# Design of a Reputation System Based on Dynamic Coalition Formation 

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#### Abstract

Reputation systems bear some challenging problems where buyers have different subjectivity in evaluating their experience with sellers and they may not have incentives to share their experience. In this paper, we propose a novel reputation system based on dynamic coalition formation where buyers with similar subjectivity and rich experience will be awarded virtual credits for helping others find trustworthy sellers to successfully conduct business. Our theoretical analysis confirms that the coalitions formed in this way are stable.


## 1 Introduction

In a multiagent-based e-commerce environment, buying agents and selling agents involved in monetary transactions have asymmetric information. Sellers know more about their products, while buyers never fully know whether the products satisfy them until receiving the products for which they have paid. On another hand, buyers' satisfaction is very important for the success of e-commerce. In addition, buyers are always, to some degree, uncertain about the future behaviors of sellers. Thus, the main motivations for introducing trust and reputation systems into e-commerce are to: i) mitigate such information asymmetry problem; ii) help buyers find trustworthy sellers to conduct satisfactory transactions; and iii) decrease the uncertainty of buyers about sellers' future behaviors.

Compared to trust models where only buyers' own experience with sellers is taken into account when modeling the trustworthiness of sellers, reputation systems are more useful especially for the new buyers that do not have much personal experience with sellers, because in reputation systems, buyers share their experience/information about sellers with other buyers [3]. However, reputation systems also face two challenging problems. One is the subjectivity problem where the information about sellers shared by other buyers is their own subjective evaluation about the products delivered by the sellers and may be biased. Another is the incentive problem in the sense that buyers may not have incentives to share their information with others.

To address the two problems, in this paper, we design a dynamic coalition based reputation system. In our system, we introduce the notion of virtual credits to provide buyers with incentives to share their information about sellers. A
novel credit allocation algorithm is proposed to allocate credits to coalition members based on the quantified subjective difference among them and the amount of information they provided. The result is that buyers with similar subjectivity will form a coalition. Well-experienced buyers will join coalitions to share their information about sellers for receiving virtual credits. Less-experienced buyers can join coalitions to gain information from buyers that have the similar subjectivity. The coalitions formed in our system are also proven to be stable.

## 2 Uncertainty and Subjectivity in Trust Modeling

Feedbacks from buyers that have ever been directly involved into transactions with a seller $s$ compose the evidence space for the trustworthiness of the seller. In the evidence space, a buyer $i$ has $\left(P_{i}^{s}, N_{i}^{s}\right)$ to express its direct experience with the seller $s$, where $P_{i}^{s} \in \mathbb{N}$ is the number of satisfactory transactions and $N_{i}^{s} \in$ $\mathbb{N}$ is the number of dissatisfactory transactions. According to the DempsterShafer theory (DST) and Jøsang's trust metric [1], the evidence space can be mapped to a trust space $T_{i}^{s}(b, d, u)$ as follows: $b_{i}^{s}=\frac{P_{i}^{s}}{P_{i}^{s}+N_{i}^{s}+2}, d_{i}^{s}=\frac{N_{i}^{s}}{P_{i}^{s}+N_{i}^{s}+2}$, $u_{i}^{s}=\frac{2}{P_{i}^{s}+N_{i}^{s}+2}$, where $b_{i}^{s}, d_{i}^{s}$ and $u_{i}^{s}$ represent belief, disbelief and uncertainty parameters, respectively. Here, $b_{i}^{s}$ represents the probability that the proposition that the seller $s$ is trustworthy is true, and $d_{i}^{s}$ represents the probability of the proposition is false. Note that $b_{i}^{s}+d_{i}^{s}+u_{i}^{s}=1$ and $b_{i}^{s} \in[0,1), d_{i}^{s} \in[0,1)$, $u_{i}^{s} \in(0,1]$. We can then define the amount of information $E_{i}^{s}$ the buyer $i$ has about the seller $s$ and link $E_{i}^{s}$ to the uncertainty $u_{i}^{s}$ as follows:
Definition 1. Amount of Information $E_{i}^{s}$ : Given that a buyer $i$ has done $P_{i}^{s}+$ $N_{i}^{s}$ transactions with a seller $s$, the amount of information $i$ has about $s, E_{i}^{s}$, is defined as $\frac{P_{i}^{s}+N_{i}^{s}+2}{2}$. Then, $E_{i}^{s}=\frac{1}{u_{i}^{s}}$.

