCLMS is due to an implicit constraint imposed by the *cross relation* cost function. This constraint requires the estimated impulse responses to be of the same energy which is often untrue in practice. We next propose a new algorithm exploiting revised cost function to improve the robustness of MCLMS to noise. Monte Carlo simulation results show that the proposed algorithm can gain significant improvement in steady-state performance.

**Index Terms**—Blind channel identification, adaptive algorithms, cross relation.

## I. INTRODUCTION

Channel identification involves the estimation of an unknown system by analyzing its input/output using a mathematical model [1]. This problem of fundamental interest arises in a variety of signal processing [2], wireless communications and body area network applications [3][4][5]. The self-recovering identification or blind channel identification problem was originally proposed by Sato [6]. Since then, this research problem has drawn wide attention by many researchers [7][8]. Although these algorithms provide satisfactory estimation results in certain scenarios, they often require relatively large number of data samples, which may limit their tracking performance in a highly time-varying environment [9]. To address this issue, many second-order statistics based algorithms have been proposed. For example, subspace algorithms utilize the null subspace of the data matrix [10][11] while the two-step maximum likelihood algorithm exploits an orthogonal complement matrix of the generalized Sylvester matrix [12].

According to [13], a blind channel identification algorithm should satisfy the following design requirements: adaptivity, fast convergence and low complexity. Therefore, adaptive blind channel identification algorithms are of high interest. It is also useful to note that a higher-order statistic (HOS) cost function is barely concave which leads to slow convergence. In addition, HOS-based algorithms cannot be computed accurately from a small number of observations [1]. Based on Karhunen-Loève transform and exploiting its separation property, authors of [14] proposed a general framework to identify a large class of nonlinear systems. The least-squares approach [9] to blind channel identification, on the other hand,

introduces the concept of *cross relation*. It has also been shown that, to uniquely identify the channels, the necessary and sufficient conditions include the polynomials of the channels being co-prime (they do not share any common roots) and that the auto-correlation matrix of the source signal being full rank [9][15].

Based on the cross relation concept, an adaptive multichannel least-mean-squares (MCLMS) algorithm has been proposed [16]. The MCLMS algorithm follows a least-mean-square (LMS) framework and similar to most of the adaptive algorithms, it suffers from performance degradation in a noisy environment. Exploiting similar cross relation concept, the authors of [17] proposed to solve the linear and non-linear parts of a single-input multiple-output (SIMO) system by kernel Hilbert space and canonical correlation analysis, respectively. It is interesting to note that the normalized multichannel frequency-domain LMS (NMCFLMS) [1], which is a direct extension of MCLMS, also suffers from the misconvergence problem in a noisy environment [18]. This noise robustness problem is due to the fact that direct minimization of the cost function does not necessarily imply good channel estimates. Although algorithms have been proposed to address this noise robustness problem and to improve the convergence performance of NMCFLMS [19][20], it is still unclear how noise affects the cross relation cost function of MCLMS.

In this paper, we analyze how noise affects the cost function of MCLMS. This analysis allows one to gain new insights into the performance of the algorithm and we show that noise degrades the convergence performance by forcing the estimated channels to have equivalent  $l_2$ -norm, which is untrue in practice. We then proceed to propose a joint optimization problem to mitigate the cross relation error due to noise, therefore overcoming degradation in convergence performance of MCLMS in a noisy environment. Results obtained from Monte Carlo simulation illustrated in Section IV show that the proposed improved MCLMS (IMCLMS) algorithm can achieve higher noise robustness and can gain significant improvement in steady-state performance.

