

Coalition Formation Game Based Reputation System

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Abstract. In e-marketplaces, reputation systems are helpful for modeling the trustworthiness of sellers, especially when buyers do not have much personal experience with the sellers. However, reputation systems bear a challenging problem: the *subjectivity problem*, where buyers have different subjectivity in providing ratings about the same seller. Such subjectivity difference impedes the validity of the reputation systems. In this paper, we propose a theoretical model based on coalition formation game to avoid the subjectivity problem. We also construct a specific coalition formation game based reputation system with a *proportional allocation algorithm*. In the proposed system, buyers contribute to building reputation systems within their coalitions and benefit from the allocated utility which is created by the reputation systems. Our theoretical analysis shows that buyers with the same subjectivity have incentives to form one coalition to avoid the subjectivity problem, if specific conditions are satisfied.

Keywords: Reputation Systems, Coalition Formation Game, E-marketplaces

1 Introduction

An electronic marketplace provides opportunities for conducting business via electronic channels usually an Internet based platform, such as eBay and Amazon [8]. E-marketplaces bear the problem that buyers and sellers have asymmetric information [3]. Sellers know more about their products and behavior, whereas buyers seldom fully know whether their transactions are satisfactory until they receive the products. Buyers may receive products under a bad delivery service, e.g. the deferred product delivery. Buyers may also receive lower quality products compared with those promised by sellers. Therefore, a buyer in e-marketplaces is not as easy to experience a satisfactory transaction as that in real shops. Reputation systems then emerge to address this issue. Reputation systems collect ratings from buyers about sellers and aggregate them as the reputation scores of the sellers. Sellers' behavior then can be predicted by reputation systems. As a result, buyers can know more about sellers before conducting transactions through the reputation of sellers. Such information asymmetry is effectively mitigated.

In reputation systems, it is observed that buyers have different subjectivity. Buyers' different subjectivity has shown that buyers have different preferences

or evaluation criteria in providing ratings about sellers. It raises a problem in aggregating those ratings together. The reputation calculated from ratings provided by buyers with one type of subjectivity may not be useful for buyers with another type of subjectivity. For example, the same product is received by buyers in five days. Some buyers may provide a positive rating for this transaction, because five days is acceptable to them. However, other buyers may provide a negative rating for the same transaction, because five days is too long to be accepted by them. This problem in reputation systems is named as the *subjectivity problem*, which has been pointed out in [2, 10, 1].

In order to cope with the subjectivity problem, a few reputation systems [7, 6, 11, 9] have been proposed. There are two main solutions in addressing this problem. The first solution is to allow buyers to express their experience using objective ontology [7]. It requires buyers to understand the ontology and take efforts to represent their experience clearly using the ontology. The other solution is that a buyer evaluates the subjectivity difference with other buyers before using the ratings provided by them. Each buyer has to build the reputation of sellers, even similar buyers hold the similar reputation scores for the same sellers. We will introduce these approaches in detail.

We propose a new solution to address the same problem. A set of buyers with the same subjectivity can form a group which is called a *coalition*. In each coalition, the reputation of sellers is built based on the ratings provided by the buyers of this coalition. If buyers in a coalition have the same subjectivity, then the reputation system will not suffer from the subjectivity problem. Therefore, how buyers form coalitions becomes crucial in our approach. Buyers in e-marketplaces are self-interested. Game theory provides a theoretical support in analyzing the interactions between self-interested individuals. In addition, buyers can achieve more utility through forming coalitions, because the reputation of sellers built within the coalitions assists buyers in choosing more suitable sellers. This motivation of forming coalitions coincides with coalition formation games in game theory. Therefore, we design a Coalition Formation Game (CFG) based reputation system in this paper to address the subjectivity problem.

In the proposed CFG based reputation system, buyers with the same subjectivity play a coalition formation game. Reputation systems are constructed within coalitions by aggregating the ratings provided by the members of the coalition. The utility created by the reputation system is *coalition value*. The coalition value is then allocated among buyers by a *proportional allocation algorithm*. We then theoretically analyze the coalition structure and buyer strategies. As we desire that buyers with the same subjectivity type will form one coalition, which is proved as the *optimal coalition structure*, our analysis focuses on this coalition structure. We prove that the optimal coalition structure can be a *strict core partition* and buyers have incentives to form the desired coalition structure, if specific condition is satisfied. Therefore, the proposed reputation system in this paper can address the subjectivity problem.