Given two buyers' modelings of the same seller in the trust space, we can also define their subjective difference in their trust modelings of the same seller:

Definition 2. Subjective Difference: Given the two respective trust tuples that the two buyers $i$ and $j$ have of the same seller, $T_{i}^{s}\left(b_{i}^{s}, d_{i}^{s}, u_{i}^{s}\right)$ and $T_{j}^{s}\left(b_{j}^{s}, d_{j}^{s}, u_{j}^{s}\right)$, the subjective difference of the buyers $i$ and $j$ regarding the seller $s$ is defined as

$$
\begin{equation*}
D_{i, j}^{s}=\frac{1}{2}\left(\frac{\left|b_{i}^{s} u_{j}^{s}-b_{j}^{s} u_{i}^{s}\right|}{b_{i}^{s} u_{j}^{s}+b_{j}^{s} u_{i}^{s}}+\frac{\left|d_{i}^{s} u_{j}^{s}-d_{j}^{s} u_{i}^{s}\right|}{d_{i}^{s} u_{j}^{s}+d_{j}^{s} u_{i}^{s}}\right) \tag{1}
\end{equation*}
$$

where $D_{i, j}^{s} \in[0,1), u_{i}^{s} \neq 1$ and $u_{j}^{s} \neq 1$. Then, the subjective difference of $i$ and $j$ is $D_{i, j}=\frac{\sum_{s \in S} D_{i, j}^{s}}{|S|}$, where $S$ is the set of sellers $i$ and $j$ both have encountered with and $|S|$ represents the number of sellers in $S$.

## 3 Dynamic Coalition Formation

To address the problems of subjectivity and incentives in reputation systems, we propose a credit allocation algorithm for dynamic coalition formation.

### 3.1 Model Overview

In a typical multiagent-based electronic marketplace, buying agents conduct business with selling agents. After the transactions are finished, buyers evaluate whether the transactions are successful. In our work, we assume that the evaluation results are binary, either successful or unsuccessful. These are precisely the experience about sellers that the buyers will later on share with other buyers in the system. In the e-marketplace, we assume that sellers sell the similar kinds of products. For sellers selling a different type of products, a different set of coalitions will be formed regarding those sellers. By this simplified assumption, we do not deal with the transformation from buyers' subjectivity on one type of sellers to that on another type of sellers. Because of this assumption, we can also assume that each buyer will be able to gain the same amount of profit if its transaction with a seller is successful, which is denoted as $\alpha \in \mathbb{R}_{+}$. But, if the transaction is unsuccessful, the buyer will lose a certain amount of profit denoted as $\beta \in \mathbb{R}_{+}$. For the purpose of numerical analysis, we also assume that every buyer has the same amount of need for purchasing products, which is represented by transaction rate, the number of transactions the buyer will conduct with sellers over a fixed period of time, denoted as $r \in \mathbb{N}$. Based on this assumption, buyers in the system have different amount of transaction history or personal experience with sellers, only because they participate in the system for different time periods. The longer the buyers participate in the system, the more experience they will be able to gain. Therefore, if the success rate of transactions is $p_{i} \in[0,1]$, then the profit $F_{i} \in \mathbb{R}$ a buyer is able to gain within a specific time period $t_{0}$ can be calculated as:

$$
\begin{equation*}
F_{i}=r t_{0}\left(p_{i} \alpha-\left(1-p_{i}\right) \beta\right) \tag{2}
\end{equation*}
$$

In our system, buyers autonomously form coalitions. Within each coalition, buyers (coalition members) can share their experience (information about sellers) with other members. To create incentives for buyers to share their experience with their coalition members, the buyers will be rewarded with virtual points if the transactions of their members with sellers are successful [2]. The number of credits rewarded to the buyers in the coalition is proportional to the profit gained by the members from successfully conducting transactions with the sellers. For the purpose of simplicity, we make the number of credits after a successful transaction with a seller equal to the amount of profit gained from the transaction, which is $\alpha$. These credits can be redeemed by buyers for discounts from sellers or privileges in the system, therefore, the attitude of buyers towards the credits is positive, i.e. the more credits the better. We assume here that a buyer's utility towards virtual credits is discounted by a constant $\theta \in(0,1)$ set for the system.