## II. PROBLEM FORMULATION

We consider a SIMO finite impulse response (FIR) system shown in Fig. 1. The observed signal of the  $i$ th channel is

$$y_i(n) = x_i(n) + v_i(n), \quad i = 1, \dots, M, \quad (1)$$

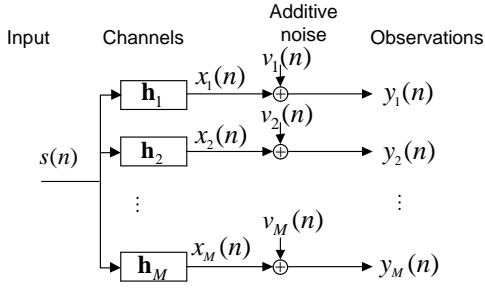


Fig. 1. Illustration of the relationship between the input  $s(n)$  and the observations  $x_i(n)$  in a SIMO FIR system.

where  $M$  is the total number of channels,  $v_i(n)$  is the (uncorrelated) additive noise,

$$x_i(n) = \mathbf{h}_i^T \mathbf{s}(n), \quad i = 1, \dots, M,$$

is the received signal,  $\mathbf{h}_i = [h_{i,0} \ h_{i,1} \ \dots \ h_{i,L-1}]^T$  is the  $i$ th channel impulse response,  $\mathbf{s}(n) = [s(n) \ s(n-1) \ \dots \ s(n-L+1)]^T$ , and  $L$  is the length of the channel while  $[\cdot]^T$  denotes the transpose operator. The aim of blind channel identification is to estimate the channels  $\mathbf{h}_i$ ,  $i = 1, \dots, M$  without any *prior* knowledge of  $s(n)$ .

The MCLMS algorithm begins by considering the cross relation [16]

$$y_i(n) * h_j = y_j(n) * h_i, \quad i, j = 1, \dots, M, i \neq j, \quad (2)$$

if the noise is absent. Expressing (2) in vector notation, we obtain

$$\mathbf{y}_i^T(n) \mathbf{h}_j = \mathbf{y}_j^T(n) \mathbf{h}_i, \quad (3)$$

where  $\mathbf{y}_i(n) = [y_i(n) \ y_i(n-1) \ \dots \ y_i(n-L+1)]^T$ . However, in the presence of uncorrelated noise, the above cross relation no longer holds and an error function can be defined as

$$e_{ij}(n) = \begin{cases} \mathbf{y}_i^T(n) \mathbf{h}_j - \mathbf{y}_j^T(n) \mathbf{h}_i, & i \neq j, i, j = 1, \dots, M, \\ 0, & i = j, i, j = 1, \dots, M. \end{cases}$$

Therefore, a cross relation based cost function is obtained as

$$\chi(n) = \sum_{i=1}^{M-1} \sum_{j=i+1}^M \hat{e}_{ij}^2(n), \quad (4)$$

$$\hat{e}_{ij}(n) = \mathbf{y}_i^T(n) \hat{\mathbf{h}}_j(n) - \mathbf{y}_j^T(n) \hat{\mathbf{h}}_i(n). \quad (5)$$

The channel impulse responses can be estimated by minimizing (4). In order to avoid a trivial estimate with all zero elements, a unit-norm constraint is imposed on  $\hat{\mathbf{h}}(n) \triangleq [\hat{\mathbf{h}}_1^T(n) \ \hat{\mathbf{h}}_2^T(n) \ \dots \ \hat{\mathbf{h}}_M^T(n)]^T$  at all time iterations such that the error signal becomes

$$\epsilon_{ij}(n) = \hat{e}_{ij}(n) / \|\hat{\mathbf{h}}(n)\|_2. \quad (6)$$

The corresponding cross relation based cost function becomes

$$J(n) = \sum_{i=1}^{M-1} \sum_{j=i+1}^M \epsilon_{ij}^2(n) = \frac{\chi(n)}{\|\hat{\mathbf{h}}(n)\|_2^2} \quad (7)$$

and minimizing (7) results in the MCLMS algorithm given by [16]