2 Related Work

In current literature of reputation systems, two types of solutions have been proposed to address the subjectivity problem in e-marketplaces. In the first type of solutions [7], buyers objectively express their experience using an ontology [7]. The ontology is designed by experts, which consists of the fundamental concepts and properties (such as demand, commitment and experience) for a specific domain. By using ontology, buyer's experience is represented as objective facts. These objective facts can then be interpreted by other buyers according to their own preference or evaluation criteria. Thus, this approach can get avoid of the subjectivity problem. The shortcoming of this approach is that it requires expert's effort to build an ontology and buyer's effort to clearly express their experience using the ontology. Since the ontology is domain-dependent, it is complex and costly to design an ontology for every domain. On the other hand, buyers have different ability in learning the ontology. Even given an ontology, it is still difficult to make sure that all the buyers are able to understand the ontology and to express their experience clearly.

In the second type of solutions, the subjectivity problem is mitigated as subjective ratings are evaluated differently by buyers according to their subjectivity. In [6], buyers re-interpret other buyer's ratings through learning others' evaluation functions. Since ratings are provided by buyers based on their own evaluation functions, by learning their evaluation function, buyers can convert these ratings into those that are fit for their own subjectivity. In [11], buyers select like-minded buyers to be their *advisors*, and request ratings from their advisors to build the reputation of sellers. As like-minded buyers have the similar subjectivity, the subjectivity problem then is effectively mitigated. In [9], communities are formed by *super agents* who are buyers having high capability in exploring sellers. Super agents select similar buyers as their members and collect ratings from them to construct the reputation of sellers. Buyers evaluate the similarity with the super agents and weighted aggregate the reputation provided by super agents. Thus, the subjectivity problem is also mitigated. However, there are two common shortcomings in this type of solutions. The first shortcoming is that the modeling is complex, because each buyer has to model every other buyer. This modeling complexity increases exponentially as the number of buyers increases. Another shortcoming is that the computation of the reputation of sellers is duplicated. As each similar buyer would calculate out the similar reputation locally, the similar reputation is then computed duplicated. This computation duplication wastes computational resource. This problem would get more significant as the number of similar buyers increases.

In this paper, we propose a new solution to address the subjectivity problem, i.e. coalition formation game based reputation system. In the proposed approach, buyers with the same subjectivity form a coalition and share ratings within the coalition to build the reputation of sellers.

Coalition formation game is widely applied in the area of computer engineering, such as Grid computing, sensor networks and tasks allocation, in order to model the cooperation among self-interested individuals. In e-marketplaces,

coalition formation game is applied to the group-buy schemes in [4]. Buyers from the same geographical location purchase products together by forming coalitions to save shipment costs and also enjoy the quantity discount offered by sellers. In the proposed CFG based reputation system, buyers are incenived to form coalitions with other buyers who have the same subjectivity, in order to address the subjectivity problem.

3 The Proposed Model

In this section, we propose a game theoretical model to abstract our approach in addressing subjectivity problem. In this model, buyers are the players and they play a coalition formation game. In the coalition formed by buyers, the reputation of sellers is built. The value of a coalition is evaluated by the utility brought by the reputation system of the coalition.

3.1 Model Description

We first provide the definition of coalition formation game [5] as follows:

Definition 1. *A coalition formation game (\mathbf{A}, v) is given by a set of players $\mathbf{A} = \{a_i | i = 1, 2, \dots, I\}$, and a characteristic function $v : 2^I \rightarrow \mathbb{R}_+ \cup \{0\}$ that for any coalition c , $v(c)$ characterizes the total utility that these agents can achieve by forming the coalition. We assume $v(\emptyset) = 0$.*

Informally speaking, a coalition formation game consists of a set of players and a function which determines the value for every possible coalition. Players form a coalition c to achieve the value $v(c)$.

In our model, buyers form coalitions and construct reputation systems within their coalition. It is noteworthy that buyers have different subjectivity. One type of buyer subjectivity characters one type of evaluation criterion or preference in providing ratings. In each coalition, the ratings provided by buyers are aggregated as the reputation of sellers for the coalition. The value of the coalition is determined by the utility created by the reputation system of the coalition. We characterize coalition value as the utility created by the coalition for a period ¹. We will show how the coalition value is derived in the next section.