Thus, the utility of a buyer $i$ has two parts, the profit gained by successfully conducting transactions with sellers and the virtual credits gained by sharing its experience with other coalition members, formalized as follows:

$$
\begin{equation*}
U_{i}=F_{i}+\theta \sum_{j \neq i} R_{j}^{i}, \tag{3}
\end{equation*}
$$

where $R_{j}^{i} \in \mathbb{R}_{+}$is the virtual credits rewarded to buyer $i$ due to buyer $j$ 's successful transactions with sellers, and $F_{i}$ is calculated using Equation (2).

In the initiation stage of our coalition formation, each buyer is a singleton coalition. It evaluates the subjective difference with other buyers. Buyers with similar subjectivity will merge to form a coalition for two reasons. One reason is to increase the success rate of conducting business with sellers so that their transaction profit $F$ will be increased accordingly. Another reason is to gain more virtual credits because their information about sellers will be more valuable to others with similar subjectivity. The number of virtual credits awarded to the buyers is determined partially by the factor of the subjectivity difference. More details about the virtual credits allocation algorithm will be presented in the next section. When both the transaction profit and virtual points are increased, the buyers' utility will also be increased, according to Equation (3).

When a new buyer joins the system, every coalition is presented to the buyer as a coalition center (defined in the next section) and the amount of information of this coalition. The new buyer can first randomly join in one coalition. One buyer can take part in only one coalition at one time. It is possible that the random choice was wrong, but later on when the buyer gains more personal experience with sellers, the buyer will be able to switch to a correct coalition where it shares the similar subjectivity with other members in the coalition.

### 3.2 Credit Allocation Algorithm

Virtual credits assigned to a coalition when a buyer in the coalition conducts a successful transaction with a seller will be allocated to other coalition members, depending on how much their information about the seller contributes to this successful transaction. It is affected by both the subjectivity of the coalition members regarding the seller and how much information the coalition members have about the seller. The subjectivity of a coalition member is measured as the subjective difference between the member and the average opinion of all members in the coalition. Thus, we first define the center of a coalition as the average opinion of all the members in the coalition, as follows:
Definition 3. Coalition Center c: In a coalition C, for any given seller s with which some members have conducted transactions, let $P_{c}^{s}=\frac{\sum_{i \in c}\left(P_{i}^{s}\right)}{m}$ be the average number of satisfactory transactions between the members and $s, N_{c}^{s}=$ $\frac{\sum_{i \in c}\left(N_{i}^{s}\right)}{m}$ be the average number of unsatisfied transactions and $m$ is the number of such members. The coalition center c regarding $s$ is defined as $T_{c}^{s}\left(b_{c}^{s}, d_{c}^{s}, u_{c}^{s}\right)$, where $b_{c}^{s}=\frac{P_{c}^{s}}{P_{c}^{s}+N_{c}^{s}+2}, d_{c}^{s}=\frac{N_{c}^{s}}{P_{c}^{s}+N_{c}^{s}+2}$ and $u_{c}^{s}=\frac{2}{P_{c}^{s}+N_{c}^{s}+2}$. The coalition center $c$ is then a collection of $T_{c}^{s}$ for each $s \in S$ with which at least one member in $c$ has interacted.

Given the center $c$, we then calculate the discounted amount of information buyer $i$ has about the seller $s$ as follows:

$$
\begin{equation*}
\hat{E}_{i}^{s}=\left(1-D_{i, c}\right) \times E_{i}^{s}, \tag{4}
\end{equation*}
$$

where $D_{i, c}$ is the subjective difference between the center and buyer $i$ (see Definition (2), and $E_{i}^{s}$ is the amount of information buyer $i$ has about the seller (see Definition (1). The detailed credit allocation rule is described in Algorithm 1, The number of credits allocated to a buyer is then proportional to the discounted amount of information it contributes to the coalition. If its subjectivity is similar to the coalition's average opinion, its information will be less discounted.