$$\hat{\mathbf{h}}(n+1) = \frac{\hat{\mathbf{h}}(n) - 2\mu[\tilde{\mathbf{R}}(n)\hat{\mathbf{h}}(n) - \chi(n)\hat{\mathbf{h}}(n)]}{\|\hat{\mathbf{h}}(n) - 2\mu[\tilde{\mathbf{R}}(n)\hat{\mathbf{h}}(n) - \chi(n)\hat{\mathbf{h}}(n)]\|_2}, \quad (8)$$

where  $\mu$  is the step-size, and

$$\tilde{\mathbf{R}}_{y_i y_i}(n) = \mathbf{y}_i(n) \mathbf{y}_i^T(n),$$

$$\tilde{\mathbf{R}}(n) = \begin{bmatrix} \sum_{i \neq 1} \tilde{\mathbf{R}}_{y_i y_i}(n) & -\tilde{\mathbf{R}}_{y_2 y_1}(n) & \dots & -\tilde{\mathbf{R}}_{y_M y_1}(n) \\ -\tilde{\mathbf{R}}_{y_1 y_2}(n) & \sum_{i \neq 2} \tilde{\mathbf{R}}_{y_i y_i}(n) & \dots & -\tilde{\mathbf{R}}_{y_M y_2}(n) \\ \vdots & \vdots & \ddots & \vdots \\ -\tilde{\mathbf{R}}_{y_1 y_M}(n) & -\tilde{\mathbf{R}}_{y_2 y_M}(n) & \dots & \sum_{i \neq M} \tilde{\mathbf{R}}_{y_i y_i}(n) \end{bmatrix}. \quad (9)$$

### III. THE IMPROVED MCLMS ALGORITHM

#### A. Analysis of MCLMS in a noisy environment

We now investigate the effect of noise on MCLMS by considering  $v_i(n) \neq 0$ . For clarity of presentation, we consider the un-normalized cost function  $\chi(n)$ , since normalization in (7) only affects the norm of the solution. The error function given by (5) is first expanded as

$$\begin{aligned} \hat{e}_{ij}(n) &= [\mathbf{x}_i(n) + \mathbf{v}_i(n)]^T \hat{\mathbf{h}}_j(n) - [\mathbf{x}_j(n) + \mathbf{v}_j(n)]^T \hat{\mathbf{h}}_i(n) \\ &= \hat{e}_{ij}^x(n) + \hat{e}_{ij}^v(n), \end{aligned} \quad (10)$$

where  $\mathbf{x}_i(n) = [x_i(n) \ x_i(n-1) \ \dots \ x_i(n-L+1)]^T$ ,  $\mathbf{v}_i(n) = [v_i(n) \ v_i(n-1) \ \dots \ v_i(n-L+1)]^T$ ,

$$\hat{e}_{ij}^x(n) = \mathbf{x}_i^T(n) \hat{\mathbf{h}}_j(n) - \mathbf{x}_j^T(n) \hat{\mathbf{h}}_i(n) \quad (11)$$

is the cross relation error due to input signals while

$$\hat{e}_{ij}^v(n) = \mathbf{v}_i^T(n) \hat{\mathbf{h}}_j(n) - \mathbf{v}_j^T(n) \hat{\mathbf{h}}_i(n) \quad (12)$$

is the cross relation error due to noise. Defining

$$\hat{\mathbf{e}}^x(n) = [\hat{e}_{12}^x(n) \ \hat{e}_{13}^x(n) \ \dots \ \hat{e}_{(M-1)M}^x(n)]^T,$$

$$\hat{\mathbf{e}}^v(n) = [\hat{e}_{12}^v(n) \ \hat{e}_{13}^v(n) \ \dots \ \hat{e}_{(M-1)M}^v(n)]^T,$$

$$\begin{aligned} \hat{\mathbf{e}}(n) &= \hat{\mathbf{e}}^x(n) + \hat{\mathbf{e}}^v(n) \\ &= [\hat{e}_{12}(n) \ \hat{e}_{13}(n) \ \dots \ \hat{e}_{(M-1)M}(n)]^T \end{aligned}$$