3.2 Coalition Values

In this section, we will formalize the coalition value that is created by the reputation system. As buyers have subjectivity and they form coalitions autonomously, a coalition can be a *pure coalition* which contains buyers with the same subjectivity, or be a *mixed coalition* which contains buyers with different subjectivity.

¹ As the coalition formed is in a long-term sense, we would like to define the coalition value as the utility that the coalition creates in a period. The period is a time unit which reflects the update frequency of the reputation system.

In the pure coalition, we model its coalition value as buyer utility surplus due to the increase of the buyer probability in conducting a satisfactory transaction. For the mixed coalition, its coalition value is qualitatively compared with the sum of several pure coalitions each of which contains buyers with one type of subjectivity of the mixed coalition.

Coalition Values in Pure Coalitions In a pure coalition c , b_i is any buyer in c . we assume that each transaction has a binary outcome, i.e. either a successful transaction or an unsuccessful transaction. If a buyer conducts a successful transaction, then the buyer would gain a positive utility $\alpha \in \mathbb{R}_+$. Whereas, if the transaction is unsuccessful, the buyer bears a certain amount of loss $-\beta$, where $\beta \in \mathbb{R}_+$. If we denote the probability of a successful transaction as p , then the buyer's expected utility in this transaction can be calculated as $p\alpha - (1-p)\beta$.

The probability of a buyer in conducting a satisfactory transaction relies on the amount of experience that the buyer has. Intuitively, if the buyer have more experience about sellers, then the probability is higher. The increase rate of this probability gets slower as the total amount of experience increases. Based on this intuition, we define a Success Rate (SR) function $p(e)$ to quantify the probability of a buyer in conducting a successful transaction. Here, e is the total amount of experience that a buyer has. There are two constrains for the SR function. The first constraint is that it should be a concave function, i.e. the first order derivation of $p(e)$ is positive and the second order derivation is negative, which are shown as:

$$\frac{dp(e)}{de} > 0, \frac{d^2p(e)}{de^2} < 0. \quad (1)$$

In addition, it should satisfy the following bound constraints:

$$p(0) = p_0, p(+\infty) = 1, \quad (2)$$

where $p_0 \in (0, 1)$ is the average probability of sellers in performing a successful transaction in our marketplace.

Based on the SR function, we would formalize buyer utility, which is denoted by u . Given that a buyer b_i has some amount of experience e_i , the probability that b_i conducts a successful transaction would be $p(e_i)$. Therefore, the utility of the buyer b_i in a period based on his own experience, denoted as u_i^i ², can be calculated as:

$$u_i^i = \gamma [p(e_i)\alpha - (1 - p(e_i))\beta],$$

where $\gamma \in \mathbb{N}_+$ is a positive integer, denoting the number of transactions the buyer conducts in a period (or transaction rate). Then, the utility of b_i gained in a period is the utility of successful transactions plus the loss from unsuccessful transactions. We can rewrite the above utility function as:

$$u_i^i = \gamma p(e_i)(\alpha + \beta) - \gamma\beta. \quad (3)$$

² In this paper, we use u_i^i to denote the utility that b_i can achieve based on his own experience, and use u_i^c to denote the utility that b_i can achieve based on c 's experience. We use u_i as b_i 's overall utility including the allocated coalition values.

By joining coalition c , the buyer b_i can obtain more experience from other buyers with whom the buyer has the same subjectivity in the pure coalition. The amount of experience that the buyer b_i can obtain in c is denoted as e_i^c which is the total experience of the pure coalition. We notice that $e_i^c \geq e_i$ where equality is true when b_i is the only buyer in c . Then, the probability of b_i in conducting a successful transaction increases to $p(e_i^c)$, and the utility of b_i increases to u_i^c . By substituting e_i^c for e_i of Eq. (3), we obtain u_i^c as:

$$u_i^c = \gamma p(e_i^c)(\alpha + \beta) - \gamma\beta. \quad (4)$$

By comparing Eq. (3) and Eq. (4), we obtain the difference between u_i^i and u_i^c :