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Alg. 1 : Credit Allocation Rule
Input : \(C\), the coalition formed by a number of buyers;
        \(e\), a transaction conducted by a member \(j\) with a seller;
        \(\alpha\), profit gained by the member \(j\) from transaction \(e\);
if \(e\) is successful then
        foreach \(i\) in coalition \(C\) and \(i \neq j\) do
            \(R_{j}^{i}=\frac{\hat{E}_{i}^{s}}{\sum_{l \neq j} \hat{E}_{l}^{s}} \alpha ; \quad / /\) credits allocated to each member other than \(j\)
    \(R_{j}^{j}=0 ; \quad / /\) no credit is allocated to \(j\) itself
else
    foreach \(i\) in coalition \(C\) do
        \(R_{j}^{i}=0 ;\)
```


## 4 Stability Analysis and Proof

Stability is an important property for dynamic coalition formation. We analyze and prove that the coalitions formed based on our proposed credit allocation rule (Algorithm (1) are stable, by proving that they are split-proof and merge-proof.

### 4.1 Analysis

According to Equation (3), a buyer's utility has two parts, its profit of conducting successful transactions and the virtual credits gained by sharing its experience with other coalition members. When the buyer has successfully conducted a transaction with a seller, a certain number of virtual credits will be awarded to other coalition members. In this case, we can transfer the profit part of the buyer's utility to the number of credits awarded to other coalition members because of the buyer's successful transactions with sellers. We perform this transformation mainly for the purpose of stability proof in the next section. According to Equation (2), Equation (3) and the credit allocation rule in Algorithm 1, Equation (3) can then be further changed to:

$$
\begin{equation*}
U_{i}=\frac{\alpha+\beta}{\alpha} r t_{0} p_{i} \alpha+\theta \sum_{s \in S} \sum_{j \neq i} \frac{\hat{E}_{i}^{s}}{\sum_{l \neq j} \hat{E}_{l}^{s}} R_{j}^{s}-r t_{0} \beta \tag{5}
\end{equation*}
$$

where $r t_{0} p_{i} \alpha$ is the number of credits awarded to other members because of buyer $i$ 's successful transactions with sellers, and $\sum_{s \in S} \sum_{j \neq i} \frac{\hat{E}_{i}^{s}}{\sum_{l \neq j} \hat{E}_{l}^{s}} R_{j}^{s}$ is the
number of credits buyer $i$ receives from successful transactions conducted by other coalition members.

In Equation (5), as $\alpha, \beta, r$ and $t_{0}$ are fixed values, the buyer $i$ 's probability of conducting successful transactions with sellers, $p_{i}$ is crucial to the buyer's utility. If $p_{i}$ is higher, the buyer is likely able to gain larger utility. This success probability is in fact affected by the total amount information the buyer has about sellers, including the buyer's own information and the information shared by other coalition members. We denote it as a function $p(E)$ where $E$ is the total amount of information about sellers, and assume that $p(E)$ is an increasing and concave function with the upper boundary of 1 . When there is little information about sellers, gaining more information will help a lot in increasing the probability of conducting successful transactions. But, when there is already a lot of information about sellers and the probability of conducting successful transactions is already high, gaining more information will not help much in increasing the probability of conducting successful transactions.

Based on the amount of information/experience about sellers a buying agent contributes to its coalition, we classify buyers into three types: senior, common and junior, defined as follows:

Definition 4. Given a coalition $C$ with $m \in \mathbb{N} \geq 2$ members/buyers and the center $c$, for any buyer $i \in C$, if $i$ meets condition $1: \frac{\hat{E}_{i}^{s}}{E_{i}^{s}+\sum_{l \neq i} \hat{E}_{l}^{s}} \geq \frac{1}{m-1}$, then buyer $i$ is a senior buyer; if $i$ meets condition 2: $\frac{\hat{E}_{i}^{s}}{E_{i}^{s}+\sum_{l \neq i} \hat{E}_{l}^{s}}<\frac{1}{m-1}$ and $\frac{E_{i}^{s}}{\sum_{l \in C} \hat{E}_{l}^{s}}>\frac{1}{m}$, then buyer $i$ is a common buyer; if $i$ meets condition 3: $\frac{E_{i}}{\sum_{l \in C} E_{l}^{c}} \leq \frac{1}{m}$, then buyer $i$ is a junior buyer, where $s \in S$ and $S$ is the set of common sellers all the members ever have ever interacted with.