and employing  $E\{\hat{\mathbf{e}}_x^T(n) \hat{\mathbf{e}}_v(n)\} = 0$ , the expectation of the cost function (4) can be expressed as

$$\begin{aligned} E\{\chi(n)\} &= E\{\hat{\mathbf{e}}^T(n) \hat{\mathbf{e}}(n)\} \\ &= E\{\chi_x(n)\} + E\{\chi_v(n)\}, \end{aligned} \quad (13)$$

where

$$\chi_x(n) = \sum_{i=1}^{M-1} \sum_{j=i+1}^M (\hat{e}_{ij}^x(n))^2, \quad \chi_v(n) = \sum_{i=1}^{M-1} \sum_{j=i+1}^M (\hat{e}_{ij}^v(n))^2.$$

It can therefore be observed that minimizing (13) is equivalent to having  $E\{(\hat{e}_{ij}^x(n))^2\} \rightarrow 0$  and  $E\{(\hat{e}_{ij}^v(n))^2\} \rightarrow 0$ ,

which can subsequently be written as

$$\begin{aligned} & \widehat{\mathbf{h}}_j^T(n) \mathbf{R}_{x_i x_i} \widehat{\mathbf{h}}_j(n) - \widehat{\mathbf{h}}_i^T(n) \mathbf{R}_{x_j x_j} \widehat{\mathbf{h}}_i(n) \rightarrow 0 \\ \implies & \widehat{\mathbf{h}}_i(n) = \mathbf{h}_i, \end{aligned} \quad (14)$$

$$\begin{aligned} & \widehat{\mathbf{h}}_j^T(n) \mathbf{R}_{v_i v_i} \widehat{\mathbf{h}}_j(n) - \widehat{\mathbf{h}}_i^T(n) \mathbf{R}_{v_j v_j} \widehat{\mathbf{h}}_i(n) \rightarrow 0 \\ \implies & \|\widehat{\mathbf{h}}_i(n)\|_2 = \|\widehat{\mathbf{h}}_j(n)\|_2, \end{aligned} \quad (15)$$

where  $\mathbf{R}_{x_i x_i} = E\{\mathbf{x}_i(n)\mathbf{x}_i^T(n)\}$ ,  $\mathbf{R}_{v_i v_i} = E\{\mathbf{v}_i(n)\mathbf{v}_i^T(n)\}$  and we have assumed  $\mathbf{R}_{v_i v_i} = \mathbf{R}_{v_j v_j}$ . We note that while (14) is desirable, (15) may not be satisfied, since  $\|\mathbf{h}_i\|_2 \neq \|\mathbf{h}_j\|_2$  in general. This implies that minimizing the cross relation error due to noise is leading the algorithm to an undesired solution with  $\|\widehat{\mathbf{h}}_i(n)\|_2 = \|\widehat{\mathbf{h}}_j(n)\|_2$ , it is therefore expected that the noise will degrade the convergence performance of MCLMS.

Figure 2 illustrates the convergence performance of MCLMS under different signal-to-noise ratios (SNRs). The normalized projection misalignment (NPM) is adopted to quantify the distance between the estimated and true impulse responses and is defined by [21]

$$\text{NPM}(n) = 20 \log_{10} \|\mathbf{h} - \alpha(n)\widehat{\mathbf{h}}(n)\|_2 / \|\mathbf{h}\|_2, \quad (16)$$

where  $\alpha(n) = [\widehat{\mathbf{h}}^T(n)\widehat{\mathbf{h}}(n)] / [\widehat{\mathbf{h}}^T(n)\widehat{\mathbf{h}}(n)]$  is the projection factor which computes the NPM up to a scaled factor between  $\widehat{\mathbf{h}}(n)$  and  $\mathbf{h}$ . In this illustrative example, we have used the same simulation setup as [16] where  $M = 2$ . A white Gaussian noise (WGN) source signal was first convolved with two third-order randomly generated impulse responses with  $\|\mathbf{h}_1\|_2 = 3.23$  and  $\|\mathbf{h}_2\|_2 = 2.72$ . A WGN was next added to each received signal to achieve SNR =  $\infty$ , 50, 40, 30, 20 and  $-10$  dB. The step-size of MCLMS was fixed as 0.01.