$$a_i^c = u_i^c - u_i^i = \gamma(p(e_i^c) - p(e_i))(\alpha + \beta). \quad (5)$$

The a_i^c is the surplus utility that is created by the coalition c in the transactions conducted by b_i in a period. Thus, the value of the coalition is calculated as the sum of a_i^c with respect to each $b_i \in c$:

$$v(c) = \sum_{i \in c} a_i^c. \quad (6)$$

Coalition Values in Mixed Coalitions If a coalition c is mixed, the coalition value of c cannot be derived as the same way as that in a pure coalition. As a buyer is choosing a seller, the experience shared by buyers with different subjectivity would mislead the buyer so as to decrease the probability of a satisfactory transaction. Suppose that c is mixed and made up of buyers with K types of subjectivity. We denote $c = \{c_i | i = 1, 2, \dots, K\}$. We have

$$v(c) < \sum_{i=1}^K v(c_i). \quad (7)$$

Therefore, from coalition values achieved by mixed coalitions, we conclude that mixed coalition is dominated by splitting it into several pure coalitions. Ideally, we assume that the coalition value of a mixed coalition is 0.

4 The System Model

A Coalition Formation Game (CFG) based reputation system is constructed in this section. We first describe the system. After that, we propose a proportional allocation algorithm for the CFG based reputation system to distribute coalition values of a coalition to its members.

4.1 The System Description

In the system, there are two types of entities: *buyers* and the *reputation center*. Buyers are self-interested. A buyer b_i has his subjectivity in providing ratings and

some amount of experience about sellers which is denoted as e_i which represents the amount of ratings ever provided by the buyer b_i . We assume that buyer subjectivity is discrete and each buyer has a specific subjectivity. Buyers with the same subjectivity have the same evaluation criteria and provide the same ratings about the same sellers. Buyer subjectivity and experience amount is common knowledge among buyers³. The reputation center is a trusted manager of the system.

In our system, any group of buyers can form a coalition. Each buyer can and only can be a member of one coalition at a time. The reputation center collects ratings provided by buyers in the coalition to build the reputation of sellers. It should be stressed that the reputation center builds reputation of sellers for each coalition based on the ratings provided by the members of the coalition. Based on the reputation system built in the coalition, buyers select sellers to conduct transactions. When a transaction happens, the reputation center would charge the buyer a certain amount of money which is equal to the utility created by the coalition. The money is then allocated by the reputation center among the buyers of this coalition. The allocation algorithm will be discussed in Section 4.2. After the transaction, the buyer can submit a rating about the seller. Buyers can leave or join a coalition at the end of each period. When a buyer leaves a coalition and joins another, the reputation center would delete the ratings provided by the buyer from the old coalition, and aggregate his ratings into the new coalition. Therefore, the reputation center would update the reputation systems of two coalitions when a buyer changes his coalition.

4.2 Allocation Algorithm

As we described that a certain amount of money was charged by the reputation center from buyers after their transactions, in this section, we describe how to allocate the money. We call the allocation algorithm as a *proportional allocation algorithm*. It is formalized in Algorithm 1.

As indicated in Algorithm 1, a buyer b_j can contribute $a_j^c = \gamma m_j^c$ to the coalition value of $v(c)$. For a buyer $b_i \in c$, the amount of allocated coalition value $r_i^c = \frac{e_i}{\sum_{k \in c} e_k} \sum_{j \in c} a_j^c$. It is in proportion to b_i 's experience. Then, the overall utility of b_i in the coalition c is

$$u_i = u_i^c - a_i^c + \sum_{j \in c} \frac{e_i}{\sum_{k \in c} e_k} a_j^c.$$

Given that $a_i^c = u_i^c - u_i^i$, we rewrite the above equation as:

$$u_i = u_i^i + \frac{e_i}{\sum_{k \in c} e_k} \sum_{j \in c} a_j^c. \quad (8)$$

³ The techniques for discovering buyer subjectivity are out of the scope of this paper, we leave them as our future investigations

Alg. 1 : Proportional Allocation Algorithm

Input : c , the pure coalition formed by a number of buyers;
 t , a transaction conducted by a member b_j with a seller;
 α , profit gained by the member j from a successful transaction t ;
 β , loss bore by the member b_j from an unsuccessful transaction t ;