According to the definition, a senior buyer is well experienced and generally has large amount of information about sellers. Its probability of conducting successful transactions is already high, and gaining more information by joining a coalition will not increase much the probability (because of the property of the probability function $p(E)$ ). Thus, the senior buyer's main purpose of joining a coalition is to gain more virtual credits in order to increase its utility. Indeed, the senior buyer's rich information about sellers will allow it to receive a lot of credits according to our credit allocation rule. On another hand, a junior buyer does not have much experience with sellers. Its little information will not bring many virtual credits to itself. Thus, its main purpose of joining a coalition is to increase its probability of conducting successful transactions with sellers by utilizing information about sellers shared by other buyers (mostly common and senior buyers), to increase its utility. All in all, we classify buying agents into the three types mainly because senior and junior buyers have different purposes for joining or leaving coalitions. In the next section, we will separately discuss their behaviors when proving the stability of our dynamic coalition formation.

### 4.2 Stability Proof

We first describe the stable status of our system and provide the properties associated with the stable status. Given a partition $\mathcal{P}=\left\{C_{1}, \ldots, C_{n}\right\}$ of $N$ (the set of all buyers in the system) and any two coalitions $C$ (with the center $c$ ) and $C^{\prime}$ (with the center $c^{\prime}$ ) in $P$, when our system is in the stable stage, the following three properties hold.
(P1) Disconnection: Defining $\tau_{c} \in(0,1]$ as the the radius of the coalition $C$, we have $\max _{i \in C} D_{i, c}<\tau_{c}$, meaning that the subjective difference between any buyer in the coalition and the center should be smaller than the radius. Also, the subjective difference between the centers of the any given two coalitions $C$ and $C^{\prime}$ should be larger than the two times of the maximum radius of these two coalitions, i.e. $D_{c, c^{\prime}}>2 \times \max \left\{\tau_{c}, \tau_{c^{\prime}}\right\}$;
(P2) Existence: In each coalition, there should be some senior buyers that have fairly large amount of information about sellers;
(P3) Equality: Given any junior buyer $i(i \in C)$ and any junior buyer $j\left(j \in C^{\prime}\right)$, their probabilities of successfully conducting transactions with sellers are similar and approach 1, i.e. $p_{i} \approx p_{j} \rightarrow 1$.

When the system evolves for a sufficiently long period of time and reaches the stable stage, the buyers that share the similar subjectivity will form a coalition because only those buyers with the similar subjectivity can provide each other with useful information about their common sellers. In other words, different coalitions will have different subjectivity towards sellers. This gives us the first property (disconnection), meaning that there is sufficient difference in subjectivity between any two coalitions so that buyers do not switch from one to another. Also, in order for a coalition to exist, the junior buyers should be able to gain information about sellers from the senior buyers to benefit from forming coalitions. Thus, in a coalition, there should exist some senior buyers that can provide information to other members for them to successfully conduct transactions with sellers, which is the second property (existence). Based on the property of existence, which expresses that it is reasonable to say that some buyers will become well experienced and gain much information about sellers to become senior members in each coalition, it is safe to assume the property of equality where junior buyers in different coalitions have the similar probability of successfully conducting transactions with sellers by gaining sufficient information from senior buyers in their coalitions, and the probability of success approaches 1. In the rest of this section, we base on the properties summarized above for the stable status of our system to theoretically prove that the coalitions formed in our system are both split-proof and merge-proof and thus stable.

Proposition 1. Given a partition $\mathcal{P}=\left\{C_{1}, \ldots, C_{n}\right\}$ of $N$ buyers (the set of all buyers in the system) that has the three properties: disconnection, existence and equality, in each coalition $C$ with coalition center $c$, any senior buyer $i$ would gain more credits than the credits $R_{i}^{s}$ generated due to buyer $i$ 's successful transactions, where $s \in S$.