As observed from Fig. 2, when no noise is presented, MCLMS converges towards  $-\infty$  implying that  $\widehat{\mathbf{h}}_i(n)$  converges to  $\mathbf{h}_i$  consistently in a noise free case. It can also be observed from Fig. 2 that the performance reduces with increasing noise as expected. Our simulation revealed that, although  $\|\widehat{\mathbf{h}}_1^T(n) \widehat{\mathbf{h}}_2^T(n)\|_2 = 1$  (due to the unit-norm constraint) for all the SNR conditions, we have  $\|\widehat{\mathbf{h}}_1(n)\|_2 = 0.6781$  and  $\|\widehat{\mathbf{h}}_2(n)\|_2 = 0.735$  for SNR = 50 dB while  $\|\widehat{\mathbf{h}}_1(n)\|_2 = \|\widehat{\mathbf{h}}_2(n)\|_2 = 0.731$  for SNR =  $-10$  dB, which justifies the validity of (15).

### B. The proposed improved MCLMS (IMCLMS) algorithm

We overcome the problem due to cross relation error of noise as described in (15) by proposing to estimate the channels via a joint optimization problem given by

$$\begin{aligned} & \min_{\widehat{\mathbf{h}}(n)} E \left\{ \sum_{i=1}^{M-1} \sum_{j=i+1}^M \widehat{\mathbf{h}}_i^T(n) \mathbf{y}_j(n) \mathbf{y}_j^T(n) \widehat{\mathbf{h}}_i(n) \right\}, \quad (17) \\ \text{s. t. } & E \left\{ \sum_{i=1}^{M-1} \sum_{j=i+1}^M \left[ \mathbf{y}_i^T(n) \widehat{\mathbf{h}}_j(n) - \mathbf{y}_j^T(n) \widehat{\mathbf{h}}_i(n) \right]^2 \right\} = 0, \end{aligned}$$

where  $\widehat{\mathbf{h}}(n) = [\widehat{\mathbf{h}}_1^T(n) \widehat{\mathbf{h}}_2^T(n) \dots \widehat{\mathbf{h}}_M^T(n)]^T$ . To understand why the above joint optimization problem can address the cross

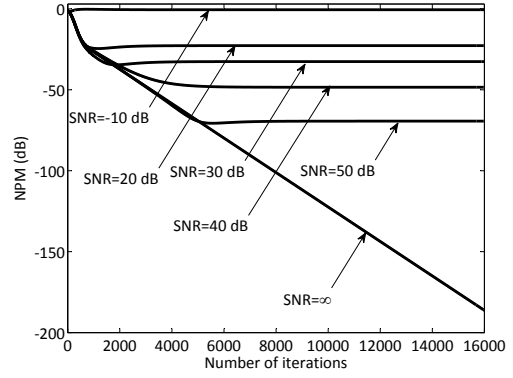


Fig. 2. The NPM performance of MCLMS with various SNRs.