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1 if  $b_j \in c$  then
2    $e_j^c = \sum_{k \in c} e_k$ ;
3 if  $t$  is done then
4    $b_j$  pays  $m_j^c = (p(e^c) - p(e_j))(\alpha + \beta)$ ;
5   foreach  $b_i$  in coalition  $c$  do
6      $r_i^j = \frac{e_i}{\sum_{k \in c} e_k} m_j^c$ ;

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Notice that in Algorithm 1, the coalition value is allocated for the pure coalition. Since the coalition value of a mixed coalition is 0, the allocated coalition value is also 0. The overall utility of a buyer b_i in a mixed coalition c is $u_i = u_i^i$. Given the proportional allocation mechanism, buyers may join coalitions to maximize their own utility. After every buyer has chosen a coalition to join, a coalition structure CS is formed, which is a partition of buyers. We denote a coalition structure as $CS = \{c_i | \bigcap_i c_i = \mathbf{A}, c_i \cap c_j = \emptyset \text{ where } j \neq i\}$.

5 Coalition Structure Analysis

In this section, we analyze a particular coalition structure which is the optimal coalition structure. We will show the conditions that buyers have the incentive to form the structure.

Definition 2. *The optimal coalition structure CS^* in the coalition formation game based reputation system is the coalition structure which maximizes the sum of coalition values:*

$$CS^* = \operatorname{argmax}_{c \in CS} \sum v(c). \quad (9)$$

In the coalition formation game based reputation system, the optimal coalition structure is described in Theorem 1.

Theorem 1. *In coalition formation game based reputation system, CS^* is the partition of buyers, where buyers with the same subjectivity type form the same coalition.*

Proof. Assume b_i with subjectivity type T_i joins a coalition c . The amount of coalition value created by b_i is $a_i^c = \gamma(\alpha + \beta)(p(\sum_{j \in c} e_j) - p(e_i))$, according to Eq.5. If c contains all the buyers with the same subjectivity as b_i , then $p(\sum_{j \in c} e_j)$

is maximized. Then, a_i^c is maximized and the sum of coalition values is maximized. Henceforth, the optimal coalition structure is the partition of buyers, where buyers with the same subjectivity type form the same coalition. \square

Since buyers are self-interested and autonomously join or leave a coalition, we intend to analyze the stability of CS^* . The stability concept in our analysis is the *strict core partition*.

Definition 3. A *strict core partition* CS is a partition of buyers where there exists no coalition $c' \notin CS$, such that $u'_i \geq u_i$ for $\forall i \in c'$ and at least one b_i satisfying $u'_i > u_i$, where u_i is b_i 's utility in $c \in CS$ and u'_i is b_i 's utility in c' .

The coalition c' in the Definition 3 is also called as a *deviation* of CS .

If buyers form CS^* which is a strict core partition, then our proposed reputation system can effectively address the *subjectivity problem*. In the following analysis, we intend to propose the conditions that CS^* is a strict core partition, given the proposed proportional allocation algorithm. The following three theorems show three conditions: a sufficient condition, a sufficient and necessary condition that CS^* is a strict core partition, and a sufficient condition that CS^* is not a strict core partition.

Given any coalition c , we denote \bar{e}_c as the average experience of buyers in the coalition c , i.e. $\bar{e}_c = \sum_{j \in c} e_j / |c|$ where $|c|$ is the number of buyers in c . The average probability of conducting a successful transaction is then denoted by $\overline{p(e_j)}_{j \in c} = \sum_{j \in c} p(e_j) / |c|$.

Theorem 2. Given the proportional allocation algorithm and $p(e)$ being a success rate function, CS^* is a strict core partition if $e_i = e_j$, for $\forall i, \forall j \in c$, and $\forall c \in CS^*$.

Proof. According to Theorem 1, in CS^* , buyers with the same subjectivity would form the same coalition. Let $b_i \in c$ for $\forall c \in CS^*$. We assume there is a deviation $c' \notin CS^*$ and b_i is any buyer in c' . Because c' can only contain buyers with the same subjectivity type with b_i , and c contains all the buyers with the same subjectivity, the number of buyers in c' is less than that in c , i.e. $|c'| < |c|$.