Proof. Without losing generality, we assume there are $m$ buyers in coalition $C$. Since buyer $i$ is a senior, $i$ 's contributed personal experience/information $\hat{E}_{i}^{s}$ should take a larger proportion than the buyers that are not seniors. According to the definition of senior agent in Definition $4, \frac{\hat{E}_{i}^{s}}{E_{i}^{s}+\sum_{l \neq i} \hat{E}_{l}^{s}} \geq \frac{1}{m-1}$ holds for any $s \in S$. Replacing $E_{i}^{s}$ by $\hat{E}_{i}^{s}$ using Equation (4), we derive $\frac{\hat{E}_{i}^{s}}{\sum_{l \in C} \hat{E}_{l}^{s}} \geq \frac{1}{m-1-\frac{D_{i, c}}{1-D_{i, c}}}$.

The disconnection property indicates that $\max _{i \in C} D_{i, c}<\tau_{c}$ and the subjective difference between any two coalitions $C$ and $C^{\prime}$ is larger than $2 \times \max \left\{\tau, \tau^{\prime}\right\}$. Since the upper boundary of subjective difference in Definition 1 is $1, \tau$ should be smaller than $\frac{1}{2}$. Therefore, $\frac{\hat{E}_{i}^{s}}{\sum_{l \in C} \hat{E}_{l}^{s}}>\frac{1}{m-1}$. According to the credit allocation rule in Algorithm 1 the number of credits allocated to $i$ due to the successful transactions conducted by any other agent $j$ in coalition $C$ in a certain period of time $t_{0}$ can be formalized as follows: $R_{j}^{i}=\frac{\hat{E}_{i}^{s}}{\sum_{l \neq j} \hat{E}_{l}^{s}} p_{j} \alpha r t_{0}$. The equality property shows that $p_{i} \approx p_{j} \rightarrow 1$. Then, we can obtain: $R_{j}^{i}=\frac{\hat{E}_{i}^{s}}{\sum_{l \neq j} \hat{E}_{l}^{s}} p_{j} \alpha r t_{0}>$ $\frac{\hat{E}_{i}^{s}}{\sum_{l \in C} \hat{E}_{l}^{s}} p_{j} \alpha r t_{0}>\frac{1}{m-1} p_{i} \alpha r t_{0}=\frac{1}{m-1} R_{i}^{s}$.

Buyer $i$ can gain credits from the successful transactions conducted by $m-1$ agents in $C$ (excluding $i$ ). Thus, the total number of credits that buyer $i$ is able to obtain $R(i)$, and $R(i)=\sum_{s \in S} \sum_{l \in C, j \neq i} \frac{\hat{E}_{i}^{s}}{\sum_{l \neq j} \hat{E}_{l}^{s}} p_{j} \alpha r t_{0}>\sum_{s \in S} R_{i}^{s}$ holds.

Theorem 1. Given a partition $\mathcal{P}=\left\{C_{1}, \ldots, C_{n}\right\}$ having the three properties: disconnection, existence and equality, any coalition $C$ in $\mathcal{P}$ is split-proof.

Proof. According to the analysis of stability, a partition is split-proof if for each group of agents $A$ in coalition $C$, there exists at least one agent whose utility in $A$ is smaller than that in $C$. We will analyze the behavior of each type of buyers (junior, common and senior) in coalition $C$.

For a junior buyer $i$ in the coalition $C$ with the center $c$, according to our analysis in Section 4.1 its main purpose of joining coalition $C$ is to increase the probability of successfully conducting transactions with sellers by gaining information about sellers from senior buyers in the coalition. Thus, it will choose a coalition that maximizes $\left(1-D_{i, c}\right) \sum_{l \neq i} \hat{E}_{l}^{s}$. If the junior agent $i$ splits out, the total amount of available information in the new coalition will decrease. This will further decrease $i$ 's utility. Therefore junior buyers do not have incentives to split out from coalition $C$ with any group of other buyers.

For a senior buyer $j$ in the coalition $C$, its main purpose of joining $C$ is to obtain more credits due to other members' successful transactions with sellers. Suppose that some of the seniors in coalition $C$ split out to form a new coalition $A$. Because the seniors have the similar amount of information about their common sellers, the number of credits generated by them is similar. Thus, the number of credits received by them will also be similar to that generated by them when those senors splits out as $A$. However, according to Proposition 1, those seniors can gain more credits than that generated by them. These seniors should be able to gain more credits in coalition $C$ than $A$. In the case where
some seniors have more information than other seniors, those seniors with less information will gain less credits in $A$ than $C$. Thus, senior buyers do not have incentives to split out to form a new coalition with other seniors.