relation error due to noise, we first note that  $\mathbf{R}_{y_i y_i} = \mathbf{R}_{x_i x_i} + \mathbf{R}_{v_i v_i}$ , and therefore (17) can be expressed as

$$\begin{aligned} & \min_{\widehat{\mathbf{h}}(n)} E \left\{ \sum_{i=1}^{M-1} \sum_{j=i+1}^M \widehat{\mathbf{h}}_i^T(n) \mathbf{y}_j(n) \mathbf{y}_j^T(n) \widehat{\mathbf{h}}_i(n) \right\} \\ & = \min_{\widehat{\mathbf{h}}(n)} \sum_{i=1}^{M-1} \sum_{j=i+1}^M \widehat{\mathbf{h}}_i^T(n) [\mathbf{R}_{x_j x_j} + \mathbf{R}_{v_j v_j}] \widehat{\mathbf{h}}_i(n) \end{aligned} \quad (18)$$

subject to the constraint

$$\begin{aligned} & E \left\{ \sum_{i=1}^{M-1} \sum_{j=i+1}^M \left[ \mathbf{y}_i^T(n) \widehat{\mathbf{h}}_j(n) - \mathbf{y}_j^T(n) \widehat{\mathbf{h}}_i(n) \right]^2 \right\} \\ & = \sum_{i=1}^{M-1} \sum_{j=i+1}^M \left[ \widehat{\mathbf{h}}_j^T(n) \mathbf{R}_{x_i x_i} \widehat{\mathbf{h}}_j(n) - \widehat{\mathbf{h}}_i^T(n) \mathbf{R}_{x_j x_j} \widehat{\mathbf{h}}_i(n) \right. \\ & \quad \left. + \widehat{\mathbf{h}}_j^T(n) \mathbf{R}_{v_i v_i} \widehat{\mathbf{h}}_j(n) - \widehat{\mathbf{h}}_i^T(n) \mathbf{R}_{v_j v_j} \widehat{\mathbf{h}}_i(n) \right] = 0. \end{aligned} \quad (19)$$

Exploiting  $\mathbf{R}_{v_i v_i} = \mathbf{R}_{v_j v_j}$ , we minimize (18) and (19) simultaneously and obtain

$$\begin{aligned} & \min_{\widehat{\mathbf{h}}(n)} \sum_{i=1}^{M-1} \sum_{j=i+1}^M \left( \widehat{\mathbf{h}}_i^T(n) [\mathbf{R}_{x_j x_j} + \mathbf{R}_{v_j v_j}] \widehat{\mathbf{h}}_i(n) \right. \\ & \quad \left. + \left[ \widehat{\mathbf{h}}_j^T(n) \mathbf{R}_{x_i x_i} \widehat{\mathbf{h}}_j(n) - \widehat{\mathbf{h}}_i^T(n) \mathbf{R}_{x_j x_j} \widehat{\mathbf{h}}_i(n) \right. \right. \\ & \quad \left. \left. + \widehat{\mathbf{h}}_j^T(n) \mathbf{R}_{v_i v_i} \widehat{\mathbf{h}}_j(n) - \widehat{\mathbf{h}}_i^T(n) \mathbf{R}_{v_j v_j} \widehat{\mathbf{h}}_i(n) \right] \right) \\ & = \min_{\widehat{\mathbf{h}}(n)} \sum_{i=1}^{M-1} \sum_{j=i+1}^M \left( \widehat{\mathbf{h}}_j^T(n) \mathbf{R}_{x_i x_i} \widehat{\mathbf{h}}_j(n) - \widehat{\mathbf{h}}_i^T(n) \mathbf{R}_{x_j x_j} \widehat{\mathbf{h}}_i(n) \right) \\ & \quad + \min_{\widehat{\mathbf{h}}(n)} \sum_{i=1}^{M-1} \sum_{j=i+1}^M \left( \widehat{\mathbf{h}}_j^T(n) \mathbf{R}_{v_i v_i} \widehat{\mathbf{h}}_j(n) + \widehat{\mathbf{h}}_i^T(n) \mathbf{R}_{x_j x_j} \widehat{\mathbf{h}}_i(n) \right). \end{aligned} \quad (20)$$