If $e_i = e_j$ for $\forall i, \forall j \in c$, then $\bar{e}_c = \bar{e}_{c'}$. The utility of b_i gained in c and c' are denoted as u_i and u'_i respectively.

$$u_i = u_i^i + \gamma(\alpha + \beta)(p(|c|e_i) - p(e_i)), \quad (10)$$

$$u'_i = u_i^i + \gamma(\alpha + \beta)(p(|c'|e_i) - p(e_i)). \quad (11)$$

Since $|c'| < |c|$ and $p(e)$ is a SR function, we have $u_i > u'_i$. Therefore, c' cannot be a deviation of CS^* , which is a contradiction with our initial assumption. As a result, there is no deviation of CS^* and CS^* is a strict core partition \square

Theorem 3. Given the proportional allocation algorithm and $p(e)$ being a success rate function, CS^* is not a strict core partition if there exists a buyer $b_i \in c$ and $c \in CS^*$, such that

$$\frac{p(\bar{e}_c)}{p(e_j)_{j \in c}} \bar{e}_c > e_i > \arg \left\{ \frac{(|c| - 1)p(|c|\bar{e}_c - e_i)}{|c|\bar{e}_c - e_i} = \frac{p(|c|\bar{e}_c)}{|c|\bar{e}_c} \right\}, \quad (12)$$

for $\exists c \in CS^*$.

Due to the limited space, we skip the proof. The main idea of the proof is that we can find a specific deviation of CS^* which contains all buyers in c but b_i who satisfies the condition in Eq. (12).

Theorem 4. *Given the proportional allocation algorithm and $p(e)$ being a success rate function, CS^* is a strict core partition if and only if there does not exist a coalition $c' \in c$ such that*

$$\frac{p(|c'|\bar{e}_{c'}) - \overline{p(e_j)}_{j \in c'}}{\bar{e}_{c'}} > \frac{p(|c|\bar{e}_c) - \overline{p(e_j)}_{j \in c}}{\bar{e}_c}, \quad (13)$$

for $\forall c \in CS^*$.

Proof. For $\forall c \in CS^*$, a buyer $b_i \in c$. According to Theorem 1, buyers with the same subjectivity type as b_i then all belong to c . The utility of b_i is

$$u_i = u_i^i + \frac{e_i}{\sum_{k \in c} e_k} \sum_{j \in c} a_j^c = u_i^i + \frac{e_i}{|c|\bar{e}_c} \sum_{j \in c} a_j^c. \quad (14)$$

For any $c' \subseteq c$ and $b_i \in c'$, the utility of b_i in c' is

$$u_i' = u_i^i + \frac{e_i}{|c'|\bar{e}_{c'}} \sum_{j \in c'} a_j^{c'}. \quad (15)$$

Supposing for $\forall c \in CS^*$, there does not exist a coalition $c' \in c$ such that Eq. (13) is satisfied. We calculate the difference between Eq. (14) and Eq.(15) as $u_i - u_i' = \frac{e_i}{|c|\bar{e}_c} \sum_{j \in c} a_j^c - \frac{e_i}{|c'|\bar{e}_{c'}} \sum_{j \in c'} a_j^{c'} = \frac{\gamma e_i}{|c|\bar{e}_c} \sum_{j \in c} \{(\alpha + \beta)(p(|c|\bar{e}_c) - p(e_j))\} - \frac{\gamma e_i}{|c'|\bar{e}_{c'}} \sum_{j \in c'} \{(\alpha + \beta)(p(|c'|\bar{e}_{c'}) - p(e_j))\} = (\alpha + \beta)\gamma e_i \left\{ \frac{p(|c|\bar{e}_c) - \overline{p(e_j)}_{j \in c}}{\bar{e}_c} - \frac{p(|c'|\bar{e}_{c'}) - \overline{p(e_j)}_{j \in c'}}{\bar{e}_{c'}} \right\}$.

Then $u_i - u_i' \geq 0$. If $u_i - u_i' > 0$, CS^* is a strict core partition. If $u_i - u_i' = 0$, i.e. $\forall i \in c'$, $u_i = u_i'$, according to the definition of the *strict core partition*, CS^* is still a strict core partition.