For a common buyer $k$ in the coalition $C$, it has some amount of experience, which is less than that of a senior buyer but more than that of a junior buyer. It can also be allocated some number of credits. Some of the common buyers may prefer to gain more information about sellers. These buyers do not have the incentive to split out, which is similar to junior buyers' behavior analyzed earlier. Some other common buyers may prefer to increase credits and want to split out with seniors. But, due to their less amount of experience about sellers compared to seniors, they will be allocated with even less credits than that when they are in coalition $C$, according to the credit allocation rule.

In conclusion, no group of buying agents splitting out from $C$ to form a new coalition $A$ can guarantee that each of the buyers in $A$ can gain more utility. Our dynamic coalition formation is proven to be split-proof.

Theorem 2. Given a partition $\mathcal{P}$ having the three properties: disconnection, existence and equality, any pair of coalitions $C$ and $C^{\prime}$ in $\mathcal{P}$ is merge-proof.

Proof. According to the analysis of stability, the pair of coalitions $C$ and $C^{\prime}$ is merge-proof if given any group of buyers $A$ from both the two coalitions, not all buyers in $A$ can gain more credits than in $C$ or $C^{\prime}$. We prove this by analyzing the behaviors of each type of buyers.

For any junior buyer $i$ in the coalition $C$, its purpose of joining a coalition is to gain more information about sellers. Therefore, it prefers to merge with a group of buyers that $i$ has less subjective difference with but that have more information about sellers. According to the equality property, junior buyer $i$ in coalition $C$ with center $c$ and another junior buyer $j$ in coalition $C^{\prime}$ with center $c^{\prime}$ can both gain sufficient amount of information about sellers in their respective coalition, therefore, $\left(1-D_{i, c}\right) \sum_{l \in C, l \neq i} \hat{E}_{l}^{s}=\left(1-D_{j, c^{\prime}}\right) \sum_{l \in C^{\prime}, l \neq i} \hat{E}_{l}^{s^{\prime}}$. The disconnection property indicates that $D_{i, c}<\tau_{c}, D_{j, c^{\prime}}<\tau_{c^{\prime}}$ and $D_{c, c^{\prime}}>2 \times \max \left\{\tau_{c}, \tau_{c^{\prime}}\right\}$. Thus, we can derive $D_{i, c^{\prime}}>\tau_{c^{\prime}}>D_{j, c^{\prime}}$ and $\left(1-D_{i, c^{\prime}}\right) \sum_{l \in C^{\prime}, l \neq i} \hat{E}_{l}^{s^{\prime}}<(1-$ $\left.D_{j, c^{\prime}}\right) \sum_{l \in C^{\prime}, l \neq i} \hat{E}_{l}^{s^{\prime}}=\left(1-D_{i, c}\right) \sum_{l \in C, l \neq i} \hat{E}_{l}^{s}$, meaning that the amount of information $i$ can gain in coalition $C^{\prime}$ will be less than that gained in coalition $C$. Junior buyers do not have incentives to merge with other coalitions.

For any senior buyer $j$ in $C$, the subjective difference between the agent $j$ with any group of agents from another coalition $C$ is larger than $\tau$ ( $C$ 's radius), making $j$ 's information less useful. In consequence, the number of credits $j$ can receive by joining coalition $C^{\prime}$ will be smaller than that in $C$. Therefore, the seniors do not have incentives to merge with buyers from any other coalition.

For a common buyer $k$ in $C$, after merging with buyers from another coalition $C^{\prime}$, either its probability of successfully conducting transactions with sellers or the number of credits it can receive may be decreased. Based on the above analysis, our dynamic coalition is also merge-proof.

From Theorems 1 and 2, we can conclude that our dynamic coalition is stable.

## 5 Conclusion and Future Work

In this paper, we design a reputation system based on dynamic coalition formation. A credit allocation algorithm is also proposed to elicit buying agents to share their personal experience/information about selling agents. In this system, buyer with different subjectivity will form disconnected coalitions. And, we theoretically prove that the coalitions formed in this way are stable. The results of our work address the two fundamental and important problems of existing reputation systems, subjectivity and incentives for sharing experience.

In our current work, we make some assumptions for the purpose of simplifying the quantitative and theoretical analysis of agents' behaviors in the system. For future work, we will relieve these assumptions in our experimental analysis to more extensively evaluate the effectiveness of our system.

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