The above minimization is therefore equivalent to

$$\begin{aligned} & \widehat{\mathbf{h}}_j^T(n) \mathbf{R}_{x_i x_i} \widehat{\mathbf{h}}_j(n) - \widehat{\mathbf{h}}_i^T(n) \mathbf{R}_{x_j x_j} \widehat{\mathbf{h}}_i(n) \rightarrow 0, \\ \implies & \widehat{\mathbf{h}}_i(n) = \mathbf{h}_i, \\ & \widehat{\mathbf{h}}_j^T(n) \mathbf{R}_{v_i v_i} \widehat{\mathbf{h}}_j(n) + \widehat{\mathbf{h}}_i^T(n) \mathbf{R}_{x_j x_j} \widehat{\mathbf{h}}_i(n) \rightarrow 0, \\ \implies & \widehat{\mathbf{h}}_i(n) = \mathbf{0}_{L \times 1} \end{aligned}$$

since  $\mathbf{R}_{x_i x_i}$  and  $\mathbf{R}_{v_j v_j}$  are positive definite matrices. It is important to note that the trivial solution  $\widehat{\mathbf{h}}_i(n) = \mathbf{0}_{L \times 1}$  can

be avoided by the unit-norm constraint as described in (6). It can now be seen that the condition  $\|\hat{\mathbf{h}}_i(n)\|_2 \rightarrow \|\hat{\mathbf{h}}_j(n)\|_2$ , as described in (15), has been avoided and therefore the proposed IMCLMS algorithm based on (17) is expected to achieve robustness against noise.

To derive the update equation, we continue from (17) and the proposed cost function can be constructed as

$$\begin{aligned} \chi_p(n) = & \sum_{i=1}^{M-1} \sum_{j=i+1}^M \left[ \hat{\mathbf{h}}_j^T(n) \mathbf{R}_{y_i y_i} \hat{\mathbf{h}}_j(n) - \hat{\mathbf{h}}_i^T(n) \mathbf{R}_{y_j y_j} \hat{\mathbf{h}}_i(n) \right] \\ & + \sum_{i=1}^{M-1} \sum_{j=i+1}^M \hat{\mathbf{h}}_i^T(n) \mathbf{R}_{y_j y_j} \hat{\mathbf{h}}_i(n), \end{aligned} \quad (21)$$

where the subscript 'p' in  $\chi_p(n)$  denotes for the proposed algorithm. Similar to MCLMS in (7), a modified cost function exploiting the unit-norm constraint can subsequently be obtained as

$$J_p(n) = \chi_p(n) / \|\hat{\mathbf{h}}(n)\|_2^2. \quad (22)$$

It is important to note that the unit-norm normalization is introduced in (22) to avoid the null estimate. The update equation of the proposed IMCLMS is then given by

$$\begin{aligned} \hat{\mathbf{h}}(n+1) &= \hat{\mathbf{h}}(n) - \mu \nabla J_p(n)|_{\mathbf{h}=\hat{\mathbf{h}}(n)} \\ &= \frac{\hat{\mathbf{h}}(n) - \mu [\tilde{\mathbf{R}}(n) - \chi(n) \mathbf{I}_{ML \times ML} - 2M \mathbf{R}_a] \hat{\mathbf{h}}(n)}{\|\hat{\mathbf{h}}(n) - \mu [\tilde{\mathbf{R}}(n) - \chi(n) \mathbf{I}_{ML \times ML} - 2M \mathbf{R}_a] \hat{\mathbf{h}}(n)\|_2} \end{aligned} \quad (23)$$

where  $\mathbf{I}_{ML \times ML}$  is a  $ML \times ML$  identity matrix,

$$\mathbf{R}_a = \begin{bmatrix} \mathbf{R}_{y_1 y_1} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_{y_2 y_2} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{R}_{y_M y_M} \end{bmatrix}_{ML \times ML}, \quad (24)$$

$\chi(n)$  and  $\tilde{\mathbf{R}}(n)$  have been defined in (4) and (9), respectively.