Conversely, suppose the optimal coalition structure CS^* is a strict core partition, then $u_i > u_i'$ or $u_i = u_i'$ for $\forall i \in c'$, $\forall c' \in c$. Then we derive $\frac{p(|c'|\bar{e}_{c'}) - \overline{p(e_j)}_{j \in c'}}{\bar{e}_{c'}} \leq \frac{p(|c|\bar{e}_c) - \overline{p(e_j)}_{j \in c}}{\bar{e}_c}$. In other words, there does not exist a coalition $c' \in c$ such that $\frac{p(|c'|\bar{e}_{c'}) - \overline{p(e_j)}_{j \in c'}}{\bar{e}_{c'}} > \frac{p(|c|\bar{e}_c) - \overline{p(e_j)}_{j \in c}}{\bar{e}_c}$. \square

6 Buyer Strategies

The coalition structure is formed by buyers, and the strategies that buyers take depend on the formed coalition structure. We discuss buyer strategies in this section. All possible actions of a buyer can be: forming a singleton where the buyer forms a coalition by himself (Action 1), joining a mixed coalition which contains buyers with different subjectivity (Action 2), joining one of the existing coalitions which contains buyers with the same subjectivity (Action 3), and forming a new coalition with other buyers (Action 4).

Proposition 1. *Both forming a singleton (Action 1) and joining a mixed coalition (Action 2) are buyers' dominated strategies.*

Proof. As a singleton, the buyer b_i is the only buyer in his coalition. According to Eq. (8), the coalition value is zero, because $a_j^c = 0$ with $j \neq i$ where c consists only b_i . For any coalition c' which is formed by b_i and other buyer b_j , $a_j^c > 0$. Then, forming a singleton is a dominated strategy. As the coalition value of a mixed coalition is zero, buyers in the mixed coalition can gain the same utility as being a singleton. Therefore, both Action 1 and Action 2 are dominated strategies. \square

6.1 When CS^* Is a Strict Core Partition

Proposition 2. *When the condition either in Theorem 4 or in Theorem 2 is satisfied, joining the existing coalition which contains all the buyers with the same subjectivity (Action 3) is buyers' dominant strategy.*

Proof. As the condition in Theorem 4 or Theorem 2 is satisfied, optimal coalition structure CS^* is a strict core partition. Therefore, the coalition in CS^* maximizes each buyer's utility. According to Theorem 1, the coalition in CS^* contains all the buyers with the same subjectivity. Therefore, joining the existing coalition which contains all the buyers with the same subjectivity is buyers' dominant strategy. \square

6.2 When CS^* Is Not a Strict Core Partition

When CS^* is a strict core partition, each buyer would choose to a pure dominant strategy which is Action 3, as discussed in Proposition 2. Then a pure strategy Nash equilibrium exists. However, when CS^* is not a strict core partition, buyers have no dominant strategy. We show that there is a mixed strategy Nash equilibrium in this case.

Proposition 3. *When the condition in Theorem 3 is satisfied or the condition in Theorem 4 is not satisfied, there exists a mixed strategy Nash equilibrium.*

Due to the limited space, we skip the proof of Proposition 3. The idea of our proof is that we can find a mixed strategy Nash strategy for buyers given the condition in Theorem 3. In this case, there is a deviation which leads the coalition structure change all the time due to the mixed strategies taken by buyers.

7 Conclusions and Future Work

In this paper, we proposed the coalition formation game based reputation system as a novel game theoretical approach to address the subjectivity problem. In the proposed solution, the ratings of buyers are separately aggregated according to the coalition they form. We show that buyers with the same subjectivity

have the incentive to form the same coalition, if specific conditions are satisfied. Then buyers with different subjectivity would be exposed with different reputation systems, and the subjectivity problem is solved. A proportional allocation algorithm is proposed to allocate the coalition value among coalition members. We also analyze the optimal coalition structure and buyer strategies under the proposed allocation algorithm.

In future work, we plan to explore other allocation algorithms and study the properties associated with different allocation algorithms. Finally, we desire to find the allocation algorithms where the optimal coalition structure is a strict core partition unconditionally. In addition, the coalition structure is non-overlapped in the proposed model, because we constrain that a buyer has only one type of subjectivity by joining only one coalition at a time. We also intend to extend the model to an overlapping coalition formation game [12] model, allowing buyers to have multiple subjectivity or mixed subjectivity.

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