#### IV. SIMULATIONS

In the following simulations, and similar to that of [16], we have used  $M = 2$  and  $\mathbf{h}_1 = [1 - 2 \cos(\pi/10) \ 1]^T$ ,  $\mathbf{h}_2 = [1 - 2 \cos(\pi/5) \ 1]^T$ . In addition, noise is added to the channel observations to achieve SNR of 20 and 12 dB. Following the same simulation setup as [16], a WGN was adopted as the source signal to verify the performance of the algorithms. Monte Carlo simulations using different source signals were carried out for MCLMS and IMCLMS. Their step-sizes were fixed as  $\mu = 0.01$  to ensure the convergence.

Figure 3 illustrates the convergence performance of MCLMS and IMCLMS for SNR = 12 and 20 dB. Each of these plots was averaged across one hundred trials of simulations. As can be seen from Fig. 3, IMCLMS achieves an improvement in steady-state NPM of approximately 5 dB compared to MCLMS when SNR = 20 dB. Similar simulation results can be observed when SNR = 12 dB using the same simulation parameters. It can be seen from Fig. 3 that the proposed IMCLMS algorithm exhibits noise robustness and gains approximately 3 dB improvement in steady-state performance compared to MCLMS when SNR = 12 dB. We

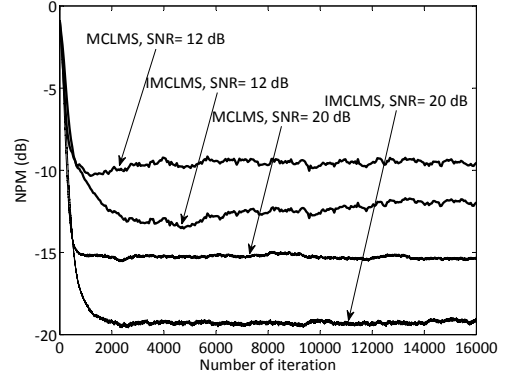


Fig. 3. The NPM performance of MCLMS and proposed IMCLMS algorithms using Monte Carlo simulations while SNR = 12 and 20 dB and  $N = 100$ .

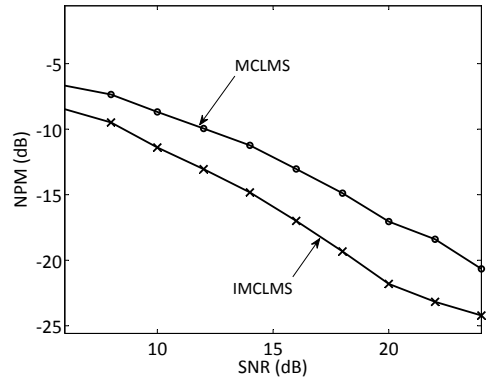


Fig. 4. Variation of steady-state NPM for MCLMS and the proposed IMCLMS algorithms using Monte Carlo simulations with 100 trials from SNR = 6 to 24 dB.

also note that at a high noise level of SNR = 12 dB, both MCLMS and IMCLMS suffer from a higher gradient noise as expected. Figure 4 illustrates the variation of steady-state NPM with different SNRs averaged over 100 trials. As can be observed from Fig. 4, the proposed IMCLMS algorithm achieves noise robustness and consistently outperforms MCLMS by achieving lower steady-state NPM values. These results justify the effectiveness of proposed joint optimization.

#### V. CONCLUSION

In this paper, the cost function of the MCLMS algorithm has been analyzed in a noisy environment and we showed that the additive noise can mis-lead the adaptive algorithm to a trivial solution. This trivial solution requires the estimated channels to have equal  $l_2$ -norm which may not be true for real channels. It has been shown in the proposed IMCLMS algorithm that minimizing the revised cost function can mitigate the cross relation error due to noise thus achieving noise robustness. Monte Carlo simulations under different SNRs have verified that the proposed IMCLMS algorithm is more robust to noise compared to the existing MCLMS algorithm.

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